

Ancient Egyptian Cubits – Origin and Evolution

by

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Abstract

This thesis suggests that prior to Ptolemaic and Roman times, ancient Egypt had two distinct and parallel linear systems: the royal system limited to official architectural projects and land measurements, and a great (ꜥ) system used for everyday measurements. A key $1/3$ ratio explains ancient Egyptian linear measurements and their agricultural origin. Emmer is $1/3$ lighter than barley, consequently, for an equal weight, a container filled with emmer will be $1/3$ greater than a container filled with barley. The lengths derived from both containers share the same $1/3$ ratio. The second chapter, *Previous Studies*, lists the work of scholars involved directly or indirectly with ancient Egyptian metrology. The third chapter, *The Royal Cubit as a Converter and the Scribe's Palette as a Measuring Device*, capitalizes on the colour scheme (black and white on the reproduction of *Appendix A*) appearing on the Amenemope cubit artifact to show the presence of two cubits and two systems: the black (royal system) and the white (great [ꜥ] system) materialized by the scribe's palette of 30, 40, and 50 cm. The royal cubit artifacts provide a conversion bridge between the royal and the great systems. The information derived from the visual clues on the Amenemope cubit artifact are tested against a database of artifacts scattered in museums around the world. The fourth chapter, *The Origin and Evolution of Ancient Egyptian Cubits*, historically relates the ancient Egyptian linear systems to the closed metrological systems

they belong to. A closed metrological system is a system in which units of length, volume, and weight are related to each other. The conclusion is that the ancient Egyptian metrological system is backward compatible as it is possible - using a *hin* as a closing volumetric unit and emmer, barley, wheat (*triticum durum*) and water as commodities - to re-construct the linear metrological systems of all ancient Egyptian periods.

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Chapter 1 - Introduction

The ancient Egyptian unit of linear measurement was the cubit (*mḥ*). Scholars, distinguish two cubits. The first is the royal cubit. Lepsius defined its length, 52.5 cm. Modern scholarship gives it a length varying between 52 and 54 cm (Girndt, 1996, pp. 56-9; Pommerening, 2005, p. 274; Zignani, 2010, p. 153). The royal cubit is divided into seven palm divisions (*šsp*) of 7.5 cm each. Each royal palm division is divided into four finger divisions (*dbꜥ*) of 1.875 cm each. The royal palm divisions and/or finger divisions are combined to create the divisions of the royal cubit. Two palm divisions make a double palm division. Five fingers make a hand (*drt*). Six fingers make a fist (*3mm*). Three palms make a small span (*pd šrj*). Three and a half palms make a great span (*pd ꜥ3*). Four palms make a *dsr*. Five palms make a *remen* (*rmn*). Six palms define a small cubit (*mḥ šrj*). Scholars (Robins and Shute, 1987, p. 13) consider the small cubit to be the second ancient Egyptian cubit providing everyday measurements. They also consider the small cubit to be a division of the royal cubit rather than a division of a second ancient Egyptian system independent from the royal system and cubit.

The royal cubit was the unit of measurement reserved for land and volumetric calculations as well as state-sponsored building projects. The reformed cubit replaced the royal cubit during the Twenty-sixth Dynasty. At the same time the small cubit disappeared. The Twenty-sixth Dynasty reformed cubit maintains the length of the royal cubit (52.5 cm) but changes the number of palm divisions and the length of each finger division within each palm division. The reformed cubit is divided into six palm divisions of 8.75 cm each. Each palm is divided into four finger divisions. The length of each finger division is increased from 1.875 cm (the length of a royal finger division) to 2.1875 cm. The Ptolemies and Romans inherited and integrated the ancient Egyptian cubits to which they added their own standards and linear measurements based on water and the specific weight of wheat (*triticum durum*) for trade compatibility purposes.

This thesis suggests the use two distinct and parallel linear systems, before the Twenty-sixth Dynasty reformation of the royal cubit. This suggestion comes from a metrological survey performed at Mendes since 2007, to be completed and published as soon as political events (2012) in Egypt allow it. The preliminary results of the survey show the use of two linear systems at Mendes. Measurements based on the royal cubit are the exception; lengths with

increments of 0.5 to 1 cm are the rule. In other words, this thesis suggests that the small cubit is more than a practical division of the royal cubit. The small cubit belongs to a system different from the royal cubit system.

The use of several cubits in the ancient Near East is common¹:

“In Babylonia, four native cubits and one foreign cubit are distinguished.”
(Powell, 1987, p.462, col. 1).

Why would ancient Egypt be different? Lepsius writes, commenting on the divisions of the royal cubit:

“Here also one would rather suspect that both cubits, initially of separate and distinct origins, late met with each other . . . But if they were to coexist in the long run, then either they would have had to adopt a simple ratio, such as 6:7 as has been found, or their use would have to be confined to distinct areas or functions . . . It remains certain at least that the difference between the two cubits must have had some historical or practical reason, and it is equally certain that the great cubit, if it existed independently from the small one, could not have had as impractical a subdivision as that in 7 palms or 28 fingers.” (Lepsius, 1865, p. 52–
Translation: J. Degreef).

This thesis relates linear measurements with agriculture. The system the small cubit belongs to comes from the cultivation of emmer. The royal cubit system comes from the cultivation of barley. The Ptolemaic and Roman Periods maintain the association of agriculture and linear measurements. The cultivation of wheat (*triticum durum*) and specifically Greek and Roman

¹ This is perhaps only a coincidence, but it is worth mentioning. The god *UU* is a hapax only found on certain cubit artifacts among the list of other Egyptian gods. The Sumerian numeric value of *UU* is 20 (twice ten); *Dingir 20* (which can be traced back to the Old Babylonian period [1800 – 1500 BC] – corresponding to the Second Intermediate period [RIA, 2011, p. 600] is the god Shamash associated with Mesopotamian metrology. What makes this coincidence intriguing is that the length of a double *remen* is 75 cm, the exact length of the Mesopotamian big cubit (Powell, 1987, vol.7, p. 462, section # 1.2.e)

linear calculations - based on the volumetric and weight characteristics of water - explain the Ptolemaic and Roman linear measurements. The methodology retained throughout this thesis is developed into three chapters.

The second chapter analyzes the work of previous scholars and lists the number of cubits defined by these scholars. The third chapter focuses on the divisions of the Amenemope cubit artifact and relates them to a STATISTICAL measurement database of artifacts found in museums around the world. The fourth chapter chronologically relates ancient Egyptian linear measurements to the metrological systems they belong to.

The second chapter, *Previous Studies*, states the works of scholars dedicated to the length of ancient Egyptian cubits. It ascertains the existence of, at least, two ancient Egyptian cubits. Previous scholars used four methods to reach the length of ancient Egyptian cubits. The first method relies on the results derived from monument and building surveys. Jomard, Howard-Vyse, Perring, Smyth, Petrie, Victor, Roik, Dieter, Carlotti, Wegner, and Zignani follow this method. The second method relies on the comparison of cubit artifacts or the interpretation of published monument and building surveys. Newton, Champollion-Figeac, Girard, Saigey, Bökh, Fenner Von Fenneberg, Thenius, Vazquez-Queipo, Lepsius, Griffith, Borchartt, Segrè, Reineke, Lorenzen, Hinz, Schlott, Hayes, Legon, Simon-Boidot, and Schmitt follow this method. The third method relates ancient Egyptian linear measurements to volumetric and area measurements. Relating linear measurements to area, volumetric, and weight measurements is called closing a metrological system. In a closed metrological system, units of length, volume, and weight are related to each other and can be derived from each other. Griffith, Segrè, Decourdemanche, and Pommerening, in the conclusion of her study of containers and ancient Egyptian volumetric units, follow this method. The fourth method is based on the divisions of the Twenty-sixth Dynasty reformed cubit. Lepsius started this method with his remarks on the number of grid squares used between the sole of the foot and either the forehead or the root of the nose in reliefs of standing or sitting human figures. This number of grid squares, varying throughout ancient Egyptian history, is called the Canon of Proportion. Lepsius, Iversen (1990, pp. 113-114), Carlotti (1995 - Annexe, pp. 127-39), and Zignani (20210, p. 154) follow this method. They consider that the royal and reformed cubits have two main divisions: $\frac{2}{3}$ (Lepsius, and Iversen) and $\frac{5}{6}$ of the royal and reformed cubits (Iversen, Carlotti, Zignani). Robins and Roik relate the

Canon of Proportion with the small cubit and the *nby* (a multiple of the royal cubit) respectively.

The four methods define five cubits:

- 1 A "Pyramid" cubit, 60 to 75 cm long.
- 2 A royal and reformed, *digital* (the basic division of the cubit is the finger [*digitus* in Latin]) cubit 52 to 54 cm long.
- 3 A royal and reformed *uncial* (the division system is duodecimal [base 12 – *uncia* means the twelfth part in Latin] *djeser* cubits, $\frac{2}{3}$ of the royal and reformed cubit, 30 and 35 cm long.
- 4 A *remen*-cubit, also *uncial*, $\frac{5}{6}$ of the royal and reformed cubit, 37.5 cm 43.75 cm long respectively.
- 5 An *uncial* small cubit, with 24 finger divisions - 45 to 47 cm long - which disappears with the reformed cubit.

The third chapter, *The Royal Cubit as a Converter and the Scribe's Palette as a Measuring Device*, capitalizes on the method used to shift from the royal cubit to the reformed cubit and applies it to the Amenemope cubit artifact. The reformed cubit and the royal cubit share a common length of 52.5 cm. Changing the number of palm divisions - six in the reformed cubit instead of seven in the royal cubit - defines the reformed cubit. The length of each reformed finger division in each reformed palm division is increased. It measures 2.1875 cm instead of 1.875 in the royal cubit. The Amenemope cubit artifact is the only complete cubit artifact showing its divisions with two different colours: black and white. This colour scheme indicates a conversion process similar to the process used to shift from the royal cubit to the reformed cubit. The length of the palm is maintained in the black and white systems, but the number and length of each finger division is different:

- Four finger divisions of 1.875 cm each in the black system;
- Three finger divisions of 2.5 cm each in the white system;
- The length of the great finger division is $\frac{1}{3}$ greater than the length of the royal finger division. This $\frac{1}{3}$ ratio explains the agricultural origin of ancient Egyptian linear measurements.

The only difference between the conversion process of the reformed cubit and the conversion process of the Amenemope cubit artifact is that the latter reduces the number of finger divisions instead of the number of palm divisions. More than an indication of divisions, this colour scheme is an indication of two distinct origins and two parallel ancient Egyptian linear systems. The visual clues provided by the black and white colour schemes respect the basic principles of

ancient Egyptian mathematics. Ancient Egyptian mathematics is essentially additive and allows doubling, taking $2/3$, halving, multiplying by ten, taking $1/10$, and rounding. The white system includes the small cubit and has a theoretical cubit length of 60 cm. The black system is the royal system. The royal cubit length is limited to 52.5 cm. The Amenemope cubit artifact offers a conversion bridge (a converter in this thesis) on par with our modern rulers. Modern rulers show both metric and imperial measurements on two different rows. The Amenemope cubit artifact shows its divisions on a single row with two different colours (white and black). The royal cubit is well documented with numerous complete cubit artifacts from the Eighteenth Dynasty onwards (see *Excursus B – Division Evolution*). There are even more numerous broken cubit artifacts from the same period. Very few authors, apart from Zivie, have considered the scribe's palette (*gstj*) as a measuring artifact. This thesis suggests that the scribe's palette - usually 30, 40, and 50 cm long - represent the white system known in this thesis as the great (ʿ3) system.

A statistical measurement database of "elite" and "non-elite" artifacts scattered in museum around the world (mostly from Cairo) ascertains the existence of two linear systems prior to the Twenty-sixth Dynasty. *Database_1* contains 3866 entries with a total of 4905 measurements (heights, lengths, widths, and depths). All objects and measurements have a catalogue number, a catalogue name, a volume number, a page number and time period. When possible, they also show their location of origin, sub-location, and dynasty. The majority of the entries is from the Predynastic to the Middle Kingdom Periods, but includes New Kingdom, Late Period, Ptolemaic, and Roman measurements. The textual references in the fourth chapter of this thesis show the use of the royal cubit as a converter mainly from the Predynastic to the Middle Kingdom Periods, hence the emphasis on these periods. *Database_6* contains 56 entries. They correspond to the lengths of the 28 fingers of the royal cubit. The lengths are incremented by half a royal finger (0.9375 cm) up to 52.5 cm (the length of a royal cubit). *Database_4* contains 120 entries incremented by 0.5 cm up to 75 cm (the length of a double *remen*). *Database_6* and *Database_4* are the scales used to calculate the percentage of the use of the royal cubit in "elite" artifacts in *Database_3*. The same scales allow the calculation of a regional percentage of the use of the royal cubit in the Abydos nome for "elite" and "non-elite" artifacts in *Database_5* and *Database_3*.

1244 entries in *Database_1* do not match the great, royal, or double *remen* measurements for several reasons. Measurements from broken artifacts or uncertain measurements were not entered in *Database_1*, but their references were, in order to maintain the integrity of the reference catalogues. Multiples of the scribe's palette, royal cubit, double *djeser*, and double *remen* have been discarded. Trends are based on the maximum length of a double *remen* (75 cm) which includes the length of the scribe's palette, the royal cubit, the great cubit, the *njb*, and the *remen*. Finally, the balance of unmatched entries (582) would require a “subjective” delta calculation between the royal and the great systems. They represent circa 12% of the 4905 total measurements. They would not drastically change the trends resulting from the analysis of the 3651 matched measurements. The very small percentage of data requiring a "subjective" delta analysis confirms the existence of two independent ancient Egyptian linear systems. It leads to four conclusions:

- 1 The length of royal cubit is limited to 52.5 cm and cannot be 52 to 54 cm long.
- 2 The royal cubit as a converter may appear at the end of the Second Dynasty or the beginning of the Third Dynasty.
- 3 The scribe's palette as a measuring artifact is used from the Old Kingdom onwards.
- 4 There is no real evidence of a specialized use of the royal cubit in the database.

Chapter 4, *The Origin and Evolution of Ancient Egyptian Cubits*, attempts to explain and trace the origin of the two ancient Egyptian linear systems. It relates volumetric, weight, and area measurements to linear measurements. In other words, chapter 4 capitalizes on the notion of closed metrological systems. A closed metrological system is a system in which units of length, volume, and weight are related to each other and can be derived from each other. *Excursus A – Closed Metrological Systems and Commodities* includes the weight and volume characteristics of emmer, barley, and wheat (*triticum durum*) introduced by the Ptolemies. The weight and volumetric characteristics of water, the basis of classical metrology, play also an important part in the linear metrology of the Ptolemaic and Roman Periods.

The weight and volume related lengths section of this chapter links the volumetric characteristics of emmer, barley, and wheat (*triticum durum*) with ancient Egyptian metrology. It illustrates the agricultural origin and evolution of the majority of ancient Egyptian linear measurements, from the Old Kingdom down to the Ptolemaic and Roman Periods. 450 *hins* is the estimated volumetric capacity leading to the definition of Predynastic, Old Kingdom, and Middle Kingdom

linear measurements. The volumetric capacity of 100 *dwt* containers, equal to 200/250 litres and rounded to 450 *hins*, is the basis of this estimation. This estimation in *hins* fits the ancient Egyptian method. The Ptolemaic pBritish Museum 10399 problems 42 and 43 (Parker, 1972, p. 57) relates, in a genuinely ancient Egyptian manner, the length of cubits with *hins*. This estimation fits the two thirds principle of ancient Egyptian mathematics. Two thirds of 450 *hins* give the volumetric capacity of a royal cubit, 300 *hins*. Applying the specific weight of emmer (440 grams per litre) and barley (660 grams per litre) to 450 *hins* defines the great and small *debens* of pBoulaq 13 and Cour-Marty's weight series. The great *deben* is related to barley and to the royal system. The small *deben* is related to emmer and the great system. The organization of the treasury in the Third Dynasty corresponds to the importance given to barley over emmer during the Old Kingdom and provides the date when the royal cubit became the “official” linear standard for central government related projects. It is fitting that the royal system, based on barley, reflects this change and becomes (and remains) the official metrological system to match the relative importance given to barley over emmer. The great system remains the everyday linear metrological system.

From the area related lengths section of this chapter, the parallel use of the great and the royal systems appears in garment and field calculations. They are difficult to quantify in the Gebelein linen inscriptions, the Abusir papyri, pMoscow 18, and the Hekanakhte Papers. The *stbt* inscriptions of the White Chapel of Sesostri confirm the conversion role of the royal cubit. They provide an indication of the cubit and system used in each ancient Egyptian nome.

Finally, the volume related lengths section of this chapter addresses the *tbt* and *nbt* divisions of the royal system. Volume related lengths show the evolution of the royal system. The *tbt* appears during the Old Kingdom in one inscription in the *mastaba* of Ptahshepses and during the Middle Kingdom in Papyrus Reisner I. The New Kingdom introduces two new metrological units. The *nbt*, linked to the *dnj* in the ostraka found in the Tomb of Sen-Mut and in the inscriptions of the Cenotaph of Seti I, replaces the *tbt*. The volume of a *dnj* equals the volume of a cubed royal cubit, linking the *nbt* to the royal system. These new units replace the *tbt* and *h3r* sack in construction projects. Their introduction matches the reversal of the importance given to barley over emmer, already traceable in the change of the volumetric capacity of the New Kingdom (*h3r*) sacks. It confirms the conversion role played by the royal system and the independent

evolution of the royal system evidenced by the black and white inscriptions of the Amenemope cubit artifact.

Chapter 2 - Previous Studies

Summary

Four methods of finding the length of a cubit are listed in the works mentioned in this chapter: a survey of lengths in monuments - the majority of authors; a comparison of cubit artifacts - Lepsius; relating volume, weight, and length measurement units (Griffith, Segrè, and Decourdemanche); using different divisions and ratios over a common length of 52.5 cm: digital, seven palms of four fingers (royal cubit) or six palms of four fingers (reformed cubit); uncial, the *djeser* (two thirds of the royal or reformed cubit) divided into twelve finger divisions instead of sixteen and the *remen* ($5/6$ of the royal and reformed cubit) divided into twenty finger divisions (Iversen, Carlotti, and Zignani).

These four methods, define five cubits: A pyramid cubit and *nby* of 60 cm to 75 cm in length, never fully integrated within the royal cubit system described by Lepsius; a digital royal cubit, 52 cm to 54 cm long; an uncial *djeser*-cubit $2/3$ of the royal and reformed cubit of the Twenty-sixth Dynasty; a *remen*-cubit, $5/6$ of the royal and reformed cubit and also part of Iversen's, Carlotti's and Zignani's uncial system; A small cubit, 45 cm to 48 cm long, also part of Iversen's, Carlotti's and Zignani's uncial system, but disappearing with the Twenty-sixth Dynasty reform.

1 Ancient Egyptian Cubits

Previous work dedicated to the study of ancient Egyptian metrology falls into two groups: studies dedicated exclusively to ancient Egyptian cubits and comparative studies where ancient Egyptian metrology is mentioned. What follows is a brief chronological outline of the major works that are at the origin of this thesis.

1.1 Cubits

1.1.1 Herodotus

Two quotations from the *ἹΣΤΟΡΙΑΙ* are generally made without any significant comments.

“These pyramids are a hundred fathoms high; and a hundred fathoms equal a furlong of six hundred feet, the fathom measures six feet or four cubits, the foot

four spans and the cubit six spans.” (Herodotus 2:149 - A. Godley translator, 1982, pp. 458–9.)

The “span” of Herodotus is the Greek word **Pal aisthōr** palm. What Herodotus describes is the reformed cubit of the Twenty-sixth Dynasty with six palm divisions instead of the traditional seven palm divisions of the royal cubit (Lepsius, 1865, p. 19).

“The Egyptian cubit being equal to the Samian.” (Herodotus 2:168 - A. Godley translator, 1982, p. 481.)

This quotation would be meaningless without a definition of the Samian cubit. The Ashmolean Museum in Oxford displays a life-size relief of Samian origin, The Metrological Relief, published by E. Lorenzen (1966, pp. 28–9; catalogue number a000103 [<http://www.ashmolean.org/ash/departments/antiquities/>]) that gives the length of the Samian cubit. According to the position given to the elbow when the arm is bent, the cubit length varies between 0.54 m (9 squares) and 0.525 m (8 and three quarter squares) from elbow to finger-tips on Lorenzen’s scale (the side of each square [added by Lorenzen on his drawing] is six cm long). The length of the foot in the relief (four and three quarter squares) makes the Samian cubit an unlikely candidate for the Egyptian royal cubit, but a likely candidate for the Achaemenid cubit (Gershevitch, 1985 p. 626).

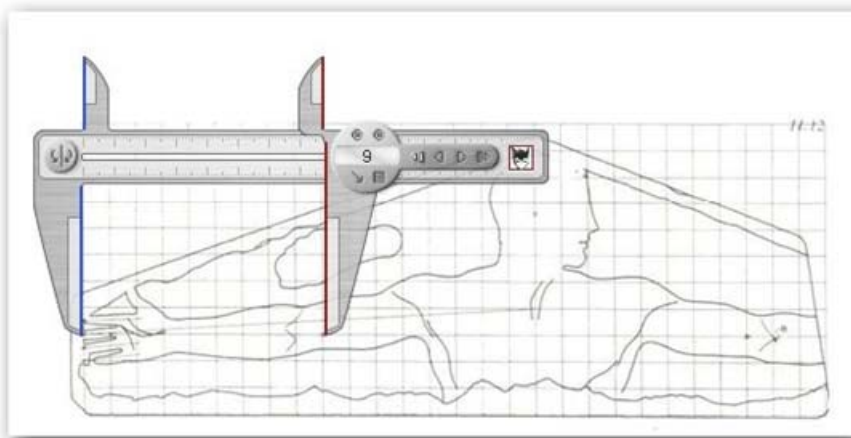
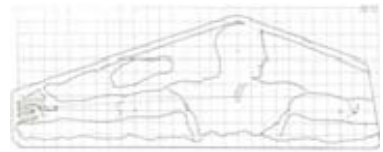


Figure 1. Metrological Relief-Ashmolean Museum a000103.

July, 1, 1798, the arrival of the first French soldiers and scientists of Bonaparte's Armée d'Orient, is the birth-date of scientific metrological measurements and discussions on ancient Egyptian linear systems, leading to the definition of different standards given to ancient Egyptian length measurements. Two methods were used: physical survey (particularly of the Great Pyramid) and studies of existing cubit artifacts.

1.1.2 E. F. Jomard

In his *Exposition du système métrique des anciens Égyptiens*, and a series of articles, "Notes sur un manuscrit égyptien sur papyrus renfermant des plans de monuments, avec les mesures écrites en chiffres hiéroglyphiques," "Description d'un étalon métrique, orné d'hiéroglyphes, découvert dans les mines de Memphis par les soins de M. le Chevalier Drovetti," and "Lettre à M. Abel Rémusat, sur une nouvelle coudée trouvée à Memphis," Jomard gave the following values for the small and royal cubits : 46 cm, and 57 cm respectively (Jomard, 1822b, pp. 432–4). Jomard's cubit lengths come from inaccurate survey calculations. Jomard did not clear completely the debris covering the baseline of the sides of the Great Pyramid. He gives 230.902 m, $51^{\circ}19'4$," and 144.194 m for the length of the base, the angle of the slope, and the height of the Great Pyramid. Following Greek sources (500 cubits for the length of the base of the Great Pyramid) he calculated the length of the ancient Egyptian cubit: 0.4618 m.

1.1.3 J. J. Champollion-Figeac

J. J. Champollion-Figeac (1824, pp. 289–329) gave a value of 52.4 cm for the royal cubit in "Observations sur les coudées égyptiennes découvertes dans les mines de Memphis."

1.1.4 P. S. Girard

P. S. Girard in his "Mémoire sur le nilomètre de l'Île Éléphantine et les mesures égyptiennes," and in "Notice sur quelques étalons de l'ancienne coudée égyptienne récemment découverts," gave an average value of the length of the royal cubit of 52.7 cm (Girard, 1809, pp. 1–48; 1824, pp. 34–40).

1.1.5 R. W. Howard-Vyse and J. S. Perring

R. W. Howard-Vyse (1839) and J. S. Perring (1839) in *Operations carried on at the Pyramids of Gizeh* gave a value of 52 cm for the cubit used for the construction of the pyramids based on in situ surveys.

1.1.6 Ch. P. Smyth

Ch. P. Smyth (1867) in his *Life and Work at the Great Pyramid of Jeezeh*, gave two values for the cubit: a “builder” cubit of 52 cm and a “pyramid” cubit of 63.5 cm similar, according to him, to the cubits used by Moses to build the tabernacle and Noah the Ark. He made the most accurate measurements of the Great Pyramid that any explorer had made up to that time, and he photographed the interior passages, using magnesium lights, for the first time.

1.1.7 R. Lepsius

Capitalizing on previous theoretical studies and publications of several cubit artifacts Jomard (1822b, pp. 432–4), Saigey (1834), Thenius (1846), Vazquez-Queipo (1859), and Champollion-Figeac (1824, pp. 289–329), Richard Lepsius reconstructed, from fourteen samples (two of which, numbers seven and eight, proved to be fakes), scattered in European museums, the length, seldom contested since, of the small, royal, and reformed cubits (Lepsius, 1865, pp. 50 ff. and Table 1, Translation: J. Degreeef).

Table 1. The Small and Royal Cubits According To Lepsius (Lepsius, 1865, Table 1).

Egyptian Division	Division	Metric Value (metres- m)
NA	1 Small Finger	0.0187 m
NA	1 Large Finger	0.0219 m
NA	2 Small Fingers	0.0375 m
<i>Šsp</i>	Palm	0.075 m
<i>3mm</i>	Fist	0.100 m
<i>Dsr</i>	2/3 Measure	0.300 m

Egyptian Division	Division	Metric Value (metres- m)
<i>Rmn</i>	<i>Remen</i>	0.375 m
<i>Mḥ šrj</i>	Small Cubit	0.450 m
<i>Mḥ nsw</i>	Royal Cubit	0.525 m
NA	Fathom	1.80 m

Two important points need to be mentioned regarding the reconstruction performed by Lepsius:

- 1 Although he admits the existence of two different cubits and finger length divisions (Lepsius, 1865, pp. 52 ff.), Lepsius gave a value of 45 cm to the small cubit based on lengths found on nilometres making it a subdivision of the royal cubit.
- 2 Lepsius also reconstructs palm division markers that do not exist on the original samples.

1.1.8 W. F. Petrie



Petrie corrected Jomard’s survey calculations (clearing all debris covering the baseline of the sides of the Great Pyramid) and gave the equivalent of 230.364 m for the base of the Great Pyramid, 51°50’35” for the angle of its slope, and the equivalent of 146.6 metres for its height. Petrie’s published report of this triangulation survey, and his analysis of the architecture of Giza therein, is exemplary in its methodology and accuracy, and still provides much of the basic data regarding the pyramid plateau to this day. Petrie is the first author to remark that the relationship between the royal cubit and the small cubit might be different from what Lepsius had previously described. In his *Ancient Weights and Measures*, he notes that, in his collection of cubits, the marker of the small cubit does not coincide with the 24th finger mark suggested by Lepsius:

“It should be noted that though the lesser cubit is usually stated to be 24 digits, it is distinctly limited as not over 23 digits on the two most detailed cubits. These various lengths are evidently other standards, approximately marked on the royal cubit . . . We must look to other sources to see what standards were known which could thus be notified.” (Petrie, 1977, p.41)

He also suggested, in *Deshasheh*, that ancient linear measures found in ancient Egypt could be foreign and related to the northern, Punic, and Persian feet.

“There are then three examples of this cubit . . . divided into two feet, I proposed to connect with the Asia Minor foot . . . It is significant that Kahun contained foreigners, and here this cubit lay in the tomb which had foreign pottery.” (Petrie, 1898, pp. 37–8)

1.1.9 F. L. Griffith

In his comprehensive *Notes on Egyptian Weights and Measures*, Griffith (1892, pp. 403–50) confirmed the length of the cubits (small and royal) given by Lepsius and went on to mention the multiples of the royal cubit (*khet* and *iterw*) before examining measurements of areas, volumes and weights. Griffith noticed that the determinative for cubit varies. When used in conjunction with length and breadth the Gardiner sign List D42 () is found. The same sign also appears when architectural units on the earliest monuments are mentioned. Elsewhere the sign used seems to be Gardiner list D36 [] (Griffith, 1892, p. 406). Following the Rhind Mathematical Papyrus, he is also the first author to give a specific equivalence between volumes and lengths: 1 *khar* sack equals two thirds of a (royal) cubed cubit (Griffith, 1892, p. 406).

1.1.10 L. Borchardt

In a study of ancient Egyptian nilometres, (Borchardt, 1906, pp. 27) Borchardt gave the following lengths for the Edfu nilometre cubits dated to Ptolemy XI:

Table 2. Borchardt’s Edfu Nilometre Cubits (Borchardt, 1906, p. 27).

Cubit Number	Cubit Length	Palm Length
20 a	53.2 cm	3.89 cm
20 b	52.5 cm	3.75 cm
19	54.4 cm	3.96 cm
18	52.5 cm	3.75 cm
17	52.5 cm	3.75 cm
16	51.8 cm	3.11 cm

In his *Gegen die Zahlenmystik an der großen Pyramide bei Gize* (Borchardt, 1922, pp. 8 ff), Borchardt remarked that the length of 0.525 metres of the royal cubit is seldom encountered and preferred a “standard” length of 63.5 cm.

1.1.11 W. F. Reineke

W.F. Reineke (1963, pp. 145–63) in *Der Zusammenhang der altägyptischen Hohl-und Längenmaße* confirmed Griffith’s data for lengths, areas, and volumes. He added a few more definitions for different containers and their use, and then gives their volume in *hins*.

1.1.12 A. Schlott

Adelheid Schlott’s doctoral thesis, *Die Ausmaßse Ägyptens nach altägyptischen Texten*, (Schlott, 1969) mainly concerns itself with a multiple of the cubit, the iterw, and the dimensions of ancient Egypt as they appear in the White Chapel of Sesostris I, as well as their integration within the ancient Egyptian religious concept of the universe. Schlott is responsible for the classification of the different cubit artifacts (group 1: operative [carrying only metrological divisions]; group 2: cubits carrying an offering formula; group 3: votive [carrying various inscriptions {nome names, Nile heights, etc.} but no offering formula]), and the alphabetical reference standardization of their sides that will be followed throughout this thesis (Schlott, 1969, p. 38; 53–63 Roik, 1993, p. 7–8).

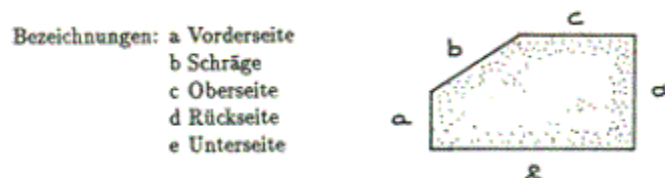


Figure 2. Schlott’s Standardization of the Sides of a Cubit-measure (Schlott, 1969, p. 38).

1.1.13 E. Iversen

Iversen’s work (Iversen, 1975, 1990, pp. 114–25) is dedicated to the Egyptian Canon of Proportion. His exposé led him to define the ancient Egyptian cubits (small, royal, and reformed) and their subdivisions. Iversen (1975, p. 17) agreed with the values given to the small and royal cubits by Lepsius.

“In most accounts of Egyptian metrology it has been generally accepted that the decimal standard lengths of the small and the royal cubits, was 0.450 m and 0.525 m respectively. These figures were originally induced from the average lengths of the cubit rods on which Lepsius based his metrological studies, and they have on the whole been confirmed by new material. Nevertheless, it is probably not wise to accept these figures unconditionally as representing the universal standard length of the two measures throughout their history.” (Iversen, 1975, p.17)


His major disagreement with Lepsius was the number of digits making a fist (Gardiner sign list D49 ). He gave five and one third fingers (Iversen, 1975, p. 32) where Lepsius gave six fingers.

Table 3. Iversen’s Canon and Metrological Divisions (Iversen, 1975, p. 17).

Anatomical Part	Number of Squares	Metrological Units	Fists Value	Palms Value	Inches Value	Fingers Value
Thumb	1/4	Inch	1/4	1/3	1	1 1/3
Palm of four Fingers	3/4	Palm	3/4	1	5	4
Fist of four fingers + thumb	1	Fist	1	1 1/2	4	5 1/3
Elbow-shoulder	3 3/4	<i>Remen</i>		5	15	20
Elbow wrist	3	2/3 measure	3	4	12	16
Elbow-thumb	4 1/2	Small Cubit	4 1/2	6	18	24
Elbow-medius	5 1/4	Royal Cubit	5 1/4	7	21	28
Base-hairline	18	Fathom	18	24	72	96



Figure 3. Iversen's Canon and Metrological Divisions (Iversen, 1975, p. 17).

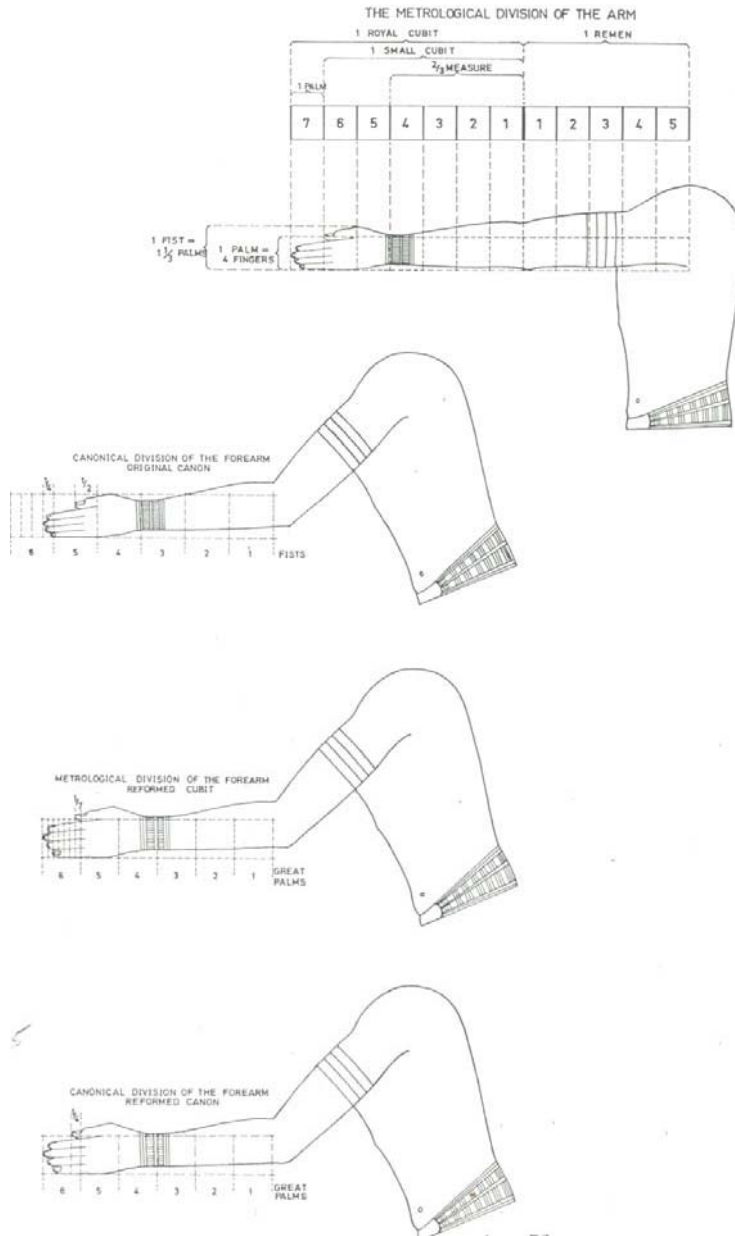


Figure 4. Iversen's Divisions of the Arm (Iversen, 1975, Plate 9).

He is also the first author to stress the importance of the *dsr* in ancient Egyptian metrology as the basis of what he called the uncial division ($2/3$ of a royal and reformed cubit, divided into twelve finger divisions instead of sixteen) rather than the traditional digital division of the royal cubit (Iversen, 1990, pp. 113-114).

1.1.14 G. Robins

Robins (1982, pp. 61–75) defended Lepsius and contested the connection that Iversen previously made between the divisions of the small and royal cubits, and the length of the arm. After a careful study of 60 mummies from the Cairo, British, and Manchester museums she concluded:

“Iversen postulated that the canonical length of the forearm from the elbow to the thumb was one small cubit of 45 cm and was the equivalent of 4 1/2 grid squares; that to the finger-tips, it was one royal cubit of 52.5 cm which equals 5 1/4 squares; and that to the wrist, it was three squares and equal to 30 cm. The data from these mummies, however, suggests that in nature the distance from elbow-bone to wrist is about 27 cm and from elbow-bone to finger-tips, it is 46 cm.”
(Robins, 1982, p. 70).

Robins also equated the fist with six metrological fingers, as Lepsius did. According to her new physical evidence (if one were to follow Iversen’s values) the distance from elbow-bone to finger-tips (Iversen’s definition of the royal cubit) would be 46 cm (incidentally, very close to the value already given by Jomard for the small cubit), while the small cubit would measure 27 cm. She also studied the natural and canonical proportions in Egyptian art (Robins, 1983a, pp. 17–25; 1983b, pp. 67–72; 1983c, pp. 91–6; 1984a, pp. 21–32; 1984b, 1985a, pp. 47–54; 1985b, pp. 51–64

1.1.15 J.F. Carlotti

Carlotti (1995 - Annexe, pp. 127-39) accepted Iversen's digital and uncial divisions. To the digital system belong the royal cubit, the finger, the palm, the hand, the sandal, and the double palm. To the uncial system belong the inch, the fist, the *djeser*, the *remen*, and the small cubit. Carlotti considered both the *djeser* and the *remen* of the uncial system as cubits, on par with the small cubit. According to this theory, there are four cubits in ancient Egypt: the royal cubit, the small cubit, the *remen*-cubit, and the *djeser*-cubit.

1.1.16 T. Pommerening

In *Die altägyptischen Hohlmaße*, T. Pommerening concluded her very complete catalogue and study of ancient Egyptian containers and volumes with the following Table (Pommerening, 2005, pp. 267–8).

Table 4. Pommerening's Containers and Volumes (Pommerening, 2005, pp. 267–8).

Unit	Volume	Divisions	Attested Period	Origin
Old Kindom-Ointment Measure	1.44 Litres	1/2, 1/4, 1/8, 1/16	Predynastic and Old Kingdom	1/100 of cubed cubit?
<i>Dw.t</i> Measure	2.2 Litres		Old Kingdom	Meat Measure
<i>D3.t</i> Measure = Handful	75–100 ccm		Old and Middle Kingdoms	Physical measure
Middle Kingdom- ¹ <i>D3</i>	75 ccm	1/2	Middle Kingdom	Handful ? Involving *1/64 of ¹ Hekat
Middle Kingdom- ² <i>D3</i>	150 ccm	1/2, 1/4	Middle Kingdom	From the MK <i>D3</i> . involving *1/64 of ² Hekat
<i>Ds</i> Measure	Unknown (between 0.8-2.6 Litre)	1/2, 1/4	Middle Kingdom	<i>Ds</i> Beer Container
<i>Hnw</i> Measure	480 ccm	1/2, 1/4, 1/8, 1/16, 1/32, 1/3, 2/3, 1/6, 1/5	New Kingdom to Greek and Roman Periods	<i>Hnw</i> Container
<i>D3</i> Measure	300 ccm	1/2, 1/4, 1/8, 1/16, 1/32, 1/64, 1/128	New Kingdom to Greek and Roman Periods	Involving *1/64 of ⁴ Hekat
¹ <i>R</i> Measure	15 ccm	Eclectic (e. g. 1/3, 1/10, 1/2190) and 2/3	Middle to New Kingdoms	*1/64 ¹ Hekat = 1 Middle Kingdom ¹ <i>D3</i> = 5 ¹ <i>R</i> -> 1 ¹ <i>R</i>
² <i>R</i> Measure	30 ccm	Eclectic and 2/3	Middle to New Kingdoms	*1/64 ¹ Hekat = 1 Middle Kingdom ² <i>D3</i> = 5 ² <i>R</i> -> 1 ² <i>R</i>

Unit	Volume	Divisions	Attested Period	Origin
3R Measure	45 ccm	Eclectic and $2/3$	Second Intermediate Period	* $1/64$ ${}^1Hekat = 1$ Middle Kingdom ${}^3D\beta = 5$ ${}^3R \rightarrow 1$ 3R
1Hekat	4.8 Litres	$1/2, 1/4, 1/8, 1/16, 1/32, 1/64, 1/3, 2/3$	New Kingdom	$1/30$ of a cubed cubit the usual common denominator of the measurement

In Table 4 above, the multiplicity of containers having the same name for different volumes at the same period is striking, and at first glance, is ambiguous until one remembers that the volume of these containers is related to the specific weight of the commodities they contained, leading to the following questions:

- 1 Which commodity goes into which container?
- 2 What is the specific weight of the commodity going into the container described?

Without going into too many details in this outline, let us consider the two Middle Kingdom *Djas*: 1Dja of 75 cubic cm and 2Dja of 150 cubic cm, one being in capacity the double of the other. One litre equals 1000 cubic cm. Let us consider two commodities, com1, the specific weight of which is 400 grams per litre, and com2 the specific weight of which is 800 grams per litre. If the 1Dja is used to measure com2 and the 2Dja is used to measure com1, the equivalence between the two *Djas* is established by their respective weights:

- One 1Dja of com2 weighs (75 cc multiplied by (800 g divided by 1000 cc)) = 60 grams.
- One 2Dja of com1 weighs (150g multiplied by (400g divided by 1000 cc)) = 60 grams.

In other words, to avoid confusion between the two *Djas*, they must be defined and associated with their respective commodity or commodities of reference, making exchanges based on weight possible: the weight of one 1Dja of com2 equals the weight of one 2Dja of com1, making the use of scales redundant in specific operations.

The actual commodities associated with a *Dja*, as mentioned by Pommerening, are water, oily products, and powdery substances including myrrh. Pommerening gives a weight between 0.42 and 0.62 grams per cubic cm of myrrh (Pommerening, 2005, p. 236. Water weighs 1 gram per cubic cm, and if one takes the average between 0.42 and 0.62 grams per cubic cm, or 0.50 gram

per cubic cm of myrrh, one ¹*Dja* of water (1 gram multiplied by 75 cubic cm) equals one ²*Dja* of Myrrh (0.50 gram multiplied by 150 cubic cm) or 75 grams.

The weakness of the system becomes obvious when one asks for one *Dja* without specifying the commodity. Does one get 75 cubic cm (¹*Dja*) or 150 cubic cm (²*Dja*) in the Middle Kingdom?

To avoid this ambiguity two solutions are possible:

- Give a unique capacity to the *Dja*: this seems to be the case, as the New Kingdom *Dja* has a volume of 300 cubic cm.
- Supply a commodity name associated either the ¹*Dja* or ²*Dja* container.

1.1.17 G. Schmitt

G. Schmitt (2005, pp. 55–72) raised the problem of the different cubit lengths during the Ptolemaic Period and their connection with the *hin* as they appear in Greek and Roman papyri (particularly in pBritish Museum 10399, problems 42 to 43 [Parker, 1972, pp. 55–57]). He gave a very useful catalogue of the lengths of the cubits in Greco-Roman Egypt. His conclusion was that several standards were used during the Greco-Roman Period but he did not define what these standards were.

1.1.18 J. Wegner

In his book, *The Mortuary Temple of Senwosret at Abydos*, Josef Wegner gives a good example of the parallel use of two linear metrological systems. The brick dimensions of the mortuary temple follow two standards: the royal cubit and a local standard of 22.5, 30, and 15 cm (Wegner, 2007, p.10) reflected in locally produced bricks of 25*15*10 cm. (Wegner, 2007, p.26).

1.1.19 P. Zignani

Dealing with the Ptolemaic temple of Denderah, Zignani finds that the cubit used has a length of 53.4 cm (Zignani, 2010, p. 158). He gives a clear definition of the difference between "digital" and "uncial," the finger division structure coined by Iversen and followed by Carlotti. The digital *djeser* is divided into sixteen finger divisions of 1.875 cm each while the uncial *djeser* is divided into twelve finger divisions of 2.5 cm each. Digital (*digitus*, finger in Latin) refers to the number

of fingers of the royal cubit (28) and the *djeser*. The uncial *djeser* is also known as the "sacred cubit" (Zigani, 2010, p. 154).

1.2 *Nbj*

1.2.1 W. C. Hayes

The first study dedicated to the *nbj* as a metrological unit appeared in William C. Hayes, *Ostraka and Name Stones from the Tomb of Sen-Mut (No 71) at Thebes* (1942, pp. 37 ff.) where the author suggested that the term *nbj* used on ostraka is linked to the length of pick handles 60 to 65 cm).

1.2.2 N. Victor

Victor (1991, pp. 101–10) gave the following definition of the length of the *nbj*:

“From a recent and detailed examination of the architectural features of more than 150 rock cut tombs at eight different Old and New Kingdom sites, I found a remarkable repetition of certain measurements representing multiples of 2.5 cm., i.e. 5 cm., 7.5 cm., 10 cm., 12.5 cm., 15. cm., 17.5 cm. and multiples of 17.5 cm., i.e. 35 cm., 52.5 cm., 70 cm. Also observed was the consistent multiple of 70 cm., i.e. 140 cm., 210 cm., 280 cm., 350 cm., etc. The regular use of these multiples is found in the principal architectural features of the tombs, that is the measurements of length, width, depth and height, as well as in all architectural details as niches, false doors, façades, lintel projections, step heights, pillars, shafts, etc.”

1.2.3 E. Roik

In a book, (Roik, 1993) preceded and followed by a series of articles (Roik, 1990, pp. 91–99 ; 1999, pp. 73–94; 2003, pp. 351–65), Roik, after a careful study of the tomb of Ta-Useret (KV 14) and the study of dimensions of bricks in the ancient Near East (Jericho, Catal Höyük, and Babylon) which she linked to the length of the Greek tetradoron (32 multiplied by sixteen multiplied 8 cm), suggested possible divisions of the *nbj*, based on the Horus-Eye system of

fractions. Roik’s “physical” evidence was a single artifact showing eight divisions and reconstructed by Petrie: the Deshasheh Rod.

Table 5. Divisions of the Nbj According to Roik (Roik, 1993, p. 5).

Unit	Division	Metric length
1 <i>nbj</i>	8 Units	65.00 cm
	7 Units	56.87 cm
<i>nbj</i> cubit	6 Units	48.75 cm
	5 Units	40.62 cm
1/2 <i>nbj</i> = 1 Doric foot	4 Units	32.50 cm
	3 Units	24.37 cm
	2 Units	16.25 cm
	1 Unit	8.12 cm

Roik’s reconstruction of the *nbj* has from two drawbacks:

- 1 The *nbj* as a metrological term and measurement unit only appears for the first time during the Eighteenth Dynasty on ostraka discovered in Sen-Mut’s tomb.
- 2 A lack of convincing physical evidence. Simon-Boidot’s (1993, pp. 157–77; 2000, p. 71) remarked that Roik’s division of the *nbj* is purely arbitrary:

“La division en huit du nbj est justifiée uniquement par les huit divisions de la règle (non conservée) de Dechacheh. Or, ces huit intervalles sont restitués par Petrie . . . Une moitié est vierge, l’autre est graduée de huit intervalles très inégaux. Ce chiffre huit reste donc très contestable.” (Simon-Boidot, 1993, p. 170).

1.2.4 J. Legon

In an article, Legon (1994, pp. 97–105) considered the *nbj* as a unit of volume. This is true only during the Ptolemaic and Roman Periods. The Greek word **naubion**, only applicable to Egypt and derived from the word *nbj*, is a unit of volume. **Naubia** are calculated using **xu ν a** (wooden rod-measures of length) and cubits. In Sen-Mut’s tomb No 71, from which Legon took his

references, the unit of volume is the *dnj* bag. The *nj* is a unit of length used in volumetric calculations.

1.2.5 C. Simon-Boidot

Simon-Boidot systematically refuted Roik's theory (Simon-Boidot, 2000, pp. 66–79 [following her 1993 article {Simon-Boidot, 1993, pp. 157–77} mentioned above]). Simon-Boidot's association of a unit of ten cm with the Canon of Proportions (Simon-Boidot, 2000, p.67) is very interesting as it links, according to this thesis, the Canon of Metrology to the scribe's palette used as a measuring artifact.

2 Comparative Studies

Several metrological studies below include ancient Egyptian systems on a comparative basis.

2.1 Cubits

2.1.1 I. Newton

In his *Dissertation upon the Sacred Cubit of the Jews*, Newton (1774, pp. 491–510) gave an accurate value for the royal cubit (52 cm) by studying the dimensions of the king's chamber in the Cheops pyramid. Newton accepted the claim made by Greaves in *Pyramidographia, or a Description of the Pyramids in Ægypt* that the pyramid was 693 English feet or 400 cubits long. Confirmation of Greaves' figures came with the dimension of the King's Chamber: 34.38 by 17.19 English feet or twenty by ten cubits. He also suggested a value of 63.5 cm for what he called a "Pyramid Cubit"

2.1.2 Authors Quoted By Lepsius In *Die Alt-Aegytsche Elle Und Ihre Eintheilung* (1865)

- **J. F. Saigey**. In his *Traité de Métrologie*, Saigey (1834, p. 15) gave the following length for the Amenemope cubit used by Lepsius : 52.35 cm.
- **A Bökh** (1838, p. 215, 217, 235, 237, 288) gave a length of 52 cm for the royal cubit and, in agreement with Jomard, 46 cm for the small cubit.

- **L. Fenner Von Fenneberg** (1859, p. 68) and **F. O. Hultsch** (1862, p. 20, 241, 280). These authors' contributions are dedicated to the Ptolemaic divisions of the cubit. Hultsch quoted Dydimus (Hultsch, 1864, p. 241): "this great cubit has six palms or 24 fingers amounting to 1 1/2 Ptolemaic or 1 4/5 Roman feet." He gave a length of 52 cm for the royal cubit.

2.1.3 R. Lepsius

In his *Die Längenmaße der Alten*, Lepsius (1884) undertook a complete study of the ancient Egyptian, Babylonian, Jewish, Persian, Greek and Roman linear measurement systems. As far as ancient Egyptian lengths are concerned, he confirmed the value given in his previous study on the cubit (52.5 cm).

2.1.4 A. Segrè

In a comprehensive study of ancient metrology Segrè (1928) summed up previous works on ancient Egyptian lengths, volumes, and weights. He confirmed the length of the cubit given by Lepsius and added a weight precision for the *deben* previously given by Griffith: 1 *deben* = 97.036 grams.

“Le unità ponderati egiziane più comuni dalla dinastia sono: il deben e il suo decimo kedet (copt. Kite); il peso del primo dedotto dal khar o 2/3 di cubito cubico egiziano è di gr. 97,036 e quello della seconda per conseguenza di gr 9,703.” (Segré, 1928, p. 13).

2.1.5 J. A. Decourdemanche

In a work very seldom quoted (Decourdemanche, 1909), the author linked linear with weight measures mathematically:

“Un fait domine toute la métrologie ancienne et celle des Arabes: tous les poids, toutes les mesures, soit des anciens, soit des Arabes sont en relation arithmétique des plus simples avec l'un quelconque des talents dits talents-types, lesquels ont également entre eux des rapports arithmétiques extrêmement simples. Il suffit en conséquence, de connaître exactement la valeur française de l'un quelconque des

éléments pondéraux ou de mesure, soit des anciens, soit des Arabes pour pouvoir en déduire, avec la certitude mathématique, la valeur de tous les autres.

Ce travail de comparaison est singulièrement facilité, notons-le, soit par les équivalences (autrement dit les parités) fournies par les métrologues grecs de l'école d'Alexandrie ou par les mathématiciens Arabes, soit encore par les données relatives aux poids, insérées à la suite des traductions Arabes ou persanes des oeuvres de Gallien ou dans les ouvrages médicaux musulmans. Il en est de même, pensons nous, des légers écarts de mesure entre certains relevés d'architectes (opérés plus ou moins exactement sur des monuments en ruine, dont ils rétablissent hypothétiquement l'état primitif) et les dimensions des pieds ou coudées établies d'après les données mathématiques fournies par les cubes en eau des poids qui ont servi à déterminer les longueurs exactes de ces mêmes pieds ou coudées." (Decourdemanche, 1909, pp. 6–7).

In other words, Decourdemanche found a definite relationship between weights and linear measurement that will be explored throughout this thesis.

2.1.6 E. Lorenzen

In his *Technological Studies in Ancient Metrology* Lorenzen (1966, pp. 28–9) attempted to establish a connection between Egyptian canon grids and the Vitruvian proportions of the human body applied to temple architecture. Vitruvius states:

“For Nature has so planned the human body that the face from the chin to the top of the forehead and the roots is a tenth part; also the palm of the hand from the wrist to the top of the middle finger is as much; the head from the chin to the crown, an eighth part; from the top of the breast with the bottom of the neck to the roots of the hair, a sixth part; from the middle of the breast to the crown, a fourth part; a third part of the height of the face is from the bottom of the chin to the bottom of the nostrils; the nose from the bottom of the nostrils to the line between the brows, as much; from that line to the roots of the hair, the forehead is given as the third part. The foot is a sixth of the height of the body; the cubit a quarter, the

breast also a quarter. The other limbs also have their own proportionate measurements. And by using these, ancient painters and famous sculptors have attained great and unbounded distinction. In like fashion the members of temples ought to have dimensions of their several parts answering suitably to the general sum of their whole magnitude.” (Vitruvius, Granger translator, 1931, pp. 159–60).

His explanation is based on a rather complicated and ingenious process that he called the *Double-Inch System* (the royal cubit is split into eight half Double-Inches; it is similar in its result to Roik’s approach [Lorenzen, 1966, pp. 102 ff]). On a metrological stand-point, Lorenzen did not contest the value of the small and royal cubits defined by Lepsius. Ironically, when Lorenzen declared:

“We shall try to show that Lepsius was correct in his metrical values, but what he called a royal cubit might better be called a small cubit.” (1966, p. 102)

He might be the first author to correctly disassociate the small cubit from the royal cubit.

2.1.7 W. Hinz

Walther Hinz’s *Islamische Masse and Gewichte* is particularly interesting as it shows the existence of two medieval Egyptian cubits (Hinz, 1955, p. 55) the “praktische Elle” of 66.5 cm and the “schwarze Elle” of 54 cm representing the legacy of previous linear metrological systems, the cubit of 54 cm being a direct descendant of the Nilometric cubit of the Ptolemaic and Roman Periods.

2.2 *Nbj*

2.2.1 A. Segrè

Segrè (1928, pp. 25–8) considered the **naubion** (*nbj*) to be a unit used for volumes. He is not certain whether it should be related to the small, royal, or Alexandrian cubits.

Previous scholars used four methods to reach the length of ancient Egyptian cubits. The first method relies on the results derived from monument and building surveys. Jomard, Howard-

Vyse, Perring, Smyth, Petrie, Victor, Roik, Dieter, Carlotti, Wegner, and Zignani follow this method. The second method relies on the comparison of cubit artifacts or the interpretation of published monument and building surveys. Newton, Champollion-Figeac, Girard, Saigey, Bökh, Fenner Von Fenneberg, Thenius, Vazquez-Queipo, Lepsius, Griffith, Borchartt, Segrè, Reineke, Lorenzen, Hinz, Schlott, Hayes, Legon, Simon-Boidot, and Schmitt follow this method. The third method relates ancient Egyptian linear measurements to volumetric and area measurements. Relating linear measurements to area, volumetric, and weight measurements is called closing a metrological system. In a closed metrological system, units of length, volume, and weight are related to each other and can be derived from each other. Griffith, Segrè, Decourdemanche, and Pommerening, in the conclusion of her study of containers and ancient Egyptian volumetric units, follow this method. The fourth method is based the divisions of the Twenty-sixth Dynasty reformed cubit. Lepsius started this method with his remarks on the number of grid squares used between the sole of the foot and either the forehead or the root of the nose in reliefs of standing or sitting human figures. This number of grid squares, varying throughout ancient Egyptian history, is called the Canon of Proportion. Lepsius, Iversen (1990, pp. 113-114), Carlotti (1995 - Annexe, pp. 127-39), and Zignani (20210, p. 154) follow this method. They consider that from the Twenty-sixth Dynasty the royal and reformed cubits have two main divisions (names unknown): $2/3$ (Lepsius, and Iversen) and $5/6$ of the royal and reformed cubits (Iversen, Carlotti, and Zignani). Robins and Roik relate the Canon of Proportion with the small cubit and the *nby* (a multiple of the royal cubit) respectively. The four methods define five cubits:

- A "Pyramid" cubit, 60 to 75 cm long.
- A Royal, digital (the basic division of the cubit is the finger [*digitus* in Latin]) cubit 52 to 54 cm long.
- A royal and reformed uncial (duodecimal [base 12] – *uncia* means the twelfth part in Latin) *djeser* cubits, $2/3$ of the royal and reformed cubit, 30 and 35 cm long.
- A *remen*-cubit, also uncial, $5/6$ of the royal and reformed cubit, 43.75 cm long.
- A small cubit, 45 to 47 cm long, which disappears with the reformed cubit.

Chapter 3 - The Royal Cubit as a Converter and The Scribe's Palette as a Measuring Device

Summary

Stationers sell rulers showing metric and imperial linear divisions on both sides. The only difference between the Amenemope cubit artifact and modern rulers is the position of the graduations: usually on top and at the bottom of our modern rulers; on the same row, but with different colours, on the Amenemope artifact. There are two spans (*pd*) on the Amenemope artifact: a small span (*pd šrj*), marked by outlined signs (white in the reproduction of *Appendix A*), and a great span (*pd ʿ3*) marked by filled signs (black in the reproduction of *Appendix A*). Why is it necessary to have a common name for two divisions and qualify them as "small" in white, and "great" in black other than to indicate two systems? The only cubit conversion information available is coming from observations made by Lepsius regarding the Canon of Proportion - followed by Iversen (1990, p.114), Carlotti (1995 - Annexe p.131), and Zignani (2010, p. 154) - and from the Denderah cubit artifacts published by Legrain (1916, pp. 149-52).

From Legrain's reproduction below both the *Mḥ Ntr* (the former royal cubit) and the *Mḥ* (the reformed cubit) can be divided into palm divisions of three or four finger divisions each.

During the Twenty-sixth Dynasty, the cubit is divided into six palm divisions instead of the traditional seven palm divisions of the royal cubit. As a result, the small cubit disappears and the palm length is increased by 1 1/6: 8.75 cm instead of 7.5 cm. The number of fingers per palm division remains constant, four fingers per palm division, with a length of 2.1875 cm per finger division. The 28 royal finger divisions of the royal cubit become 24 reformed finger divisions. This is the explanation for the small cubit disappearance: 24 finger divisions now define the length of the reformed cubit; in the royal system these 24 finger divisions represent the small cubit of 45 cm. This change in the number of palm divisions brings also the definition of two "new" cubits: the *djeser*-cubit and the *remen*-cubit. The *djeser*-cubit equals to 2/3 of the reformed cubit, or 16 reformed finger divisions, or 35 cm in the reformed system. Zignani (2010, p. 154) defines the *djeser*-cubit as 16 fingers of "a cubit," without mentioning which cubit is involved (royal or reformed), divided by 12. The *remen*-cubit

is defined as $5/6$ of a cubit. The reformed *remen*-cubit corresponds to 20 reformed finger divisions with a length of 43.75 cm. This length is very close to the length of the small cubit (45 cm) disappearing with the reformed cubit. There is a similarity between the number of palm divisions of the reformed cubit and the reformed *remen*-cubit on one hand, and of the number of palm divisions of the royal cubit and of the small cubit on the other: the reformed *remen*-cubit is shorter by one palm division than the reformed cubit; the small cubit has also one less palm division than the royal cubit on the royal cubit artifacts.

The difficulty with the *djeser*-cubit and the *remen*-cubit is that $2/3$ and $5/6$ give an even number of expected finger divisions (16 or 20) only when the length of the royal cubit is divided into 24 uncial reformed finger divisions. Carlotti (1995 - Annexe p.131) defines the *djeser*-cubit as the ordinary *djeser* division of the royal cubit: one half plus $1/14$ of a royal cubit or 16 royal finger divisions (one half of a royal cubit = 14 royal finger divisions + $1/14$ of a royal cubit or two royal finger divisions). He defines the *remen*-cubit as $1/2 + 1/7 + 1/14$ of the royal cubit or $14 + 4 + 2$ royal finger divisions or 20 royal finger divisions which is the definition of a royal *remen* of 37.5 cm. Legrain has published three Ptolemaic cubits found at Denderah and probably used for the construction of the Denderah temple or parts of it. These three cubits have a length varying between 53 cm and 53.5 cm. Although this thesis considers that these cubits are not royal (The first row probably represents the *Mh Ntr*, the Ptolemaic descendant of the royal cubit – The third row probably represents the *Mh*, or reformed cubit of the Twenty-sixth Dynasty) they are relevant to this discussion as they show divisions based on 3, 4, and 5.

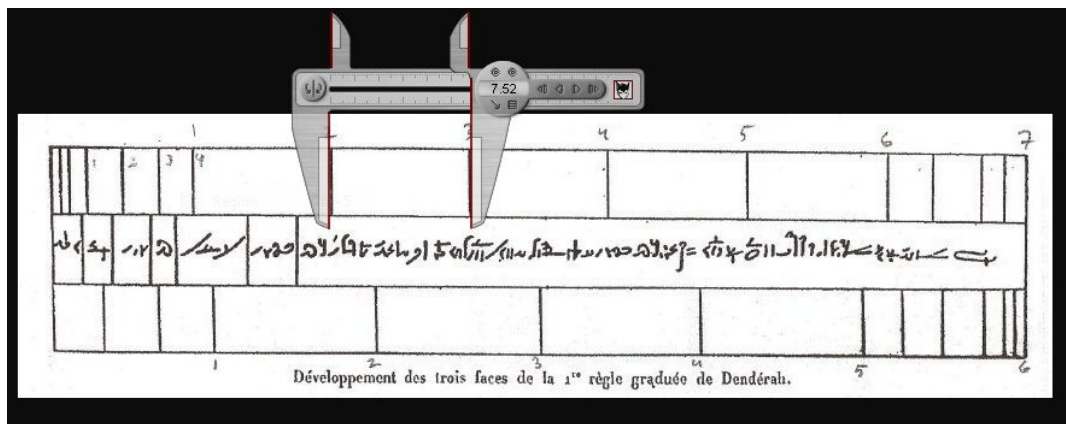


Figure 5. Length of the *Mh Ntr* palm division (7.5 cm) with two possible finger divisions: four finger divisions (left) or three finger divisions (right) per palm division.

The Denderah cubit can be divided, in the first row, into palms of four or three finger divisions with a length of varying between 7.5 and 7.6 cm² which are the characteristics of either the *Mh Ntr*, the Ptolemaic equivalent of the royal cubit, or seven palms of the Ptolemaic or Roman Alexandrian cubit which explains why the artifact is 53 cm long instead of the expected 52.5 cm. The third row represents the *Mh* (the reformed cubit) with a palm division of 8.75 cm (see Chapter 4 - the origin and evolution of ancient Egyptian cubits and Excursus B – division evolution).

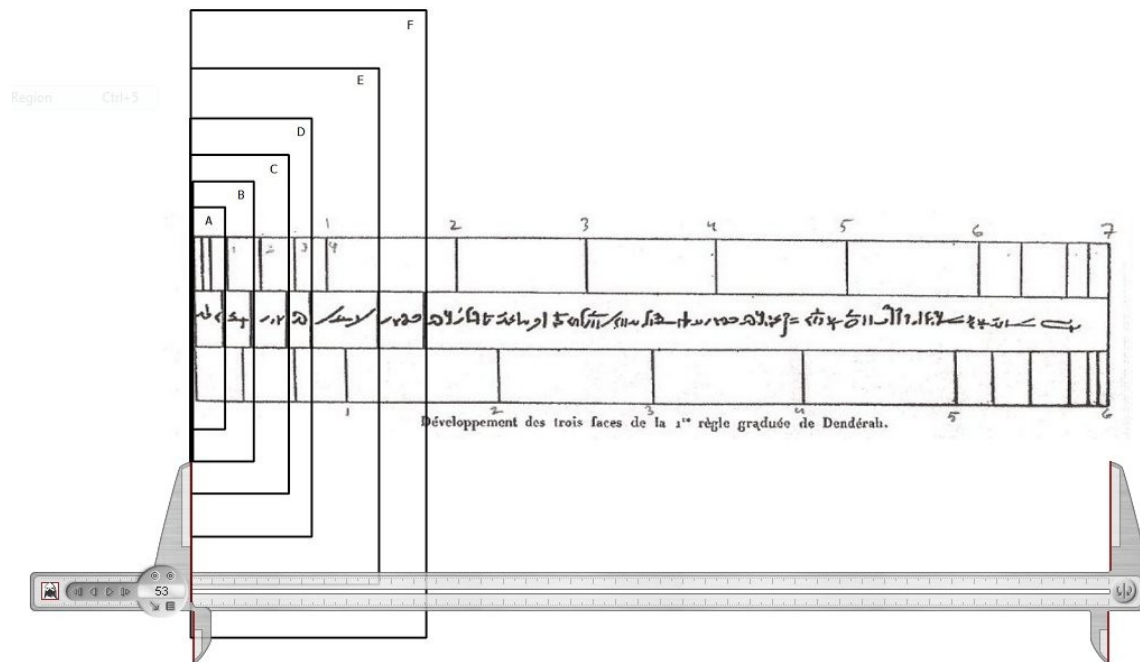


Figure 6. The divisions of seven palms of the Alexandrian cubit (53 cm).

The second row of Figure 6 shows divisions - the length of the width of the rectangles (A = 1/32, B = 1/16, C = 1/10, D = 1/8, E = 1/5, F = 1/4) rectangles - based on the numbers 3, 4, and 5.

- 3: Palm of three fingers. The last palm division of the first row of Figure 6.
- 4: Palm of four fingers (the first palm division of the first row of Figure 6) and the 1/8, 1/16, 1/32 divisions of the second row of Figure 6.

² The screen calipers appearing on Figures 5 have been calibrated as 818 screen pixels for 53 cm. The slight measurement discrepancies (1/10 or 2/10 of a millimetre) are due to the figure reproductions, not to the calipers: some vertical lines are not absolutely straight.

- 5: The 1/5, 1/10 divisions of second row of Figure 6.

The purpose of this chapter is to investigate whether the black and white markers on the Amenemope cubit artifact indicate a division pattern similar to the uncial divisions of the reformed cubit or to the divisions appearing on the Denderah cubit artifact (Figure 6, published by Legrain) and to test the results against a database of measurements of everyday objects scattered in museums around the world³. If the Amenemope royal cubit artifact is a converter - an artifact showing two metrological systems - it must follow the basic principles of ancient Egyptian mathematics: be “essentially additive” (Neugebauer, 1969, p. 73) and allow “doubling, taking 2/3, halving, multiplying by ten, and taking 1/10” (Clagett, 1999, p.20). The divisions and cubit markers on the Amenemope cubit artifact comply with these requirements. One would also expect that two parallel linear systems would be represented by two distinct artifacts as our modern rulers do: the divisions of the imperial system correspond to the yard; the metric system divisions correspond to the metre. The black markers on the Amenemope cubit artifact correspond to the royal system while the white markers correspond to the great (ꜥ) system). To shift from the black (royal system) to the white (great [ꜥ] system), one needs to increase the length of the white finger division to 2.5 cm. This is done by adding one third of the length of a royal finger division to the royal finger division itself (1.875 cm divided by 3 = 0.625 cm; 1.875 cm + 0.625cm = 2.5 cm) and decreasing the number of finger divisions to three per palm. This 1/3 ratio is significant as it matches the difference between the specific weights of barley and emmer: barley is 1/3 heavier than emmer and explains the agricultural origin of both systems developed in *Chapter 4 - The Origin and Evolution of Ancient Egyptian Cubits*. As a result, the length of the royal cubit (52.5 cm) equals 21 great finger divisions, which explains why the royal cubit is divided into seven palm divisions. 21 great finger divisions in seven palm divisions of three finger divisions each equal 28 royal finger divisions in seven palm divisions of four finger divisions each; seven palms in both systems show a common length of 52.5 cm, the length of the royal cubit. Showing the great [ꜥ] system of 60 cm on a royal cubit artifact becomes redundant. In both systems adding one palm division gives a length of 60 cm.

³ The methodology followed in the database is based on G. Grasshoff’s article on finger divisions that are not necessarily in any fixed relation to the cubit (GrassHoff, 1999, pp. 97-148).

What is remarkable in the statistical database of measurable everyday objects scattered around the world is that only 582 entries would require a “subjective” delta calculations between the royal and the great systems to match them to either system; they represent only almost 12% of the 4905 total valid measurements in the database. Such a small percentage ascertains the parallel use of the royal and great [3] systems before the Ptolemaic and Roman Periods.

3 The Amenemope Royal Cubit Artifact as A Converter

Figure 7 is taken from Lorenzen’s book (Lorenzen,1966,Plate I) The picture reproduces the cubit and drawing used and made by Lepsius. The markers A, B, and C as well as the numbering of fingers from 16 to 28 have been added to the original reproduction

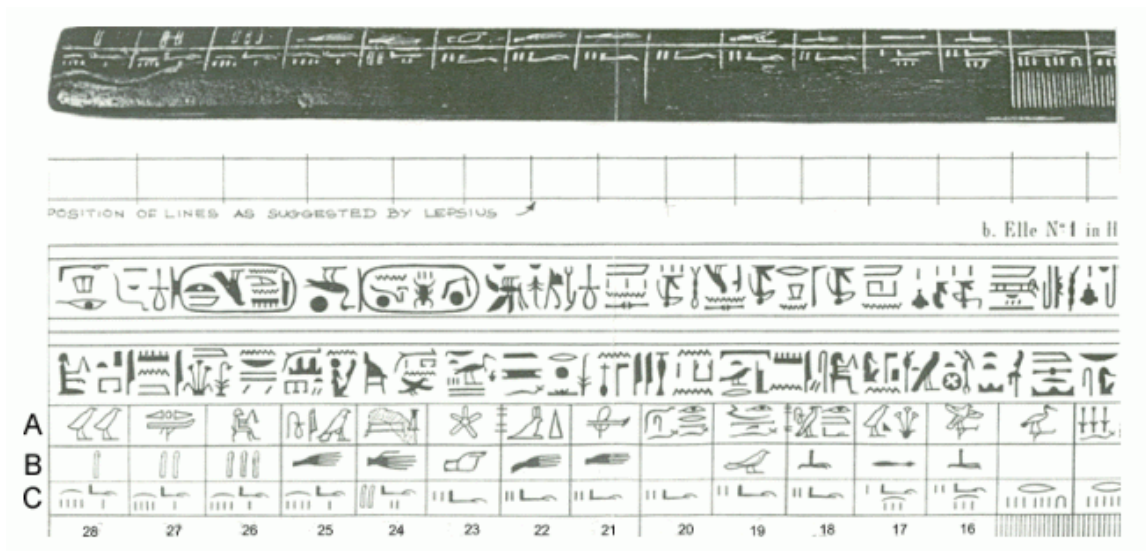


Figure 7. Amenemope Division Markers (Turin museum #6347 – Lorenzen, 1966, Plate I – Scale 1/1 - see Excursus A).

3.1 Division Markers

Summary

There are two systems indicated on the Amenemope cubit artifact: A “black” system including the palm of four finger divisions, the hand, the double palm, the great span, the *djeser*, the *remen*, and the royal cubit and a “white” system including a palm of three finger divisions, the

fist, the small span, and the small cubit. The existence of two systems on the same artifact implies that the Amenemope cubit artifact and the royal cubit is a converter.

- The Amenemope cubit artifact (Turin museum #6347), thanks to its dedicatory text, can be dated to the end of the Eighteenth Dynasty (Horemheb). What is uniquely remarkable on the Amenemope cubit artifact is the colour difference used for the division markers:
 - Outlined signs (white in the reproduction of *Appendix A*) are used for one, two, and three fingers, the fist, the small span (*pd šrj*), and the small cubit (*mḥ šrj*).
 - Filled signs (black in the reproduction of *Appendix A*) are used for the palm of four finger divisions, the hand, the double palm, the great span, the *djeser*, the *remen* and the royal cubit.

The majority of the black and white divisions can be doubled when the resulting value is shorter than the length of the royal cubit itself. When doubled, the black palm of four finger divisions gives a double palm, twice a doubled palm gives a *djeser*; and doubling a great span gives a royal cubit. The white markings follow a similar pattern: doubling the white palm of three finger divisions gives a fist; two fists equal a small span, two small spans equal a small cubit (*mḥ šrj*). Two parallel patterns marked in black and in white indicate two systems and two cubits.

The only black divisions that are not doubled are the hand, the *djeser*, and the *remen*. Four hands, however, make a *remen*. The *djeser* and the *remen* are not doubled as they would exceed the length of a royal cubit artifact (52.5 cm, defined by Lepsius). The only white division which is not doubled is the small cubit as doubling would also exceed the length of a royal cubit artifact. This does not imply that the *djeser*, the *remen*, and the small cubit cannot be doubled. Doubling the *djeser*, the *remen*, and the small cubit becomes important as these divisions are the uncial divisions of the reformed cubit of the Twenty-sixth Dynasty. By doubling the *djeser*, *remen*, and small cubit, one would expect to find a common factor or denominator respecting ancient Egyptian mathematical principles and explaining the reason for the reformation of the royal cubit during the Twenty-sixth Dynasty, particularly its reduction of finger divisions from 28 royal finger divisions to 24 finger divisions. One would also expect that the doubling of the *djeser*,

remen, or small cubit would explain the origin of the *nbs*, the length of which is estimated to 65 cm.

3.2 Doubling the Small Cubit, the *Remen* and the *Djeser*

Summary

60 cm becomes the common length denominator when doubling the small cubit, the *remen*, and the *djeser*. These 60 cm provide a conversion bridge between the white and black systems appearing on the Amenemope cubit artifact. 60 cm comply with the “additive” and “taking 2/3” principles of ancient Egyptian mathematics.

Doubling a small cubit (marked white) gives a length of 90 cm or three royal *djesers* (marked in black). Doubling a *djeser*, obviously, gives a double *djeser* of 60 cm. Doubling a *remen* gives a length of 75 cm, or one double *djeser* and a quarter. The double *djeser* of 60 cm appears in all the doubling operations above and respects the “taking 2/3” principle of ancient Egyptian mathematics: 2/3 of a doubled small cubit gives a length of 60 cm, the value of a double *djeser*. Furthermore 2/3 of a small cubit (marked white) provides a conversion bridge between the white marking of the small cubit and the black markings of the *remen* and the *djeser* on the Amenemope cubit artifact. The black *djeser* of the Amenemope artifact equals half of 2/3 of a doubled small cubit or a double *djeser* divided by two. The black *remen* of the Amenemope artifact equals 5/6 of the doubled small cubit divided by two, or half a double *djeser* and a quarter.

3.3 Cubit Markers on The Amenemope Cubit Artifact

Summary

The cubit markers on the Amenemope cubit artifacts define two systems and two cubits.



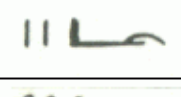
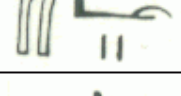

Line C of the Amenemope cubit artifact has always been problematic, particularly to Lepsius who declares:

“In my opinion what we have here is an incomplete execution, a sloppiness deriving from the unconventional use of these cubit rods and the general

carelessness in crafting such artifacts, similar to many other very obvious examples of negligence.” (Lepsius, 1865, p.27 - Translation: J. Degreef)

The glyphs of Figure 7 are difficult to read as cardinals. They have to be read as ordinals in conjunction with line B of Figure 7. By their respective position, the reading of C16: two cubits and three palms; C17: 1 cubit and three palms; C18 to C22: two cubits; C24: two cubits and two fingers; and C25: 1 cubit and four palms, make no sense in the position in which they appear on the Amenemope cubit artifact, as the cardinal reading would exceed the length of the royal cubit artifact itself. Read as ordinals, even if one would have liked to have the *nw* jar (perhaps missing on account of the limited space on the artifact), to confirm this reading, they suggest the existence of two cubits and match the two colour schemes (black and white) used on the glyphs of the Amenemope cubit artifact palm divisions (Figure 7).

Table 6. Amenemope Cubit Markers (Antoine Hirsch, 2012).

Cubit	Position
	C16
	C17
	C18 to C22
	C24
	C25

C16

should be read as second cubit of palm of three finger divisions.

- C17 should be read as first cubit of palm of three finger divisions.
- C18 to C22 should be read as second cubit.
- C24 should be read as second cubit of/with two (white) finger divisions.
- C25 should be read as first cubit with a palm division of four finger divisions.
- Related to line B on Figure 7, and from left to right:

- B25 shows the royal palm division of the royal cubit in black, implying a division by four, or four finger divisions in a palm.
- B24 shows the royal hand division of five fingers (five finger divisions between C28 and C24), also in black, implying a division by five or five fingers in a palm.
- B23 shows the fist (in white) of six finger divisions (twice three white finger divisions from B28 to B26) related to the second white cubit, implying a division by six, or six finger divisions in a palm. The palm with six finger divisions is shown from C23 to C18. The small span (with a white quail chick) is also related to the second cubit as, immediately underneath the small span glyphs on the Amenemope artifact (B19 and B18), the second cubit marker appears (C19 and C18).

3.4 Converting from the Black Royal System to the White Double *Djeser* System

Summary

The conversion to the double *djeser* system can be performed on the Amenemope royal cubit artifact using a three to four finger division ratio in a royal palm.



The royal great span is one of the two divisions that are not shared by both systems once the conversion is performed; this indicates that the double *djeser* system is a system parallel to the royal system. The great span is an “add-on” to the royal system. Two great spans equal the length of a royal cubit; in the same way two small spans equal the length of a small cubit. The other division not shared in the conversion is the royal hand. Four royal hands have the length of a royal *remen*. The double *djeser* system will be called henceforth the great (♁) cubit following pBoulaq 13 which mentions a great and a small *deben* related to two different linear metrological systems.

The conversion from the black royal system (C17) to the white double *djeser* system is done by adding one royal palm to the royal cubit. Eight royal palm divisions (60 cm) correspond to eight double *djeser* palm divisions of three finger divisions each in the white system in Table 7. This

method differs slightly from the conversion method of the royal cubit to the reformed cubit. In the reformed cubit conversion, the number of palm divisions is decreased to six from seven while four finger divisions per palm are maintained, but the length of the finger division is increased from 1.875 cm to 2.1875 cm. In the conversion from the Amenemope royal cubit artifact to the double *djeser* the number of palm divisions is increased from seven to eight while the number of finger divisions is decreased from 4 to 3, and the finger division length is increased from 1.875 cm to 2.5 cm. Using a three to four ratio is the only possible way to convert from the royal system to the double *djeser* system on the Amenemope cubit artifact. This indicates that the double *djeser* system is a system parallel to the royal system as the other royal cubit marker, C25, is simply the cubit marker of seven palm divisions of four finger divisions; the remaining “white” cubit markers are divisions of the double *djeser* system. Two parallel systems explain why the great span is one of the royal divisions not shared by both systems: it is a halving “add-on” of the royal system. Two great spans equal one royal cubit on par with two small spans making a small cubit on the double *djeser* system. The *djeser* and the *remen* divisions are shared by both systems, even if they are marked in black on the Amenemope artifact. The conversion leaves the royal hand of 9.375 cm without parallel in the double *djeser* system. Four hands give the value of a royal *remen* of 37.5 cm. The integration of the *remen*, a unit of length as well as of area is related to the great system and discussed further in Chapter 4 - *The Origin and Evolution of Ancient Egyptian Cubits*.

(See Table 7 – Next Page)

Table 7. Divisions of Amenemope Cubit Artifact (Antoine Hirsch, 2012).

Palm of 4 Fingers		Royal Conversion		Palm of 3 Fingers	
	Length	Divisions			
32	60.000	Double Djeser		24	
28	52.500	Royal Cubit		21	
24	45.000	Small Cubit		18	
20	37.500	Remen		15	
16	30.000	Djeser		12	
14		Great Span			
12	22.500	Small Span		9	
8	15.000	Double Palm			
7					
6	15.000	Fist		6	
5	9.375	Hand			
4	7.500	Palm			
3	7.500	Palm		3	
	1.875	2.500			
Royal	cm	cm		Double Djeser	

Royal	Fingers	Double Djeser	Fingers	Length cm
<u>Palm</u>	4	<u>Palm</u>	3	7.5
<u>Double</u>	8	<u>Fist</u>	6	15
<u>Palm</u>				
<u>Small Span</u>	12	<u>Small Span</u>	9	22.5
<u>Djeser</u>	16	<u>Djeser</u>	12	30
<u>Remen</u>	20	<u>Remen</u>	15	37.5
<u>Small Cubit</u>	24	<u>Small Cubit</u>	18	45
<u>Royal Cubit</u>	28	<u>Royal Cubit</u>	21	52.5
<u>Double Djeser</u>	32	<u>Double Djeser</u>	24	60

The double *djeser* will be called in this thesis the great (ꜥ) cubit named after the “great” *deben* of Pap. PBoulaq 13; the origin of its length, 60 cm, will be traced in the fourth chapter of this thesis when ancient Egyptian units of length are put into their closed metrological system context. The 1/3 royal finger division length added to the royal finger division to convert from the royal system to the great [ꜥ] system is significant as it matches the 1/3 difference ratio between the specific weights of barley and emmer: barley is 1/3 heavier than emmer and explains the agricultural origin of both the royal and the great [ꜥ] systems developed in see Chapter 5 - *The Origin and Evolution of Ancient Egyptian Cubits*.

3.5 Converting from the Great Cubit to the Royal Cubit and the Scribe’s Palette

Summary

The great cubit markers on the Amenemope cubit artifact can be split into two categories. The first category has three or six finger divisions per palm division and facilitates the conversion of the great system to the royal system. The second category, being decimal, answers the “multiplying by ten and taking 1/10” requirement of ancient Egyptian mathematics. The scribe’s palette is the “physical” measuring artifact linked to the great cubit system.

The great cubit markers on the Amenemope cubit artifact can be divided into two categories. The first category is based on a division of the great cubit into palms of three and six finger divisions: C16 and C18 to C22. The second category, C24, can be split either into palms of two white finger divisions, or into palms of five finger divisions as C24 is located immediately under the royal hand of five fingers (B24).

Table 8. Great Divisions of the Amenemope Cubit Artifact (Antoine Hirsch, 2012).

Cubit Marker	Number of Finger Divisions per Palm	Number of Palm Divisions	Length of Palm Division	Length of Finger Division per Palm
C16	3	8	7.5 cm	2.5 cm
C18 to C22	6	4	15 cm	2.5 cm
C24	2	12	5 cm	2.5 cm
C24	5	12	5 cm	1 cm

The first category is a conversion from the great cubit to the royal cubit. C16 is the result from the royal system conversion to the great system explaining why the royal cubit has seven palm divisions: seven royal palm divisions of four finger divisions each correspond in length to seven great palm divisions of three finger divisions each. C18 to C22 corresponds to the division of the great cubit into four parts where one palm division of six finger divisions equals, on the Amenemope royal cubit artifact, a double royal palm division of eight finger divisions, two great palm divisions (12 finger divisions) equal a royal *djeser* of 16 finger divisions, and three great palm divisions (18 finger divisions) equal a royal small cubit of 24 finger divisions.

The second category is far more interesting because it complies with the last requirement of ancient Egyptian mathematics “multiplying by ten and taking 1/10.” The division of C24 into 12 palms of either two or five finger divisions gives decimal values easily multiplied by ten or divided by 1/10. More importantly they explain the majority of decimal measurements found at Mendes. Mendes is not the only place where such measurements are found. In the introduction of this thesis there is a reference to an article of N. Victor (1991, pp. 101-10) which is worth repeating here:

“From a recent and detailed examination of the architectural features of more than 150 rock cut tombs at eight different Old and New Kingdom sites, I found a remarkable repetition of certain measurements representing multiples of 2.5 cm., i.e. 5 cm., 7.5 cm., 10 cm., 12.5 cm., 15. cm., 17.5 cm. and multiples of 17.5 cm., i.e. 35 cm., 52.5 cm., 70 cm. Also observed was the consistent multiple of 70 cm., i.e. 140 cm., 210 cm., 280 cm., 350 cm., etc. The regular use of these multiples is found in the principal architectural features of the tombs, that is the measurements of length, width, depth and height, as well as in all architectural details as niches, false doors, façades, lintel projections, step heights, pillars, shafts, etc.”

The only remark that can be made is that Victor’s comments do not apply to the *njb* but to the great cubit. A similar comment applies to *Database_1*. Out of 465 entries, 50 to 60 cm long, only 12.26% match the range defined for a royal cubit (52 to 52.9 cm). It would therefore be

extraordinary not to find a measuring artifact associated with the great cubit; this measuring instrument is the scribe's palette.

4 The Scribe's Palette as a Measuring Artifact

Summary

The scribe's palette is the "physical" representation of the great cubit. Its maximum length of 50 cm corresponds to the divisions of the C24 great cubit marker on the Amenemope cubit artifact. 50 cm do not exceed the length of a royal cubit and matches the length of a forearm or "cubit."

Zivie (1977a, pp. 36–7) is one author who considers the scribe's palette (*gstj*) as a measuring instrument on par with the cubit; Schott (1967, pp. 95–113), sees it more as an instrument to calculate grain quantities. The association of the scribe's palette with metrology is suggested by the Ptolemaic Famine Stela (Barguet, 1953, p.12, 20 and 35; Pl. IV, line 1; 1953, p.35; Haying, 1998, pp. 515–21) where the scribe's palette is a device listed for land measurement operations. The word cubit (*mḥ*) is used in the Edfu inscriptions to designate the scribe's palette (Wilson, 1997, p. 451; Zivie, 1977a, pp. 24–5-36–7) confirming the existence of two parallel systems: the royal cubit on the one hand, and the scribe's palette. on the other, representing the great system. In Coptic, the word ⲄⲟⲤⲧ derived from *gstj* (Westendorf, 1965, p. 468) is a unit of length (value unknown).

The data published by Glanville (1932, pp. 53–61) indicates that the lengths of the scribe's palettes follow the great system rather than the royal system. Examples, dated from the end of the Old Kingdom (British Museum 52942) to the Ramesside Period (British Museum 55140), but with most examples dating from the Eighteenth Dynasty, show that the system used for these artifacts follows the C24 divisions of the great cubit with lengths varying between 30, 40, and 50 cm. Anatomically speaking, 60 cm for a cubit might be too long, 40 and 50 cm are not, as both lengths are close to the lengths of the small and royal "cubits." It is not really surprising to note that the 50 cm scribe's palette (a division of the 60 cm great cubit) matches the maximum height of a papyrus roll (47 cm [Cerný, 1985, p. 8; Leach and Tait, 2000, pp. 227–53]).

The palette of Smendes (Twenty-first or Twenty-second Dynasties) published by Hayes (1948, pp. 47–50), 48.6 cm long and worn smooth by evidence of long usage, was probably originally 50 cm long. Each “palm” is divided into five parts as C24 of Table 8.



Figure 8. Smendes' Palette (Schott, 1967, p. 100).

As Hayes points out The Smendes palette provides a ratio of 1:6 used in the layout of canonical squares in artistic representations and matches the increase of palm length in the reformed cubit of the Twenty-sixth Dynasty.

5 The *Remen* and the *Nbj*

Summary

The *remen* and the *nbj* are units of length. The *remen* is also a unit of area; the *nbj* becomes a unit of volume during the Ptolemaic Period. The *remen* is indirectly related to the great cubit. A squared royal cubit equals 0.2756 square metres. A great cubit of 60 cm multiplied by a small cubit of 45 cm, belonging both to the great system, gives an area of 0.27 square metres, very close to the result obtained by a squaring royal cubit. The sandal and its New Kingdom multiple, the *nbj*, is related to the royal system. Half a great span defines a sandal, four sandals equal a royal cubit, and five sandals equal the length of a *nbj*.

Two sets of divisions must be mentioned before being addressed within their closed metrological system context in the fourth chapter of this thesis. The first set includes the hand and the *remen*, the second set covers the sandal and the *njb*.

5.1 *Remen*

On the Amenemope artifacts (and other cubit artifacts - see *Excursus B*) the *remen* (37.5 cm) is made of four hand divisions of 9.375 cm each. There is no direct relation between the *remen*, the hand, and the other divisions of the royal cubit. A double hand is not marked on any artifact while a double palm (twice a royal palm division of four finger divisions) and a fist (twice three white finger divisions) are.

A - Fist Division



B - Double Palm Division

Doubling the length of the royal *remen* (37.5 cm) and dividing it into palms of three great finger divisions, shows the palm, fist, small span and *djeser* as divisions of the double *remen* and of the great cubit. This would indicate that the *remen* is related, but distinct, from the great cubit. The length of the *remen* on a royal cubit (37.5) cm is anatomically too long to represent a forearm, the collar bone, or the shoulder (the meaning of the word *remen*) of a normal person. Carlotti (1995 - Annexe, 134) mentions the analogy between ancient Egyptian and Roman linear divisions. He does not mention the Roman *jugerum* as his article deals only with linear measurements and cubits. The *jugerum* in Roman metrology (Gaffiot, 1934, p.871) is both a unit of area (240 Roman feet long by 120 feet wide) and of length (104 Roman feet).

The same applies to the *remen*. As a measure of area it represents 1/2 of an *aroura* (Gardiner, 1988 [grammar], p. 200). Galán (1990, pp. 161-4) notes that the *remen* sign always appears after a rectangle in the Rhind Mathematical Papyrus problems 51 and 52, with the meaning of half a rectangle, in opposition to *gs* referring to the "abstract" calculated half of the same rectangle. An *aroura* is made of 100 cubits of land (*mḥ t3*) each 100 cubits in length with a depth of one cubit. 100 royal cubits in length multiplied by one cubit in depth give an area of 27.565 square metres or an area for each squared royal cubit of 0.2756 square metres. If the double *remen* is related to the great cubit, a similar area could be calculated using the great cubit divisions. The great cubit of 60 cm multiplied by the small cubit of 45 cm, belonging both to the great system, gives an area of 0.27 square metres, very close to the result obtained by a squaring a royal cubit. The hypotenuse of such rectangle equals 75 cm, the length of a double *remen*. This in turn explains Galan's remark: an *aroura* can be also divided into two triangles, hence the physical *remen* sign. The division of the *aroura* into two triangles is perhaps the reason why the triangular "tongue of land sign" Gardiner sign N 21 appears as a determinative after *3ḥt* (field).

The *remen* is followed in texts with either a *gs* sign or a rectangle determinative which indicates two possible methods of calculating an *arouras*.

The royal cubit confirms its role as a converter. A double *remen* divided into palm divisions of four finger divisions, following the 3/4 ratio appearing on the first palm division of the Amenemope cubit artifact, integrates the great system and gives the following divisions:

Table 9. Remen Divisions (Antoine Hirsch, 2012).

Number of Double <i>Remen</i> Divisions	Length Cm	Royal Palm Name
1	9.375	Hand
2	18.75	
3	28.125	
4	37.5	<i>Remen</i>
5	46.875	
6	56.25	
7	65.625	<i>Nbj</i>
8	75	Double <i>Remen</i>

5.2 *Nbj* and Sandal

The *nbj* is usually associated with the ceremony of the stretching of the cord where it does not carry any particular metrological meaning, at least during Ptolemaic times (Zába, 1953, p.59, Plate 1, line Aa; Wilson, 1997, pp. 501–2 ff.). The *nbj* is associated with heights in pAnastasi III, 5, 6-7 with “recruits of one *nbj* high” and with volume in ostraka from Sen-Mut’s tomb [Hayes, 1942, pp. 21–30 and 40] and in the Cenotaph inscription of Seti I (Frankfort, 1933, pp. 93–4, Plate XC, 1). It has a length of approximately 65 cm according to Roik (1993, p.5) and Hayes (1942, pp. 37 ff.). Such a length appears as seven hands on the double *remen* projection of Table 9.

The sandal (*tbt*) is mentioned in a graffito in the *mastaba* of Ptahshepses (Verner, 1981, pp. 479) and in Papyrus Reisner I (Simpson, 1963, Section G, Plate 13A, lines 16–8 and Section H, Plate 14A lines, lines 32–4). Verner (1981, pp. 479 ff.) gives the *tbt* a length of about 10 cm and Roik (1993, pp. 62-63) thinks it is a division of the *nbj* with a length of 8.125 cm). It is associated with volumetric calculations.

On the Amenemope cubit artifact, the great span of 14 fingers (26.25 cm) has no division. Doubling a great span gives a royal cubit of seven palms of four fingers. Dividing the great span by two would give a "palm" division 13.125 cm long. Three of these "palm" divisions correspond to the length of a royal cubit (52.5 cm), four to the length of a *nbj* 65.625 cm (respecting the 3/4 ratio appearing on the first palm of the Amenemope cubit artifact). The length of a sandal corresponds to the length of seven royal finger divisions or 13.125 cm. The only place where this division might be marked on the Amenemope artifact is over three lines: between A22 (with the special four stroke marker added by Lepsius to the left of the *Soped* glyphs) and before the division of a finger in 13 parts (C13). Perhaps such marker is not necessary as the length of a *nbj* is also given by seven hands of five fingers.

6 Conclusions from the Database

Summary

The great system seems to be more “productive” than the royal system limited to a maximum length of 52.1 to 52.9 cm in architectural projects but with exceptions. The next chapter, chapter 4, linking measures of length, volume, and weight in a closed ancient Egyptian metrological system may provide a more satisfactory answer to the purpose of the royal system and the royal cubit.

(1) The database (*database_1*) on the CD is a Microsoft Excel 2010 spreadsheet containing 3866 entries with a total of 4905 measurements (heights, lengths, widths, and depths) of measurable objects from museums in the world (mostly from Cairo). This is a statistical database including published measurements of unbroken objects from museum catalogues. Its purpose is to verify whether both the royal and great systems measurement patterns are common occurrences, regardless of the type of object measured. All objects and measurements are referenced by their catalogue number, catalogue name, volume, page, period, and when available, location, sub-location, and dynasty. The objects have been classified by periods from Predynastic to Roman and have been labeled Pre to 3 (inclusive) covering objects from the Predynastic to the Third Dynasty, Old Kingdom, First Intermediate Period, Early Middle Kingdom, Middle Kingdom (early?), Middle Kingdom, Second Intermediate Period, New Kingdom, Third Intermediate Period, Bubastite to Saite, Late Period, Ptolemaic, Late Ptolemaic-Early Roman, and Roman Period⁴. The majority of the entries are from the Predynastic to the

⁴ The statistical object categories included in the Database_1.xlsx spreadsheet table are: Alabaster block, Arrow point, Arrow-head, Axe, Axe head, Base of Bowl, Base of statue, Beads, Blade of adze, Blade of dagger, Blade of hoe, Block of stone, Boat, Bowl, Bracelet, Bull's hind leg, Cartonnage, Cartouche, Chest, Chisel, Cist, Clay-cone, Clay-seal, Coffin, Coffin inscription, Comb, Cone, Copper tool, Crescent, Cubit type used, Cubit type used in room, Cylinder, Dish, False door or part of, Figure, kneeling, Fish, model of, Flat dish, Flat disk, Flint, Flint chisel, Flint dagger, Flint flake, Flint knife, Flint scraper, Foot of bull, Foot of chair, Foot of chair leg, Foot of offering table, Foot of statuette, Fore limb of an articulated statuette, Forefoot of bull (model), Foundation Stone, Funerary inscription or part of, Grindstone, Hair pin, Harpoon, Hawk's head, Head, Head of statuette, Head rest, Hind leg of bull (model), Hippopotamus (model), Inner coffin, Inscription, Ivory object, Ivory plaque, Jar with sealing, Jar-sealing, Knife, Knob, Lid of jar, Lion, Mace-head, Mask, Mastaba or part of, Obelisk, Offering, Offering dish, Offering table, Offering trough, Outer coffin, Pad, Painting, Palette, Part of Statue, Part of cartouche, Part of coffin, Part of false door, Part of offering list, Part of sarcophagus, Part of temple wall, Part of tomb, Part of wall, Pebble, Pendant, Pin, Painting, Plaque, Polisher, Pot, Rattle, Relief, Relief or part of, Rod, Rod of faience, Rod of ivory, Round offering table, Rubber, Sarcophagus, Sarcophagus/Coffin, Statue, Saucer, Schist plaque, Seal, Seal of a jar, Sealing, Shell shaped pendant, Sign, Slag of pottery, Slate, Slate palette, Small chisel, Small tablet, Spinning- whorl, Spoon, Stand, Stand of offering table, Statue, Statuette, Stela, Stela/Inscription, Stone block, Strip of copper, Strip of thin copper, Thin plate, Three armed tool, Tile, Tool, Triangular plate, Trough, Tube, Vase, and Vase-stand.

Middle Kingdom Periods as the use of a royal cubit as a converter, according to textual references in the fifth chapter of this thesis, corresponds to this time period.

(2) The *database_6* spreadsheet contains 56 entries representing the lengths of 1 to 28 finger divisions of the royal cubit by increments of half a royal finger division up to 52.5 cm. The *database_4* spreadsheet contains the lengths of 1 to 40 royal finger divisions necessary to reach the length of a double *remen* (75 cm). It also contains the 120 lengths corresponding to the length of the great cubit (60 cm) of one palm division of five finger divisions (C24) divided into half finger divisions of 0.5 cm each. The lengths they contain have been applied to *database_2*, *database_3*, and *database_5*.

(3) The *database_2* spreadsheet contains the percentage of 3651 measurements⁵ matching finger divisions or divisions (without any delta adjustment) of either the scribe's palette, the royal cubit, the great cubit, or the double *remen*.

(4) The *database_3* spreadsheet contains the percentage of royal cubits used for "elite" objects in the database.

(5) The *database_5* spreadsheet contains the percentage of royal cubits used in the database at Abydos.

Two addenda (*Addendum 1* and *Addendum 2*) palliate the statistical effects of *database_1*. *Addendum 1* performs an analysis of the heights and widths of stelae in the British Museum contrasting the results given by the measurements of stela and inscriptions not clearly demarcated by register lines in the elite "Stela/Inscription" category in *database_1*. *Addendum 2* performs an analysis of the clearly defined items (51) of the "False door or part of" category of

⁵ 1244 entries in Database_1 do not match great, royal, or double *remen* measurements for several reasons. Measurements from broken artifacts or uncertain measurements were not entered in Database_1, but their references were to maintain the integrity of the catalogues used. Multiples of the scribe's palette, royal cubit, great cubit, and double *remen* have been discarded as trends are based on the maximum length of a double *remen* (75 cm) which includes the length of the scribe's palette, the royal cubit, the great cubit, the *nby*, and the *remen*. Finally, the rest of the entries (582) would require a "subjective" delta calculations between the royal and the great systems to match them to either system; they represent 12% of the 4905 total measurements and would not drastically change the trend results given by the analysis of the 3651 matched measurements.

database_1. Both addenda show the combined use of both the royal and great systems, labelled as "mixed," in single objects in both "elite" sub-categories.

Statistically, what is remarkable in the database of measurable everyday objects scattered around the world is that only 582 entries would require a subjective delta calculations between the royal and the great systems to match them to either system; they represent only 12% of the 4905 total valid measurements in the database. Such a small percentage ascertains the parallel use of the royal and great [?] systems before the Ptolemaic and Roman Periods.

6.1 First Conclusion

The royal cubit cannot be 52 to 54 cm long.

The length of the royal cubit (52.5 cm) can only vary between 52.1 and 52.9 cm. The lengths between 50 to 52 cm and 53 to 54 cm are explained by the great cubit C24. During the Ptolemaic and Roman Periods adapted and adopted length units may overshadow the great cubit.

6.2 Second Conclusion

The royal cubit as a converter may appear at the end of the Second Dynasty or the beginning of the Third Dynasty.

In the *database_2* Excel spreadsheet, there is no significant increase of the percentage of occurrences of the royal cubit from the Predynastic to the Middle Kingdom Periods⁶.

Table 10. Percentage of Royal Cubit Entries in Database_2 (Antoine Hirsch, 2012).

Period	Percentage
Pre to 3 (inclusive)	7.86%
Old Kingdom	9.93%
First Intermediate Period	17.24%
Middle Kingdom	8.24%

⁶ All the entries with the mention "shared" in the database have been treated as royal cubits or part of the royal system, although they are shared by both the "decimal" great and royal systems.

6.2.1 Examples

The Palermo Stone may partially answer when the royal cubit was first used as a converter.

6.2.1.1 The Palermo Stone

The Palermo Stone may indicate a change of cubit systems during the Old Kingdom. This change can be suggested, but not ascertained, from the sudden introduction of finger division fractions to record the heights of the Nile during the twelfth year of Nebka, if this twelfth year is indeed Nebka's. The stone is broken and what is attributed to Nebka's reign could correspond to his predecessor Khasekhemwy. The data of Table 11 are taken from W. Helck (1966, pp. 74–9), the de Cénival (1965, pp. 11–17), London (Petrie, 1916, pp. 114–20), and Cairo (Sethe, 1932, vol. 1, pp. 240–5) fragments.

Whether the records of the Nile heights are accurate is questionable: Nile heights are missing in the five years referring to the reign of Aha and in Djer's year two (Bell, 1970, p. 571).

Furthermore, the inscriptions may not be accurate, as they are in all likelihood transcriptions on stone of "field" records. Nevertheless, the important point in the Palermo Stone is the sudden transition to the use of finger fractions to record the heights of the Nile flood.

In Table 11, which is listing the Nile heights in the Palermo Stone, the black line shows the change of cubit divisions likely happening during the twelfth year of Nebka. There are two possible interpretations of the calculations:

- 1 A decision is suddenly made to record finger fractions, in which case the calculations in Table 11 are irrelevant.
- 2 There is a change in the cubit system. The cubit used, starting with Nebka's twelfth year, must be the royal cubit of seven palm divisions. The cubit used previously must have had divisions that match the length of a royal cubit (0.525 m) of seven palm divisions, otherwise the heights of the Nile could not be comparable between years and dynasties. The only possibility is seven palm divisions (52.5 cm) of three finger divisions each 2.5 cm long, corresponding to the length of a royal cubit divided into seven palm divisions of four finger divisions of 1.875 cm each. There is no mention of which span is used in year nine of Djer and years 31, 37, and 39 of Adjib. The cubit involved must have a span division that matches the length common to both systems which confirms the previous suggestion: only the small span (22.5 cm) is shared by both systems

(great and royal). The discrepancies between the heights using seven palms (52.5 cm) of C16, without the addition of the length of the fraction of finger divisions, and the heights using royal cubit, with the addition of the length of the fractions of finger divisions, are highlighted in grey in the Table 11. They are minimal.

Table 11. Palermo Stone Entries (Antoine Hirsch, 2012).

Palermo Stone Entries Legend

RC: Royal Cubit

Cubit X: Great Cubit – C 16

Dynasty	King	Year	Cubit	Span	Palm	Finger	Fraction	Cubit X	RC
I	Djer	1	6	0	0	0	0	3.150	3.150
		3	4	0	1	0	0	2.175	2.175
		4	5	0	5	1	0	3.025	3.019
		5	5	0	5	1	0	3.025	3.019
		6	5	0	1	0	0	2.700	2.700
		7	5	0	0	0	0	2.625	2.625
		8	6	0	1	0	0	3.225	3.225
		9	4	1	0	0	0	2.325	2.325
		(26?)	3	0	0	0	0	1.575	1.575
		I	Den	x+2	5	0	0	0	0
x+4	5			0	1	2	0	2.750	2.738
I	Adjib (?)	(28?)	3	0	1	2	0	1.688	1.688
		(29?)	4	0	1	0	0	2.175	2.175
		(30?)	8	0	0	3	0	4.256	4.256
		(31?)	3	1	0	0	0	1.800	1.800
		(32?)	5	0	2	0	0	2.775	2.775
		(33?)	5	0	1	2	0	2.750	2.738
		(34?)	4	0	2	0	0	2.250	2.250
		(35?)	2	0	0	0	0	1.050	1.050
		(36?)	5	0	0	0	0	2.625	2.625
		(37?)	4	1	0	0	0	2.325	2.325
		(38?)	6	0	1	2	0	3.275	3.263
		(39?)	2	1	0	0	0	1.275	1.275
		(40?)	3	0	5	2	0	2.000	1.988
		I	Semerkhet	1	4	0	4	0	0
3	4			0	0	0	0	2.100	2.100
5	4			0	4	0	0	2.400	2.400
6	4			0	0	1	0	2.125	2.119
7	5			0	0	2	0	2.675	2.663
II	Nynetjer	7	3	0	4	2	0	1.925	1.913
		8	4		0	2	0	2.150	2.138
		9	4	0	1	2	0	2.225	2.213
		10	4	0	4	0	0	2.400	2.400
		11	3	0	4	2	0	1.925	1.913
		12	4	0	0	3	0	2.175	2.156
		13	4	0	0	3	0	2.175	2.156
		14	1				0	0.525	0.525
		15	3	0	4	3	0	1.950	1.931
		16	3	0	5	2	0	2.000	1.988
		17	2	0	0	2	0	1.100	1.088
		18	2	0	0	2	0	1.100	1.088
		19	3	0			0	1.575	1.575

Dynasty	King	Year	Cubit	Span	Palm	Finger	Fraction	Cubit X	RC
III	Nebka (?)								
		4 (?)	3	0	5	2	0	2.000	1.988
		5	3	0	1	0	0	1.650	1.650
		6	1				0	0.525	0.525
		12	2		4	1	1/2	1.375	1.378
		13	2	0	3	1	0	1.300	1.294
		14	3	0	0	0	2/3	1.575	1.588
		15	2	0	6	2	1/2	1.550	1.547
		16	4	0	2	2	2/3	2.300	2.300
		17	4		2			2.250	2.250
III	Djoser (?)								
		1	4	0	2	2	2/3	2.300	2.300
		2	4	0	1	0	2/3	2.175	2.188
		3	2	0	3	2	3/4	1.325	1.327
		4	3	0	3	2	0	1.850	1.838
IV	Snofru								
		4 (?)	3					1.575	1.575
		5	3	0	5	0	0	1.950	1.950
IV	Schepeskaf								
		1	3	0	3	2	1/2	1.850	1.847
V	Userkaf								
		3	3	0	2	2	1/2	1.775	1.772
V	Sahure								
		4	2	0	2	0	1/4	1.200	1.205

6.3 Third Conclusion

The scribe's palette as a measuring artifact is used from the Old Kingdom onwards.

Table 12 shows no decrease of the use of the great system from the Predynastic to the Middle Kingdom Periods. The Canon of Proportion shows a steady use of the great system from the Old Kingdom onwards.

Table 12. Great Cubit Percentages (Antoine Hirsch, 2012).

Period	Percentage
Pre to 3 (inclusive)	92.14%
Old Kingdom	90.07%
First Intermediate Period	82.76%
Middle Kingdom	91.76%

6.3.1 Examples

6.3.1.1 The Canon of Proportion

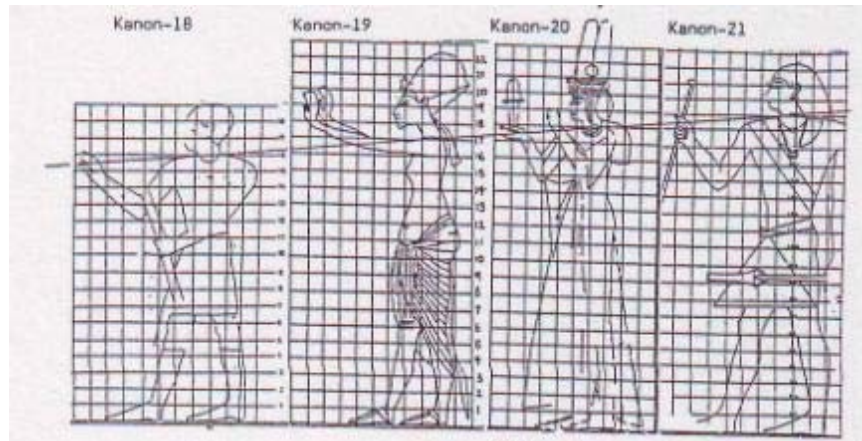
In the Canon of Proportion, grid squares are used to facilitate the representation of the proportions of the human body in reliefs of standing (or sitting) figures. The number of grid squares between the sole of the foot and either the hairline of the forehead, or the root of the nose, determines the canon used. The difficulty with the Canon of Proportion is to determine which linear metrological system is used, as conflicting measurements and interpretations do not seem to yield a comprehensive system. The Canon of Proportion is more likely related to the scribe's palette rather than the royal cubit from the Old Kingdom onwards.

Iversen (1975, pp.38–87), following Lepsius (1884, p. 99), recognizes two main canons: the “classical” Canon 18 (18 squares from the sole to the hairline of the forehead) in standing figures (14 squares in sitting figures) from the Old Kingdom to the Twenty-sixth Dynasty; and the “late” Canon 21 (21 squares from the sole to the root of the nose) in standing and sitting figures. Robins expands the definition of the two main canons (18 and 21) by adding two other canons, 19, and 20 (19 or 20 squares from the sole to the hairline of the forehead) to the previous list based on grids dating from:

- Eighteenth Dynasty 20 squares (Robins, 1984a, pp. 21–32; 1983, pp. 91–95).
- Amarna Period 19 squares (Robins, 1983, p.54; 1983b, pp. 67–72).
- Amarna Period 20 squares (Robins, 1983b, pp. 67–72, 1985b p. 54; 1985b, p.56.).
- Roik (1993, pp. 138–167) wants to add the *n_{bj}* to the Canon of Proportion. She gives the following number of squares (the number of squares between the sole and the hairline of the forehead in the artistic relief representation of standing or sitting male and female figures):
 - Old Kingdom from 16 squares to 18 squares (Roik, 1993, pp. 148–63) for standing figures.
 - Middle Kingdom 18 squares (Roik, 1993, p. 140, Table 13) for standing figures.
 - Eighteenth Dynasty 20 squares (Roik, 1993, p.140, Table 13) for standing figures.
 - Amarna Period 19 squares (Roik, 1993, p.140, Table13) for standing figures.

- Nineteenth Dynasty 20 squares (Roik, 1993, p.140, Table 13) for standing figures.
- Twenty-sixth Dynasty 21 squares (Roik, 1993, p.140, Table 13) for standing figures.

Figure 9. Representation of the Canon of Proportion (Roik, 1993, p. 163).



The relationship between the different canons and linear metrology has been the subject of different interpretations. Lepsius never concluded that there was a direct link between canon and metrology (Lepsius, 1884, p.99). Iversen, Robins, and Roik think, and rightly so, that there is a relationship between the two. Iversen and Robins see the relationship based on the cubit, but do not agree on the canonical length of the forearm from elbow to outstretched fingers representing the metrological length value of the cubit. Robins thinks it is one small cubit of 45 cm (Robins, 1983a, p. 20); for Iversen, it is one royal cubit of 52.5 cm (Iversen, 1990, p. 46).

Five squares are used to represent the forearm from elbow to the stretched fingers (Robins, 1983a, p.20). Five squares with a total length of 45 cm give a length of 9 cm (Robins, 1983a, p.20) for the side of one square, while Iversen gives ten cm as the length of side of the same square (Iversen, 1990, p.123). Roik wants to relate the length of side of the grid square to 1/8 of a *nbsj* of 65 cm, or 8.125 cm, following her interpretation of the early canon of the Old Kingdom (Canon 16) based on the Narmer Palette (Roik, 1993, pp. 147–60) and her interpretation of the canon of the tomb of Ta-Useret (Roik, 1993, p. 140, Table 18).

The length of a side of a square according to Iversen is ten cm (Iversen, 1990, p.123). In Iversen's classical canon 14 squares have a length of 140 cm, 18 squares measure 180 cm, and

21 squares equal 210 cm. The scribe's palette is 50 cm long and is divided into ten "palms" of 5 cm each; consequently, 14 squares equal 14 scribe's palette double palm divisions, 18 squares equal 18 scribe's palette double palm divisions, and 210 cm equal 21 scribe's palette double palm divisions. In other words, ten cm, the length of a grid square according to Iversen, corresponds to two palm divisions of the scribe's palette. As ancient Egyptian scribes were "drafting" texts and grid squares before they were carved out, it seems more logical to associate the Canon of Proportion with a scribe's basic instrument (the palette) rather than a cubit (small or royal), or a *nj*.

6.3.1.2 Brick Dimensions

The difficulty with brick dimensions is the ambiguity resulting from approximate measurements and decay. Spencer writes:

"The vast majority of bricks sizes recorded from Egypt are only measured to the nearest cm, or half cm at best, and until more accurate values are obtained these dimensions have to suffice . . . The basic pattern of Egyptian brick sizes is as follows: the bricks of the Archaic Period are all small, then comes an increase in size until the Middle Kingdom, followed by a fluctuation until the Twenty-sixth Dynasty, after which there is a decrease until modern times." (Spencer, 1979, pp. 147 ff.)

The ratio between brick length, width, and height remains, in theory, constant throughout the ages: one for length, one half of the length for width, and 1/3 of the length for height (Spencer 1979, pp. 147 ff.). In practice the Variations column of Table 13 below does not necessarily confirm Spencer's statement. There is another ambiguity: the dimensions of bricks according to the Kahun Papyri (Griffith, 1898, p.59) are given in palms; do these palms reflect the width of the wall of the mold used to make the bricks? In other words, when the Kahun Papyri mention bricks of 5 or 6 palms, is it necessary to include or exclude the width of the brick mold from the brick dimensions? Furthermore, which system are the palms referring to? Six palms in the royal system equals a small cubit related to the great cubit system. Is this an indication that the system retained is the royal system of seven palms of four fingers expressing great cubit lengths? Five palms in the great system correspond to the length of a scribe's palette. The examples published

by Roik (1993, p. 347, Table K-21) illustrate her idea, in the Average column, of relating brick dimensions to *nj* palms of 8.125 cm, with the objection that the *nj* does not appear as a metrological unit before the Eighteenth Dynasty. The association of bricks with the great system can be found in J. Wegner's book where bricks of 25 cm * 15 cm * 10 cm (Wegner, 2007, p. 61 – note 21; p.62), of local origin, are a clear exception to all other mortuary temple measurements which follow royal standards.

Table 13. Brick Dimensions – Roik Table K-21.

Reference	Period	Groups	Building type	Average (cm)	Variations (cm)
A	Predynastic	Group 1	General	24 x 12 x 8	21–28 x 11–14 x 5.5–8
D	Old Kingdom	Group 1	Mastaba niches	24 x 12 x 8	22–28 x 10–13.5 x 5–8.5
B	First Dynasty	Group 1	Palace facades and Mastabas	24 x 12 x 8	21–29 x 11–14 x 5.5–8
C	To First Dynasty	Group 1	Palace facades and Mastabas	24 x 12 x 8	20–24 x 10–13 x 7–8.5
B	First Dynasty	Group 4	Palace facades and Mastabas	16 x 8 x 6	15–17.5 x 5–9 x 5–7
P	Palaces (in general)	Group 3b	Palaces	40 x 20 x 12	38–43 x 18–22 x 11.5–14
Q	Up to Dynasty 12	Group 1	Private Buildings	24 x 12 x 8	25–26 x 10.5–13 x 6–7
Q	New Kingdom-Amarna	Group 2b	Private Buildings	32 x 16 x 8	30–34 x 14–17 x 8–10
R	Dynasty 21 to 30	Group 2b	Private Buildings	32 x 16 x 8	30–33 x 15–18 x 7–9
S	Ptolemaic-Roman	Group 2b	Private Buildings	32 x 16 x 8	29–34 x 12–18 x 7–11

The only conclusion that can be drawn from Table 13 is that both the great and royal systems are parallel systems as they can be applied to brick dimensions, making compulsory an *in situ* analysis, building by building. The only references that might be considered purely royal are reference B on account of the length of the royal double palm (15 cm) and of the royal hand (9.375 cm), reference Q which goes against all expectations, as Q is associated with private buildings, on account of the sandal of 13.125 cm and the royal palm of 7.5 cm. Reference S could be purely Ptolemaic and linked to the *Mh Ntr* which replaces the royal cubit. If one accepts Roik's average, Reference P corresponds to the divisions of a scribe's palette.

The existence of bricks following the great system can be found in the book published by Wegner (2007, p. 61 – note 21; p.62) already mentioned. Two sets of brick dimensions are used in the Mortuary Temple of Senwosret III at Abydos. The dimensions of the first set are 11.5-12 cm multiplied by 19-20 cm multiplied by 38-39 cm which he considers royal and translates into five royal palm divisions multiplied by two and a half royal palm divisions multiplied by 1.5 royal palm divisions. The second, local, set follows the great [ʕ] system and the scribe's palette with dimensions of 25 cm multiplied by 15 cm multiplied by 10 cm.

6.4 Fourth Conclusion

There is no real evidence of a specialized use of the royal cubit in the database.

According to Carlotti (1995 - Annexe, p.128) the use of the royal cubit is limited to architectural projects and the construction of the monuments themselves. There are also exceptions to Carlotti's theory, a few examples of which are listed below. *Database_4* lists in an Excel spreadsheet the percentages of royal cubit used in the measurements of stelae and inscriptions considered as "elite" artifacts. Even if the percentage increases from the Predynastic Period to the Middle Kingdom in Table 14, it is not what one would qualify as a specialized use of the royal cubit.

Table 14. Elite Royal Cubit Percentages (Antoine Hirsch, 2012).

Period	Percentage
Pre to 3 (inclusive)	6.25%
Old Kingdom	10.33%
First Intermediate Period	15%
Middle Kingdom	15.12%

6.4.1 Examples

6.4.1.1 Exceptions In Carlotti's And Cauville's Publications

Carlotti (1995, pp. 78, 86, 89) publishes the following lengths:

- 4.8 m for the height of the White Chapel of Sesostris I (Middle Kingdom, p. 78) which translates into eight great cubits of 60 cm rather than nine and $\frac{1}{4}$ royal cubits.
- 66.60 m for the width of the Tenth Pylon at Karnak (New Kingdom, p.83) which translate into 111 great cubits of 60 cm rather than 125 cubits of 0.532 m. A length of 0.532 m is not really royal as the definition of the royal cubit, according to this thesis, varies between 52.1 and 52.9 cm.
- 3.01 m for the E (axis dimension) length of the column of the North-East room of the *Akh-menu* of Thutmoses III (New Kingdom, p.89) which translate into five great cubits.
- In Cauville (1990, p. 84, Note 5, column 1) the length of the Edfu and Denderah Naos is given as 60 m and the length of the exterior wall of the Edfu temple is given as 120 m, translating into 100 and 200 great cubits of 60 cm respectively. Zignani retains a length of 53.4 cm for the cubit used at Denderah. This length corresponds to the length of the artifacts published by Legrain (see excursus b – division evolution).

6.4.1.2 Dressing Blocks In the Pyramid of Djoser

In his publication of Djoser's Step Pyramid, Lauer (1936, Tome 1, p. 26) has the following two illustrations in which none of the dimensions match the royal system. 2.5 cm is the basic finger length of the scribe's palette and great cubit system. All dimensions belong to the great system

with the exception, perhaps, of 65 cm which could be the length of five sandals (*tbwt*) equal to the length of the Eighteenth Dynasty *nbt*.

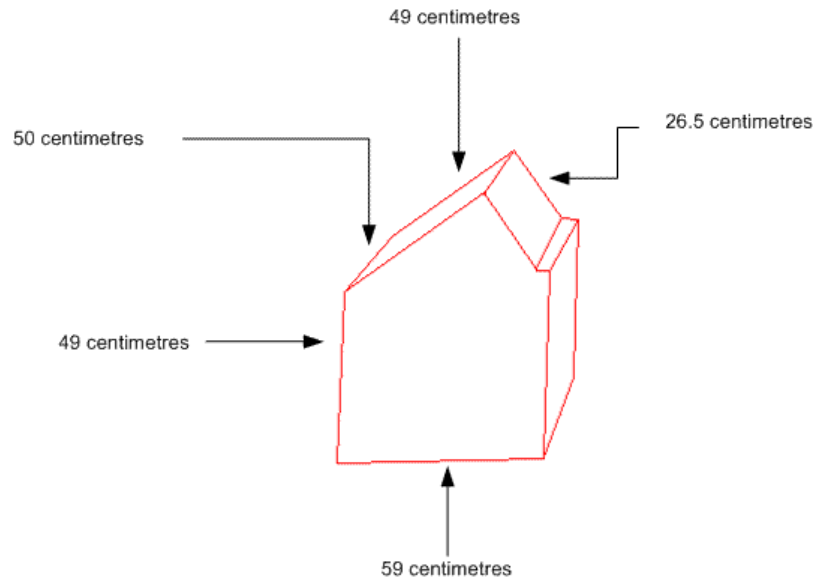


Figure 10. Block 1 (Redrawn from Lauer, 1936, Tome 1, p.26).

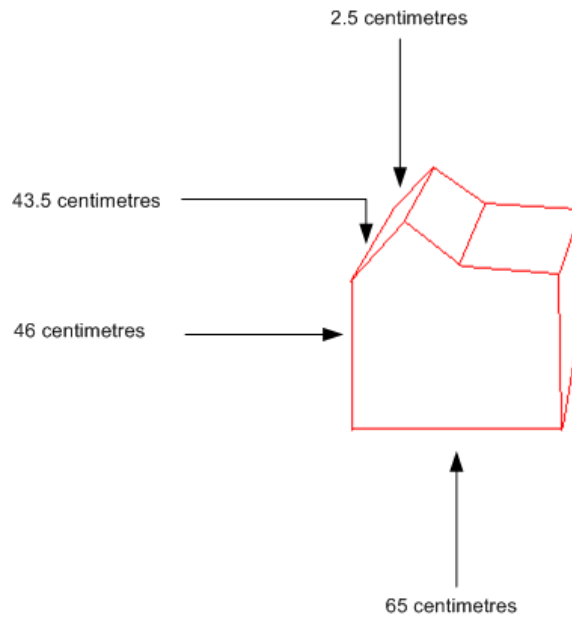
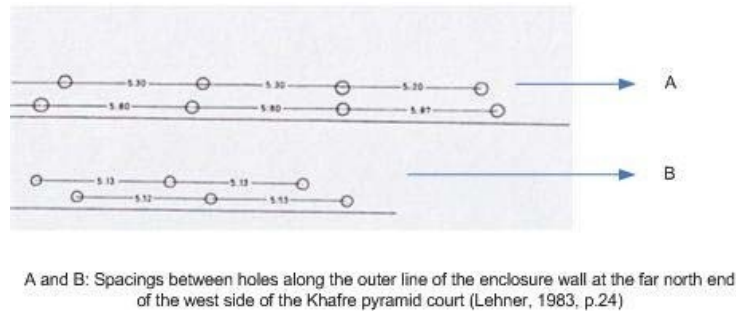


Figure 11. Block 2 (Redrawn from Lauer, 1936, Tome 1, p.26).

6.4.1.3 The Northern Enclosure Wall of Khafre's Pyramid.

The simultaneous use of the royal and great cubits is illustrated in the lengths found between holes along the outer line of the enclosure wall at the far northern end of the west side of Khafre's pyramid (Lehner, 1983, pp. 9 and 24). In the first set (figure 10) the cubits used are the great and royal cubits, the distance between the holes varies between 52 and 53 cm (royal system) and 58 and 59 cm (great system); in the second set (figure 11) the system is the great system (51 cm).

Figure 12. Lehner's Lengths in Khafre's Pyramid Northern Enclosure Wall (Lehner, 1983, p. 24).



6.4.1.4 The Pyramid of Amenemhet III

Dieter Arnold (1987, p. 28) publishes dimensions of the pyramid complex of Amenemhet III. Among them, Room 08 (Section 1.2.9) is one of the many measurements that matches more closely the great cubit system than the royal system.

Table 15. Pyramid Complex of Amenemhet III – Room 08.

Room 08	Dimension m	Number of Great System Units	Number of Royal Cubits
Length	3.89	6 and a half	7 cubits 3 Hands
Width	2.25	4 and a half scribe's palette	4 cubits 2 Hands
Height	2.75	5 and half scribe's palette	5 cubits 2 Hands

The great system solution in Table 15 offers a more practical approach for calculating dimensions than the royal cubit system: halving either a great cubit or a scribe's palettes is in conformity with ancient Egyptian mathematics principles.

The Amenemope royal cubit artifact black and white divisions match the basic division numbers found on the Denderah cubit published by Legrain: 3, 4, 5. In this chapter, using the Amenemope cubit artifact alone and its unique colour scheme it has been possible to suggest that the royal cubit is a converter between two systems: the royal and the great systems. The great system is materialized by the scribe's palette used as a measuring device. Both systems are used in parallel in ancient Egypt. As far as linear measurements are concerned, the great system seems to be more "productive" than the royal system limited to a maximum length of 52.1 to 52.9 cm in "official" architectural projects but with exceptions. The justification of the conversion must therefore be found elsewhere. The next chapter, linking measures of length, volume, and weight in a closed ancient Egyptian metrological system may provide a more satisfactory answer to the purpose of the royal system and the royal cubit.

Chapter 4 - The Origin and Evolution of Ancient Egyptian Cubits

Summary

The previous chapter of this thesis has suggested that the royal cubit was a converter between two systems, the great and royal systems. Isolating both systems and testing them against the database included with this thesis has also suggested that the royal cubit, as a measuring device, was less "productive" and common than the great system materialized by the scribe's palette. The royal cubit appears to be restricted, with exceptions, to architectural projects and the construction of buildings (Carlotti - Annexe, 1995, p. 128). Such a limited scope does not justify two linear systems but provides a clue. Major architectural projects are the domain of the state, therefore the hypothesis that the royal system is a legacy administrative system needs to be explored.

Wilkinson (1999, pp. 125-8) mentions that the organization of the treasury responsible for grain taxation seems to have taken a long time to be unified and centralized. It is only under Sekhemkhet that it becomes, and stays, the *Prwj ḥd* reflecting a consolidation of two previously separate institutions in charge of taxes for the two halves of the country the *Pr ḥd* of Upper Egypt and the *Pr dšr* of Lower Egypt. According to Brewer (Brewer, Redford and Redford, 1994, p. 28, col. 2) During the Old and Middle Kingdoms barley was the predominant cereal. Emmer was second until the relative importance of the two cereals was reversed in the New Kingdom. From the data in *Excursus A*, there is no real evidence that barley was cultivated before emmer, or emmer was cultivated before barley. The final organization of the treasury during the Third Dynasty and the predominant role of barley during the Old Kingdom is enough to create an official legacy metrological system. In turn this explains the

Ancient Egyptian linear measurements are linked to agriculture: from the Rhind Mathematical Papyrus, the volume of a *ḥ3r* sack equals the volume of 2/3 of a cubed royal cubit. For tax purpose *ḥ3r* sacks contain either emmer or barley. Emmer is 1/3 lighter than barley, consequently, for an equal weight, a container filled with emmer will be 1/3 greater than a container filled with barley. The lengths derived from both containers share the same 1/3 ratio: the great (ꜥ) finger division is 1/3 longer than the royal finger division. The great (ꜥ) system is therefore an independent linear system linked to the cultivation of emmer while the royal system is linked to the cultivation of barley.

conversion role of the royal system: the great (ϵ_3) system is based on emmer, the royal system is based on barley. The specific weights of the two commodities follow the two thirds principle of ancient Egyptian mathematics: barley is $1/3$ heavier than emmer which implies that for an equal weight of emmer and barley, the container containing emmer will be $1/3$ greater than the container containing barley. In a closed metrological system this $2/3$ ratio has major consequences on the lengths derived from volume units, particularly when the length of one great finger division is greater by $1/3$ than the length of a royal finger division as mentioned in Chapter 3 – *The Royal Cubit as a Converter and The Scribe's Palette as a Measuring Device*. For an equal weight, the size of a container filled with emmer will be greater than the size of the container filled with barley. This thesis retains 440 grams per litre for emmer and 660 grams per litre for barley (see Excursus A) and will suggest that the origin and evolution of ancient Egyptian linear metrology is agriculture. The following sections weight and volume related lengths, area related lengths, and volume related lengths will test this theory in the context of a closed metrological system and provide a history of the origin and evolution of the great, royal, Ptolemaic, and Roman linear systems.

7 Weight and Volume Related Lengths

7.1 Middle And Old Kingdom

Summary

This section must, unfortunately, use Middle Kingdom references before Old Kingdom data as the definition of a great *deben*, a small *deben* and a ($\underline{h}3r$) sack does not appear before the Middle Kingdom. The closed metrological system principle applied in this section is based on a weight unit related to a volume unit which in turn defines a length unit. With the weight of two *debens*, (90 and 140 grams), the volume of a Middle Kingdom ($\underline{h}3r$) sack, 96 litres, the number of *debens* in a ($\underline{h}3r$) sack, and the specific weights of two commodities, emmer and barley, filling the ($\underline{h}3r$) sack, it is impossible to match the weight of the two debens using a Middle Kingdom ($\underline{h}3r$) sack.

This is an indication that the weight definition of the two debens is anterior to the Middle Kingdom. There is no evidence that a sack was used during the Old Kingdom. During the Old Kingdom, a commodity, the *sft* oil, is associated with 300 hekats which is a specialized grain

volumetric unit. 1/10th of this special container of 300 *hekats* correspond to 30 *hekats* which is the volumetric definition of a royal cubit. The same *sft* oil is also associated with a second container, a *dw.t*, 100 of which (220 to 250) litres are linked, at a later date, to a well-known container in Ugarit a *kurru*. Linear measurements are associated with volumes in vase inscriptions of the second and third dynasties. Why cannot the cubic root of 450 *hins* or 100 *dw.t*, circa 60 cm – the length of the great cubit – be the origin of the great system, particularly when 220 to 250 litres, filled with emmer or barley give the weights of the small and the great *debens* which the Middle Kingdom (*h3r*) sack fails to provide?

Furthermore 2/3 of 450 *hins* give 300 *hins* which is the volume of a cubed royal cubit or 30 *hekats* which are 1/10th of the Old Kingdom specialized grain volumetric unit. This estimation in *hins* fits the ancient Egyptian method: the Ptolemaic pBritish Museum 10399 problems 42 and 43 (Parker, 1972, p. 57) relates, in a genuinely ancient Egyptian manner, length of cubits and *hins*.

Following publications by Petrie (1883, 419-27), and studies by Weigall (1908, pp. i-xvi) and Hemmy (1937, pp. 39–56), M-A. Cour-Marty (1990, p. 21; 1985, pp. 189–95; 1983, pp. 27–30) distinguishes two series of weights and three phases during which these two series appear and disappear (Cour-Marty, 1990, pp. 21–4):

- Phase 1-From the Old Kingdom to the Eighteenth Dynasty, when series of weights of 12, 14, 90, and 9 grams coexist. The first series (12 – 14 grams) correspond to 13 gram Predynastic weights found at Naqada (Petruso, 1981, p.44).
- Phase 2-From the Eighteenth Dynasty to the end of the Twenty First Dynasty, when only the *deben* of 90 grams and its division, the *kedet* of 9 grams, are used.
- Phase 3-From the Twenty Second Dynasty to the Roman Period, when the *deben* and *kedet* are used in parallel with other Near Eastern standards, before disappearing and being replaced by Roman standards.

The existence of two *debens* during the Middle Kingdom is attested in pBoulaq 18 (Thirteenth Dynasty-Scharff, 1922, pp.20–1, Plate XLIII): a *deben* ³ and a *deben* *šrj*. Both debens are linked to sacks (*h3r*):

Table 16. pBoulaq 18

Line	Transliteration	Translation
Line 9	<i>prt dbn ʕ3 ʕnw ḥ3r 1</i>	Seed: great <i>deben</i> of (lime)stone, 1 (<i>ḥ3r</i>) sack.
Line 10	<i>prt (šn n ?) dbn ʕ3 [ḥ3r] 1</i>	Seed: 1 great <i>deben</i> , 1 (<i>ḥ3r</i>) sack.
1 (the <i>šn n</i> is difficult to understand)		
Line 12	<i>q33w (dqrw) dbn šrj ḥ3r 6</i>	Mixed grain (fruit): small <i>deben</i> , 6 (<i>ḥ3r</i>) sacks.

The Rhind Mathematical Papyrus, dated by Spalinger to the late Hyksos Period but with close connection to the Middle Kingdom (Spalinger, 1990, p. 337) gives a clear volumetric definition of a sack (*ḥ3r*), based on volume and length associating cubits and sacks. The calculation methods to obtain the volume of a sack are similar to our modern methods, whether it is applied to cubes, parallelepipeds, or cylinders (Peet, 1923, Plate N; Robins & Shute, 1987 pp. 44–6). Problem 44 has been retained for calculation comments as it represents a cube, the volume of which is very simply obtained by the multiplication of a length (10 cubits) by a width (10 cubits) by a height (10 cubits).

Table 17. Rhind Mathematical Papyrus Problem 44

Transliteration	Translation
<i>tp n njs š3ʕ ifd n 10</i>	Method of reckoning a square container of 10
<i>m 3w=f 10 wsh=f 10 k3w=f 10</i>	its length consisting of 10, its width 10, and its height 10.
<i>pty ḥ33.t r=f m ššrw</i>	What is its volume in grain?
<i>w3ḥ m 10 sp 10 ḥpr.ḥr=f m 100</i>	Multiply ten by 10, it becomes 100
<i>w3ḥ-tp m 100 r sp 10 ḥpr.ḥr m 1000</i>	Multiply 100 ten times, it becomes 1000
<i>Jrj.ḥr=k gs n 1000 m 500</i>	Take half of 1000, that is 500
<i>ḥpr.ḥr=f m 1500</i>	It becomes 1500

Transliteration	Translation
$hr.t=fpw m h3r.w$	This is its content in sacks.
$jrj.hr=k 1/20 n 1500$	Take one twentieth of 1500
$hpr.hr=f m 75$	It becomes 75.
$h33.t pw r=f m hk3.t s3rw hk3.t 7500$	This is the volume that will go in it in quadruple hekats i.e. 7500 quadruple hekats of grain.

The Middle Kingdom ($h3r$) sack has a volume of 96.5 litres⁷ or 20 *hekats*. 96.5 litres of emmer weigh 42.46 kilograms (96.5 multiplied by the specific weight of emmer: 440 grams per litre) and 96.5 litres of barley weigh 63.69 kilograms (96.5 multiplied by the specific weight of barley: 660 grams per litre). There are ten *hins* per *hekat* (Pommerening, 2005, pp.74–5). According to Segrè (1928, p.13) there are 1000 *debens* in a ($h3r$) sack. The weight of this *deben* would be either 63.69 grams based on a sack of barley (63690 grams divided by 1000 *debens*) or 42.46 grams based on a sack of emmer (42460 grams divided by 1000). This conflicts with the Old and Middle Kingdoms weight values mentioned by M-A Cour-Marty.

The word *hin* appears in the Third Dynasty (Kahl, 2003, p. 279) and the association of *hins* and sacks ($h3r$) is common from the Middle Kingdom onwards (Schack-Schackenburg, 1900, pp. 135–40; 1902, pp. 65–6; Eyre, 1980, pp. 108–19; Spalinger, 1992, pp. 87–94). In the Rhind Mathematical Papyrus *hins* are also mentioned in problems 80 and 81 (Gentet and Sweydan, 1992, pp. 179) with the already mentioned ratio of ten *hins* to the *hekat*. They also appear in the pKahun LV, 4 (Gillings, 1982, pp. 162–65) with the same ratio of *hins* per *hekat*.

7.1.1 The 2/3 Ancient Egyptian Mathematics Principle Applied to Sacks and Cubits In the Middle Kingdom.

The 2/3 ancient Egyptian mathematics principle is respected in the Rhind Mathematical Papyrus problem 44: 1000 cubed royal cubits correspond to 1500 $h3r$ sacks; in other words the volume of

⁷ The cubit root of 96.5 litres gives the length of a small cubit, linking the Middle Kingdom sack to the great system.

one *h3r* sacks corresponds to the volume of two thirds of a cubed royal cubit (Griffith, 1892, p. 421; Pommerening, 2005, p. 137).

The 2/3 ratio of barley to emmer is confirmed by Letter III of the Hekanakhte Papers where one jar of oil could be exchanged for a weight of either barley or emmer expressed as two volumes of barley, or three volumes of emmer. (Allen, 2002, p.18). The Hekanakhte Papers Account V provides a calculation of sacks (*h3r*) of barley and emmer entrusted by Hekanakhte to his steward Merisu (James, 1962, Plate 10A, Allen, 2002, Account 5 [MMA 22.3.520 Recto]).

Table 18. Hekanakhte Papers-Account V

Line	Transliteration	Translation
1	<i>h3t-sp 5 3bd 2 smw sw 9</i>	Year 5, month 2 of summer, day 9
2	<i>ss n jt mhj n hk3nht swd.n=f n mrj.sw</i>	Writing of the barley of Lower Egypt of Hekanakhte that he has entrusted to Merisu
3	<i>jt mhj m3c 112</i>	New barley of Lower Egypt 112
4	<i>bd t m3c.t 63</i>	New emmer 63
5	<i>jt mhj 10</i>	Barley of Lower Egypt 10
11	<i>m3w=f n k3w jt mhj fqur bdt 10.5</i>	Its produce for the bulls: barley of Lower Egypt 4, emmer 10.5
12	<i>jr m jt mhj jrj.n hk3nht n hwtjw=f</i>	Consisting of barley of Lower Egypt that Hekanakhte has made for his farmers
13	<i>S3.hwt-hr 46</i>	Sihathor 46
14	<i>Mrj.sw 50</i>	Merisu 50
15	<i>S3-nb-njw t 46</i>	Sinebniut 46
17	<i>12</i>	12

James (James, 1962, pp. 56–57) balances the number of sacks in account V by “translating” the number of sacks of emmer using a ratio of three parts of emmer for to two parts of barley (Table 19).

Table 19. Amount of Sacks (James, 1962, pp. 56-57).

Account V-Balance of totals according to James

Account V, 1-First Part

line 3	New barley	112 (khar)
line 4	New emmer 63 (khar)	In terms of barley 42 (khar)
line 5	[Old barley]	10 (khar)
Total		164 (khar)

Account V, 1-Second Part

line 11	Barley	4 (khar)
line 11	Emmer 10.5 (khar)	In terms of barley 7 (khar)
line 13	Barley	46 (khar)
line 14	Barley	50 (khar)
line 15	Barley	46 (khar)
line 17	Barley	12 (khar)
Total		165 (khar)

This relationship between two commodities (barley and emmer) weight and volume explains the mention of two *jpt* measures in the Hekanakhte Papers. In letter III, 6 (James, 1962, Plate 8A, Allen, 2002, Plate 34) it is written $rdj.n=j\ jn.t=sn\ t3\ jp.t\ h3.t\ s.t\ jm=s\ jw=s\ ^c n.tj\ hn.t\ km.t$ (“I have arranged that they bring the measure with which it should be measured (weighed). It is adorned with black hide”). The $^c n.tj\ hn.t\ km.t$ is interesting. The mention of hide is perhaps a coincidence, but it fits the determinative of a sack of *sšrw* used throughout the Rhind Mathematical Papyrus, the hide (Gardiner’s sign list F27), particularly when in the Hekanakhte Papers, the sign used for a *h3r*, when it is mentioned, is Gardiner’s sign list V19.

A great *jpt* measure is mentioned in the Hekanakhte Papers: $m\ h3t\ m\ jpy.y\ ^c3[.t]$ as measured (weighed) by the big measure (James, 1962, Plate 13A, Allen, 2002, Plate 44), implying the existence of a “regular” *jpt* measure during the Middle Kingdom. James considers this $jpy.y\ ^c3[.t]$ as a double sack (James, 1962, p.64) while Allen takes it as a full sack of ten hekats, giving a volume of four hekats for the “regular” *jpt* measure. It is perhaps better to relate the volumes of

the great and “regular” *jpt* measures to the specific weight of the commodities they contain in the Hekanakhte Papers: barley and emmer. The *ipy.y* ʿ3[.t] is in volume one third bigger than the “regular” measure, respecting thus the two thirds ratio of volume and weight between barley and emmer. Furthermore *h3j* (*h3t* used in both instances where the word *jpt* is mentioned), as Allen points out (Allen, 2002, p.50) quoting WB III, 223, 9–11, carries more a notion of weighing than measuring, confirming the weight and volume relationship mentioned in the weight calculations of a Middle Kingdom sack (*h3r*) above.

7.1.2 Old Kingdom Volumetric Calculations - The 2/3 Principle of Ancient Egyptian Mathematics.

Very little is known about the volume of an Old Kingdom sack, or even if a *h3r* sack was used. The Abusir papyri (Posener-Kriéger, 1976, pp. 378; 408–9 note 2) do not mention (*h3r*) sacks. In account 51 2 b and 95 A b, the *dmd* sign is used for the transportation of grain and charcoal, which Posener-Kriéger relates to *tm3* (WB V, 307, 15–17), a sack for cereals and or fruits, usually written with the \dagger determinative. Pommerening mentions one container in the Old Kingdom, the *dw.t*, used for *sft* oil and meat (Pommerening 2005, pp. 75-78) with a volume of 2.2/2.5 litres⁸. Tomb inscriptions dating from the Fifth Dynasty up to the Twelfth Dynasty mention reckonings in 50, 102, 103, 105, 104, 1000, 200, and 23600 *dw.t*. (Pommerening, 2005, pp. 76–7). A container of 220/250 litres corresponds to 100 *dw.t*. Lacau and Lauer have published inscriptions found on vases of the Second and Third Dynasties recovered from Djoser’s Step Pyramid showing finger, palm, and *djeser* corresponding to their respective volumetric capacities (Lacau and Lauer, 1965, Tome V, pp. 22–8). This is one rare example where linear measurements seem to be associated with volumes⁹.

⁸ For the volumetric reconstruction, see Pommerening, 2005, p. 77, for the inscription sources, see Pommerening 2005, pp. 76-77.

⁹ What is interesting is the volume given for 1 “*oipe*” (*jp*) associated with a *remen* in #43. This “*oipe*” (*jp*) would give 52734.37 cubic cm from a cube the side of which equals the length of a *remen*. Four of these *oipes* represent 210.94 litres, within the volumetric range of 100 *dw.t*. Four *oipes* is the definition of the volumetric content of a New Kingdom (*h3r*) sack. Pommerening (2005, p. 73) is reluctant to associate linear divisions with volumetric divisions during the Old Kingdom. She suggests three alternative readings of the inscription. The inscription itself is written on a piece of a broken vase which she considers to be, probably rightly so, a cylinder physically unable to contain 52734.37 ccm. She, however, has no problem

Table 20. Lacau's Vase Volumes (Lacau and Lauer, 1965, Tome V, p. 23).

Reference	Divisions	Fraction	Diametre (cm)	Height (cm)	Volume (ccm)
No 36					
#1			29.2	53	34989.80
#2			28	62	38157.28
#3			26.4	53	28997.02
No 37					
#1	1 <i>Djeser</i>	1 Palm	30	7.5	5298.75
#2	1 <i>Djeser</i> (?)	2 Fingers	30	3.75	2649.38
#3	1 <i>Djeser</i> one Finger	3 Fingers	31.875	5.625	4486.35
#39 bis	1 <i>Remen</i>				52734.37
# 43	1 <i>Oipe</i>				

Volumes of 220/250 litres are common in the ancient Near East: at a later date (corresponding to the New Kingdom): a *kurru* of 220/250 litres can be found in Ugarit. Barley is measured in *kurru*. A *kurru* was divided into 300 *qû* (Heltzer, 1978, p.72); 1.2 *qû* almost equal 1 litre, making the volume of a *kurru* equivalent to 250 litres (Heltzer, 1976, p.41). There is also mention of a *dd* (Heltzer, 1987, p.109) the capacity of which is unknown, but is probably linked to the Egyptian *djdj* or *ddwt* (Hannig, 2001, pp. 970, 990), and perhaps also linked to the Egyptian oil and wine

relating royal cubit divisions to their volumetric capacities (2005, pp. 271-276). In principle, why deny the simple reading of 1 *remen* and 1 *oipe* and associate only the royal system to volumetric capacities?

container *dw* or *dwjt* (*dw.t*) (Hannig, 2001, pp. 1000–1) if the assimilation of the Ugaritic *dd* to *d* is possible.

Sft oil is also reckoned in *hekats* (300) in the tomb of *Jrj* (Sixth Dynasty or First Intermediate Period [Pommerening, 2005, p. 115 quoting Jéquier, 1933, 41]). According to Pommerening, 300 *hekats* is a specialized grain volume unit associated with *Sft* oil (Pommerening, 2005, p. 115); one tenth of it (30 *hekats*) is related to the royal cubit (Pommerening, 2005, p. 75) in the following manner: 30 *hekats* equal 300 *hins* of 0.48 litres which correspond to the volume of a cubed royal cubit, 144 litres. We have therefore a double system of containers for the same *sft* oil; if the *sft* oil is calculated in volumetric units defining the royal cubit (the 300 *hekats* of *Jrj*'s tomb inscription) why cannot the same commodity associated with 100 *dw.t* measures define the great cubit, particularly when units of length are associated with volumes in Lacau and Chevrier's vase inscriptions of Table 20? Taking the average volumetric value of 100 *dw.t* measures (235 litres) 452 *hins* would define the Old Kingdom equivalent of a "sack" and the length of the great cubit 60.12 cm. The great cubit equals two royal *djesers* or 60 cm; 450 *hins* fit the 2/3 principle of ancient Egyptian mathematics: two thirds of 450 *hins* correspond to a cubed royal cubit of 300 *hins* or 144 litres. This estimation in *hins* fits the ancient Egyptian method: the Ptolemaic pBritish Museum 10399 problems 42 and 43 (Parker, 1972, p. 57) relates, in a genuinely ancient Egyptian manner, length of cubits and *hins*.

216 litres filled with emmer weigh 95.04 kilograms; 1000 *debens* per *h3r* sack give a weight per *deben* of 95 grams, the small *deben* of pBoulaq 13, 1/10th of which corresponds to the weight of the *deben* kedet. 216 litres filled with barley weigh 142.56 kilograms; 1000 *debens* give a weight per *deben* of 142 grams, the great *deben* of Pboulaq13, 1/10th of which equals 14 grams and corresponds to the weights indicated in Cour-Marty's series. Furthermore, the gradual disappearance of the great *deben* from the Eighteenth Dynasty to the end of the Twenty First Dynasty matches the trend mentioned by Brewer, the reversal of importance of commodities. The other conclusion is that the definition of the weight of the two *debens* dates from, or is anterior (Petrucci, 1981, p.44), to the Old Kingdom and is not affected by the sack definition of the Middle Kingdom. The Middle Kingdom *h3r* can be considered as a purely "administrative" Middle Kingdom multiple of 200 *hins* or 96 litres as the volume of a sack changes from 96 to 76 litres during the New Kingdom. 450 *hins* define the volume from which the length (circa 60 cm)

of the great cubit and its physical representation, the scribe's palette of the previous chapter, are derived.

7.2 New Kingdom

Summary

During the New Kingdom, the volume of a (*h3r*) sack is reduced to 76/77.5 litres. Filled with emmer this (*h3r*) sack weighs between 33.44 and 33.98 kilograms, a little more than half the weight of a Middle Kingdom (*h3r*) sack of barley (63.69 kilograms). If emmer becomes more important than barley during the New Kingdom, aligning the volume of (*h3r*) sacks on the weight of smaller and lighter (*h3r*) sacks of emmer makes sense and explains the volumetric reduction of the (*h3r*) sack. During the late New Kingdom, water is associated with (*h3r*) sacks. As water cannot be carried into canvass sacks, the (*h3r*) sack from the Middle Kingdom onwards is an accounting unit as well as a container for grain.

During the New Kingdom, the volume of a sack changes from 96–96.5 litres to 76.8–77.2 litres or four *oipe* of 40 *hins* or 160 *hins* (Pommerening, 2005, pp. 142 and 153). This change does not affect New Kingdom linear measurements, but its motivation is not clear: why modify a tax system based on a (*h3r*) sack of 96/96.5 litres? In the New Kingdom, the (*h3r*) sack is standardized to four quadruple *hekats* making an *oipe*. This transition from the Old Middle Kingdom (*h3r*) sack appears to have been gradual (Spalinger, 1987, pp. 306–8) and somehow problematic, judging by the introduction of the *House Oipe* of the Horemheb decree: 50 *hins* of *sšrw* (Urk. IV, 2152, 18–19).

A suggestion for the explanation of the change is the already mentioned reversal of the importance of barley over emmer in the New Kingdom (Gardiner, 1941, pp. 27–8; Brewer, Redford and Redford, 1994, pp. 28–9; Murray, 2000, pp. 511–13). Applying the halving principle of ancient Egyptian mathematics 76/77.5 litres (the volume of a New Kingdom sack) of emmer weigh between 33.44 and 33.98 kilograms, a little more than half the weight of a Middle Kingdom sack of barley (63.69 kilograms divided by two equal 31.84 kilograms). If emmer becomes more important than barley during the New Kingdom, aligning the number of (*h3r*) sacks on the weight of smaller (*h3r*) sacks of emmer makes sense. The 2/3 weight difference

between the two commodities remain, respecting the $2/3$ principle of ancient Egyptian mathematics and the Middle Kingdom (*h3r*) sack ratio. The weight of two Middle Kingdom (*h3r*) sacks of barley corresponds to the weight of three Middle Kingdom (*h3r*) sacks of barley. Reducing the weight of a sack is an added advantage for handling and transportation: the New Kingdom (*h3r*) sack of emmer weighs about 33 kilograms instead of 42 kilograms during the Middle Kingdom; the New Kingdom (*h3r*) sack of barley weighs about 50 kilograms instead of 63 kilograms during the Middle Kingdom.

The pLouvre E3226 of the early 18th Dynasty is a good example of inventory reckoning, handling, and transportation operations to and from the same warehouse. In the papyrus, parallel accounts of dates (*bnr*) and grain (*sšr*) are given in sacks (*h3r*). Megally (1977, p. 225) mentions that dates (*bnr*) are also reckoned in *hekats*. The weight/volume ratio between barley (*jt*) and dates (*bnr*) is similar to the ratio between barley and emmer: $2/3$. This indication is given by pBerlin 10078 (Spalinger, 1988, pp.271 ff.). *bnr hḳ3t 60 3/4 jr m jt šmᶜ hḳ3t 40 1/2*: dates $60 \frac{3}{4}$ *hekats* make in barley $40 \frac{1}{2}$ *hekats*; 195.4 litres ($40 \frac{1}{2}$ *hekats*) of barley weigh (195.4 multiplied by 660 grams per litre, the specific weight of barley) 128.96 kilograms. 128.96 kilograms represent 293.11 litres ($60 \frac{3}{4}$ *hekats*) of dates, therefore the specific weight of dates, according to pBerlin 10078, is 128.96 kilograms divided by 293.11 litres or 440 grams per litre, equivalent to the specific weight of emmer. The ratio of three (*h3r*) sacks of emmer to two (*h3r*) sacks of barley becomes applicable to dates: two (*h3r*) sacks of barley equal in weight three (*h3r*) sacks of emmer or three (*h3r*) sacks of dates facilitating handling, transportation, and perhaps accounting and inventory operations of three commodities.

7.2.1 The Association of (*h3r*) Sacks with Water

With the year 29 of Ramesses III comes the first association of a (*h3r*) sack with water (pTurin 1880-Gentet and Sweydan, 1992, pp. 177–85). The authors provide three hypotheses for the association of a sack (*h3r*) with water:

- The *h3r* sack is used for the payment of water (Gentet and Sweydan, 1992, pp. 180–81).
- The *h3r* sack is used for the payment of the transportation of water (Gentet and Sweydan, 1992, pp. 181–82).

- The *h3r* sack is used as an accounting unit to standardize the administrative costs associated with delivering water. This third hypothesis is illustrated by the authors, quoting pTurin 1880 (RAD 51, Gardiner, 1995, p. 51, lines 6, 6–10).

pTurin 1880	Transliteration	Translation
Line 6,6	<i>ʕ3 n js.t Hʕw mw h3r 17 1/2 m h3w</i>	The foreman Khaw, water, 17 and a half (<i>h3r</i>) sacks in attribution.
Line 6,7	<i>P3b3s3 h3r 8 3/4</i>	Pabasa eight and three quarters (<i>h3r</i>) sacks.
Line 6,8	<i>Hʕy-m-wj3 h3r 8 3/4 dmd h3r 35</i>	Khamwja eight and three quarters (<i>h3r</i>) sacks, total 35 (<i>h3r</i>) sacks.
Line 6,9	<i>sš Jmḥ-nḥt h3r 8 3/4 whm h3r 2 1/2 dmd h3r 11 1/2 dmd wd3.w. t mw h3r 47</i>	The scribe Amennakht 8 and 3/4 (<i>h3r</i>) sacks, renewal two and a half (<i>h3r</i>) sacks, total 11 and a half (<i>h3r</i>) sacks, remainder total water 47 (<i>h3r</i>) sacks.
Line 6,10	<i>Hʕy-m-W3st h3r 12 1/2 Mry-Rʕ h3r 12 1/2 Ms.w h3r 17 1/2 Nfr-ḥtp h3r 8</i>	Khaemwaset 12 and a half (<i>h3r</i>) sacks, Meryra 12 and a half (<i>h3r</i>) sacks, Mesw 17 and a half (<i>h3r</i>) sacks, Neferhotep 8 (<i>h3r</i>) sacks.

It is impossible to transport water in a canvas (*h3r*) sack, therefore the (*h3r*) sack is an administrative volumetric unit, the first definition of which appears during the Middle Kingdom.

7.3 Late Period

The Late Period shows a preference for the *oipe* of 40 *hins* over the New Kingdom (*h3r*) sack as a volumetric standard. The 25th Dynasty pLouvre E3228b, (Cerný and Parker, 1971, pp. 127–31; Pommerening, 2005, p. 159) reflects this preference: *jnk dj.t n=k p3 (h3r) 22 T 2 T ?bd.t t3 jp.t 40 hn h3.t-sp 13 jbd 4 3ḥ.t ʕrk* (“I have given you the 22 *2 * ? (sacks) of emmer measured with the *oipe* of 40 *hins*, year 13, month four of Summer, day 30 (last day).”) 40 *hins* equal one fourth of a New Kingdom sack (*h3r*) or 19.3 litres. There is no change in linear metrology.

7.4 Ptolemaic Period

Summary

The Ptolemaic *artaba* corresponds to half a New Kingdom sack filled with wheat (around 30 kilograms). In Greek and Hellenistic metrology, water is used as a reference commodity. 30 kilograms correspond to 30 litres which in ancient Egyptian terms is rounded to 60 *hins*. With a volume of 60 *hins* it is possible to re-construct both the royal and great cubit systems, using Greek and Hellenistic metrological principles and barley and emmer as commodities. The Alexandrian cubit is defined using the specific weight of wheat (*triticum durum*): 800 grams per litre.

From the Persians, the Greeks inherited and maintained the name *artaba*, mentioned in ration payments for the military garrison of Elephantine (Briant, 1996, pp. 426-27) and related to the Egyptian *h3r*. The Persians always tried to integrate within their own system the metrological systems of the regions making their empire (Briant, 1996, pp. 963-64). The Achaemenid cubit of perhaps 54 cm (the Samian relief in the introduction of this thesis) is compatible with the great system; perhaps no adjustment was deemed necessary.

The classical approach to the Ptolemaic *artaba* is far from being simple on account of the different Hellenistic metrological standards used to define it. Segrè (1928, pp. 22-3) uses classical metrological references and gives a value of 39.30 litres for the Ptolemaic *artaba*. The Ptolemaic *choinix* has a volume of 0.808 litres while the Attic *choinix* has a volume of 1.078 litres, leading to unconvincing approximations of the Ptolemaic *artaba* based on the volume of 48, 40, 36, 30, 29, and 24 Hellenistic *choinikes* ranging between 38.78 litres and 23.62 litres (Pommerening, 2005, pp. 158–73), and ending with a common consensus that the Ptolemaic *artaba* equals 80 *hins* or two Late Period *oipe* of 40 *hins* (Duncan-Jones, 1979, p.349).

There is simpler solution to define 80 *hins* as the volume of a Ptolemaic *artaba*, dividing a New Kingdom sack (*h3r*) of 160 *hins* by two, respecting the halving principle of ancient Egyptian mathematics. The major Ptolemaic agricultural innovation is the replacement of emmer by wheat (Rostovtseff, 1989, p.259). The specific weight of wheat is 800 grams per litre. The weight of wheat of a New Kingdom sack is 76/77.2 litres multiplied by the specific weight of wheat equals

60.8/61.76 kilograms. Half a New Kingdom sack equals 80 *hins* weighing 30.4/30.88 kilograms or 62.68/63.67 *hins* of 0.485 litres each. The 62.68/63.67 *hins* are usually rounded to 60 *hins* (or three quarters of 80 *hins* [Pommerening, 2005, p.170]). With a volume of 60 *hins* it is possible to re-construct a royal cubit using Greek and Hellenistic principles and, at last, water as a reference commodity. The Ptolemies never abandoned the original ancient Egyptian royal cubit. 60 *hins* have a volume of (60 multiplied by 0.485 litres or) 29.1 litres. 29.1 litres equal 29.1 kilograms of water. To match 29.1 kilograms of water with 29.1 kilograms of barley, the volume of barley needed corresponds to 29.1 kilograms divided by the specific weight of barley, 660 grams per litre, or 44.09 litres. The foot (cubic root) of 44.09 litres is 35.32 cm. A cubit equals one foot and a half, 35.32 plus 17.66 cm equal 52.98 cm, the length of a royal cubit (with a slight discrepancy due to the specific weight of barley retained in this thesis). To obtain 29.1 kilograms of emmer, a volume corresponding to 29.1 kilograms divided by the specific weight of emmer (440 grams per litre) is needed; its value is 66.136 litres. The foot (cubic root) of 66.136 litres has a length of 40.44 cm, corresponding to a cubit of (40.44 plus 20.22 cm) 60.66 cm, the length of a great cubit. A cube of 60.66 cm of length, height, and width has a volume of 223.32 litres corresponding to the volume of 100 *dw.t* or four *oipes* of the Old Kingdom. The ancient Egyptian metrological system is therefore backward-compatible.

7.4.1 The Alexandrian Cubit

The major Ptolemaic agricultural innovation is the replacement of emmer by wheat (*triticum durum* - Rostovtseff, 1989, p.259). The specific weight of wheat (*triticum durum*) is 800 grams per litre. The weight of a New Kingdom sack is 76/77.2 litres multiplied by 800 grams (the specific weight of wheat) is 60.8/61.76 kilograms. The New Kingdom sack contains 160 *hins*. Half of a New Kingdom sack equals 80 *hins* weighing 30.4/30.88 kilograms. This change of commodity has a direct bearing on Ptolemaic linear metrology. The fundamental principle of Greek and Hellenistic linear metrology, as previously mentioned, is that a foot (the basis of Greek and Hellenistic length measures) is related to the weight of a talent of water. Therefore, a talent of 30.4/30.88 kilograms corresponds to a volume of 30.4/30.88 litres or 30400/30880 cubic cm of water. The Alexandrian foot is the cubic root of 30.4/30.88 litres or 62.68/63.67 *hins* of 0.485 litres each, equal to 31.2/31.37 cm. The Alexandrian cubit corresponds to an

Alexandrian foot and a half (31.2/31.37 divided by two equal 15.6/15.7 cm) with a length between 46 and 47 cm. Seven Alexandrian palms of circa 7.6 cm give a length of 53/54 cm.

7.4.2 Ptolemaic Cubits

One consequence of the alignment of Greek and Hellenistic metrology with water is that weight calculations based on the specific weight of three commodities (barley, emmer, and wheat) are replaced by water as a single commodity and are simplified as one litre of water weighs one kilogram. The Ptolemaic pCarlsbeg 30, pBritish Museum 10399, and pCairo JE 89127–30,89137–43 published by Parker (1972, p. 10) mention four linear cubits: a *mḥ* corresponding to the reformed cubit, a *mḥ Dḥwtj* of seven palms and 54 cm, a *mḥ Ntr* corresponding to the former royal cubit, and a *mḥ-ḥbs* used for cloth length and area measurements.

7.4.2.1 The Ptolemaic *Mḥ*

The Ptolemaic cubit is an adaptation, following Greek and Hellenistic metrological standards, of the Twenty-sixth Dynasty reformed cubit. Four palms of the reformed cubit (35 cm) are taken as the Egyptian foot mentioned in the Roman pOxyrhynchus 669. One Ptolemaic cubit equals in length one foot and a half (the Hellenistic definition of a cubit) or 35 cm plus (35 cm divided by two or 17.5 cm) or 52.5 cm.

7.4.2.2 The *Mḥ Dḥwtj* (Nilometric Cubit)

The Ptolemaic texts confirm the Old Kingdom association of *hins* and cubits. This can be found in the Ptolemaic pBritish Museum 10399, problems 42 to 43, where the *mḥ Dḥwtj* is associated with *hins* of water in the volume calculations of a mast.

7.4.2.2.1 Problem 42 (Parker, 1972, P. 55)

Problem 42	Transliteration	Translation
Line 10	<i>ḵw=k rḥ p³ r³ šsp nty šm r p³ mḥ Dḥwtj r 7</i>	You know the palm divisions that will go in the cubit of Thoḥ amounting to seven

Problem 42	Transliteration	Translation
Line 11	<i>ᵚw=k ᵚr 7 r sp 7 r 49</i>	you will make 7 seven times: 49
Line 12	<i>ᵚw=k ᵚr 49 r sp 7 r 343</i>	you will make 49 times seven: 343
Line 13	<i>ᵚw=k ᵚᵚ p3 hn šsp 1 ᵚr-ᵚt 1 mty=f 1</i>	you shall say; “the hin is one palm by one, its depth 1”
Line 14	<i>ᵚw=k ᵚᵚ ᵚr šm hn [343 r mᵚ 1 ᵚr-ᵚt 1 mty=f 1]</i>	you shall say: [343 hins usually go [into a cubed cubit, 1 by 1, its depth 1]

7.4.2.2.2 Problem 43 (Parker, 1972, P. 57)

Problem 43	Transliteration	Translation
Line 25	<i>p3 gy n gm p3 r3 hn nty šm r p3 mᵚ 1 ᵚr-ᵚt 1 mty=f 1</i>	The method of finding the number (parts) of hin(s) that will go in the cubit, 1 by 1, its depth 1.
Line 26j	<i>ᵚw=k rᵚ p3 r3 šsp nty šm r p3 mᵚ ᵚᵚwtj r 7 ᵚw=k ᵚr 7 r sp 7 r 49</i>	You know the palm divisions that will go in the cubit of Thoth amounting to seven; you will make 7 seven times: 49
Line 27	<i>ᵚw=k ᵚr 49 r sp 7 r 343 ᵚw=k [ᵚᵚ] p3 hn šsp 1 ᵚr-ᵚt 1 mty=f 1</i>	you will make 49 times seven: 343; you shall [sa]y; “the hin is one palm by one, its depth 1”.
Line 28	<i>ᵚw=k ᵚᵚ ᵚr šm hn [343 r mᵚ 1 ᵚr-ᵚt 1 mty=f 1]</i>	you shall say: [343 hins usually go into a cubed cubit, 1 by 1, its depth 1].

Parker gives 7.833 cm as the length of one *mᵚ ᵚᵚwtj* palm on the basis a *hin* of 0.4805 litres. The length of the palm division corresponds to the cubic root of the *hin*, the length of which is 7.833 cm. Seven of these palm divisions give the length of a *mᵚ ᵚᵚwtj*: 54.831 cm. This cubit is known during the Roman Period (pOxyrhynchus 669) as the Nilometric cubit. This construction, based on the cubic root of one *hin* of water to define a palm, or 343 *hins* of water to define a cubed cubit, goes against the principles of Greek and Hellenistic metrology where a cubit is defined as a foot and a half, the foot being the cubic root of a talent of water. The cubit construction of pBritish Museum 10399 problems 42 and 43 is therefore genuinely Egyptian. 343

hins to define the volume of a cubit is not an easy number. As indicated, the traditional definition of the length of a royal or reformed cubit is 52.5 cm, corresponding to 300 *hins* or 144.7 litres, since the Middle Kingdom. 144.7 litres divided by 343 *hins* give a volume of circa 0.42 litre for a *hin*, which does not go against Pommerening's or Schmitt's records (Pommerening, 2005, p. 207; Schmitt, 2005, p. 63). It is difficult to reconcile 343 *hins* with the traditional 300 *hins* defining a royal or reformed cubit. There is one possibility: 300 *hins* of 0.42 litres each give 126 litres. 126 litres of water weigh 126 kilograms. 126 kilograms is the weight of two Middle Kingdom sacks (*h3r*) of 96.5 litres each filled with barley. This would imply an association of *h3r*, *hin*, and water before or as early as the Middle Kingdom. Such an association, although possible, can only be traced back to the Ramesside Period.

7.4.2.3 The *Mḥ Ntr*

The *mḥ Ntr* shares seven palms with the *mḥ Dhwtj*. The *mḥ Ntr* is not found before the Ptolemaic Period (Lüddeckens, 1968, p. 19) and as the length of the *mḥ Dhwtj* is greater than 52.5 cm, it seems logical to equate the *mḥ Ntr* with the former royal cubit of 52.5 cm.

7.4.2.4 The *Mḥ-ḥbs* (Cloth-Weaver Cubit)

The *mḥ-ḥbs* is a measure of area. According to Parker (1972, p.10; p.19 note 16) it is wrongly used in the Ptolemaic pCairo JE 89127–30, 89137–43, problems 10 and 12 as a unit of length to express a width in problem 10 (Parker, 1972, p. 22, line 19) and a height in problem 12 (Parker, 1972, p. 23, line 4). The only linear association with the *mḥ-ḥbs* appears in the Roman pOxyrhynchus 669.

7.4.3 Examples

7.4.3.1 Edfu Nilometre

The Ptolemaic (Ptolemy XI) Edfu nilometres cubit lengths published by Borchardt (1906, pp. 25–9) match the lengths of the different cubits used during the Ptolemaic Period.

Table 21. Lengths of Nilometre Cubits (Borchardt, 1906, p. 27).

Cubit Number	Cubit Length
20 a	53.2 cm
20 b	52.5 cm
19	54.4 cm
18	52.5 cm
17	52.5 cm
16	51.8 cm

Cubits 20 b, 18, and 17 correspond in length to the *mḥ* and/or *mḥ Ntr* of 52.5 cm. The length of cubit 20 a (53.2 cm) belongs to the Alexandrian system and is an example of the ambiguity of the system represented on the nilometres: cubit 19 (54.4) is the true Nilometric cubit (*mḥ Dḥwtj*) aligned on one cubic palm of water while 53.2 cm can represent seven palms in the Alexandrian system. Cubit 16, with a length of 51.8 cm, belongs to the former great system and is perhaps an example of the word *mḥ* (cubit) used for the word *gstj* (palette) of the Edfu inscriptions (Wilson, 1997, p. 451).

7.4.3.2 Edfu Temple

The dimensions of the Edfu temple rooms, published by Cauville (1984, pp. 6–34), match the length of seven Alexandrian palms (53/54 cm) or one Nilometric cubit and correspond to the Denderah cubits published by Legrain (see *Excursus B*).

Table 22. Edfu Temple Measurements (In Cauville, 1984, p. 34).

N° pièces (cf. fig. n° 1)	Dimension coudées	E/O mètres	Calcul de la coudée (* = inadéquation)	Dimension coudées	N/S mètres	Calcul de la coudée (* = inadéquation)
1	8 $\frac{1}{2}$	4,43	0,531	6 $\frac{2}{3}$	3,57	0,536
2	7 $\frac{1}{6}$	4,37	*	6 $\frac{2}{3}$	3,57	0,536
7	7 $\frac{1}{6}$	4,15	0,530	6 $\frac{2}{3}$	3,52	0,528
3	6 $\frac{2}{3}$	3,57	0,536	6 $\frac{2}{3}$	3,55	0,533
8	6 $\frac{1}{6}$	3,72	*	6 $\frac{2}{3}$	3,55	0,533
4	8	4,70	*	8	4,43	*
5	8	4,67	*	8	4,10	*
6	8	4,70	*	8	4,12	*
14	8	4,70	*	8	4,65	*
9	8	4,40	*	8	4,53	*
10	8	4,42	*	8	4,18	*
15	8	4,45	*	8	4,57	*
16	8	4,41	*	8	4,07	*
11 ⁽¹⁾	3 $\frac{1}{6}$	2,05	0,535			
12	10 $\frac{1}{2}$	5,50	0,532	19 $\frac{2}{6}$	10,60	0,534
13	23 $\frac{2}{3}$	12,82	0,541	9	4,75	0,527
17	25 $\frac{1}{6}$	13,56	0,524	8	4,52	*
18	10	5,82	*	9	4,57	*
19	(texte en lacune)					
20	10	5,60	0,56	8 $\frac{1}{2}$	4,60	0,541
21	37 $\frac{1}{6}$	20,37	0,538	25 $\frac{1}{6}$	14	0,542
22	4	2,40	*	10	5,40	0,54
23	4	2,10	0,525	13	7,60	*
24	4	2,10	0,525	7	3,75	0,535
25	4	2,10	0,525	11	5,72	0,52
26 ⁽²⁾	63	33,65	0,534	105	56,20 ⁽³⁾	0,535
27	75	40	0,533	36	19,12	0,534
28	90	47	0,522	113	?	
29	80	42,44	0,530	90	47,18	0,524
30	90	47	0,522	240	126,23	0,525
31 ⁽⁴⁾	120	63,84	0,532	21	11,05	0,526
32	26 $\frac{1}{3}$	12,40	*			

⁽¹⁾ largeur du couloir.

⁽²⁾ dimensions extérieures du naos.

⁽³⁾ y compris le mur englobé par le pronaos.

⁽⁴⁾ largeur du pylône sans la porte.

There are only a few exceptions corresponding to the length of the Ptolemaic *Mh Ntr* of 52.5 cm: the dimensions of rooms 23, 24, 25, 28 and 30.

7.5 Roman Period

From pOxyrhynchus 669 (285–6 A.D. [Grenfell and Hunt, 1904, Part IV, p. 118, col. ii, lines 31–37]), it is possible to identify the types of cubits used during the Roman Period and relate them to previous ancient Egyptian cubit divisions.

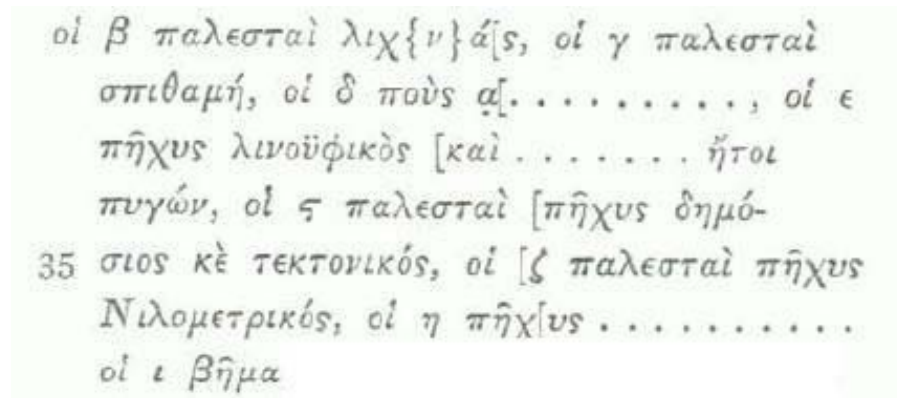


Figure 13. pOxyrhynchus 669-Col. ii-lines 31 to 41 (Grenfell and Hunt, 1904, Part IV, p. 118).

“2 palms make a likha[s, 3 palms a spithame, 4 palms an E[gyptian? foot, 5 palms a cloth-weaver’s cubit [and . . . truly a pugon, 6 palms [a public and a carpenter’s cubit, [7 palms a Nilometric cubit, 8 palms a . . . cubit . . . ten palms a bema ”

In Table 23 the divisions of pOxyrhynchus 669 are compared (in metres) with the length of the *Mh Ntr* (the former royal cubit), the *Mh* (the reformed or Ptolemaic cubit of six palm divisions), the *Mh Dhwtj* (of seven palm divisions), and the Alexandrian cubit.

Table 23. pOxyrhynchus 669 Divisions in Metres (Antoine Hirsch, 2012).

Royal Cubit		<i>mḥ</i>	<i>mḥ</i>	<i>mḥ</i>	Alexandrian	Oxyrhynchus
Divisions	Fingers	<i>Ntr</i>		<i>Dhwtj</i>	Cubit	Divisions
	40	0.75	0.875	0.78	0.76	Bema
	39	0.73	0.85	0.76	0.74	
	38	0.71	0.83	0.74	0.73	
	37	0.69	0.80	0.72	0.70	
	36	0.67	0.78	0.7	0.69	
Nbj	35	0.65625	0.76	0.68	0.67	
	34	0.63	0.74	0.66	0.65	
	33	0.61	0.72	0.64	0.63	

Royal Cubit		<i>mḥ</i>	<i>mḥ</i>	<i>mḥ</i>	Alexandrian	Oxyrhynchus
Divisions	Fingers	<i>Ntr</i>		<i>Dḥwtj</i>	Cubit	Divisions
Great Cubit	32	0.6	0.7	0.62	0.61	8 Palms Cubit
	31	0.58	0.67	0.6	0.59	
	30	0.56	0.65	0.58	0.57	
	29	0.54	0.63	0.567	0.55	
Royal Cubit	28	0.525	0.61	0.548	0.53	Nilometric Cubit
	27	0.50	0.59	0.52	0.51	
	26	0.48	0.56	0.50	0.49	
	25	0.46	0.54	0.48	0.47	
Small Cubit	24	0.45	0.525	0.47	0.46	Public or Carpenter's Cubit
	23	0.43	0.50	0.45	0.44	
	22	0.41	0.48	0.43	0.42	
	21	0.39	0.45	0.41	0.40	
Remen	20	0.375	0.4375	0.39	0.38	Cloth-weaver Cubit
	19	0.35	0.41	0.37	0.36	
	18	0.33	0.39	0.35	0.34	
	17	0.31	0.37	0.33	0.32	
Djeser	16	0.3	0.35	0.31	0.30	Egyptian Foot
	15	0.28	0.328	0.29	0.28	

Royal Cubit		<i>mḥ</i>	<i>mḥ</i>	<i>mḥ</i>	Alexandrian	Oxyrhynchus
Divisions	Fingers	<i>Ntr</i>		<i>Dḥwtj</i>	Cubit	Divisions
Great Span	14	0.2625	0.30	0.27	0.26	
	13	0.24	0.28	0.25	0.24	
Small span	12	0.225	0.2625	0.23	0.23	Spithame
	11	0.20	0.24	0.21	0.21	
	10	0.18	0.21	0.19	0.19	
	9	0.16	0.19	0.17	0.17	
Double Palm	8	0.15	0.175	0.15	0.15	Lichas
Sandal	7	0.13125	0.15	0.13	0.13	
	6	0.11	0.13	0.11	0.11	
	5	0.093	0.10	0.097	0.095	
Palm	4	0.075	0.0875	0.07833	0.076	
	3	0.056	0.065	0.058	0.057	
	2	0.037	0.043	0.039	0.038	
Finger	1	0.018	0.021	0.019	0.019	

The length of the divisions of the Alexandrian and the Nilometric (*Mḥ Dḥwtj*) cubits are so close that it is almost impossible to distinguish them, except on nilometre inscriptions (Schmitt, 2005, pp. 66–9). In Schmitt’s catalogue, which includes the length of all the artifacts and nilometre inscriptions of the Ptolemaic and Roman Periods, one set of lengths between 50 and 51 cm cannot be matched exactly by any of the lengths given in Table 23. One possible explanation is that these multiples of ten are based on a scribe’s palette, the divisions of which (ten cm) are a remnant of the great system.

8 Area Related Lengths

Summary

Area related lengths provide clues about ancient Egyptian geometry applied to garments and field measurements. Amongst the textual references used in this section, the *st3t* inscriptions of the White Chapel of Sesostris I clearly justify the use of the royal cubit as a converter; with a royal cubit, it is possible to define the cubit used in each nome of ancient Egypt. Triangular area calculations in the Rhind Mathematical Papyrus link the *remen* to the great system.

8.1 The Cloth-Weaver Cubit

8.1.1 Old Kingdom

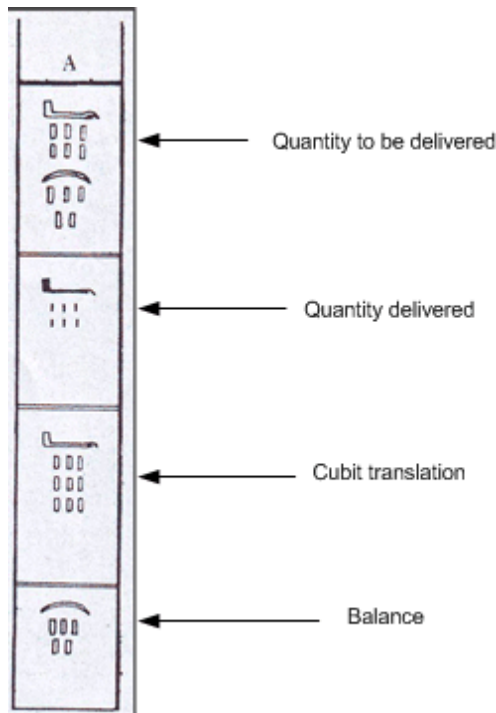
Ancient Egyptian garment lengths from the Eighteenth Dynasty to the Twenty Fifth Dynasty follow the great system using the scribe's palette or the great cubit. Accounts 47 A and 50 – 52 A of the Abusir papyri seem to suggest an equivalence between cloth-weaver cubits and royal or great cubits; applied to Posener-Kriéger's article, tentative calculations may indicate the use of the great system in the measurement of linen.

The cubit associated with linen is of particular interest. Posener-Kriéger mentions (1977a, pp. 86–96) the existence of special signs for the measurement of linen and cloth. When cloth and ancient Egyptian metrology are associated, the Ptolemaic and Roman Periods come to mind. The Old Kingdom cubit used for linen measurements is the ancestor of the *mḥ-ḥbs* of pCairo JE. 89127–30, 89137–43 and of the cloth-weaver cubit of pOxyrhynchus 669.

The physical evidence (Kemp and Vogelsang-Eastwood, 2001, p. 440; Vogelsang-Eastwood, 1993 pp. 181–2; Janssen, 1975, pp. 249–98) shows garment lengths based on multiples of half the length of a scribe's palette, 25 cm (50, 75, 100, 125) and multiples of half the length of a great cubit (*djeser*), 30 cm (30, 60, 120, 180) from the Eighteenth Dynasty to the Twenty Fifth Dynasty. These garments do not belong to the Old or Middle Kingdoms. There is, however, no reason for a change in the length of garments starting with the Eighteenth Dynasty, particularly when the ground loom used to produce them does not seem to change over time: the width of the ground loom from the Predynastic Period onwards varies between two scribe's palette (100 cm)

and two and a half scribe's palette (125/130 cm [Vogelsang-Eastwood, 2000, p. 277, c01.1; 1993, p. 6]).

The linen calculation method proposed by Posener-Kriéger is logical: the first (red) square at the top of a column (Figure 14) indicates the amount ordered, the second (black) square indicates the amount delivered, while the fourth (red) square indicates the balance. The implication of Posener-Kriéger's solution is that the second (black) square cubit system is the same as the amount ordered (first [red] square), otherwise no balance (the fourth [red] square) could be calculated. The third (red) square is not clearly explained by Posener-Kriéger, who considers it as an almost unrelated accounting entry (*quantité pour mémoire*).



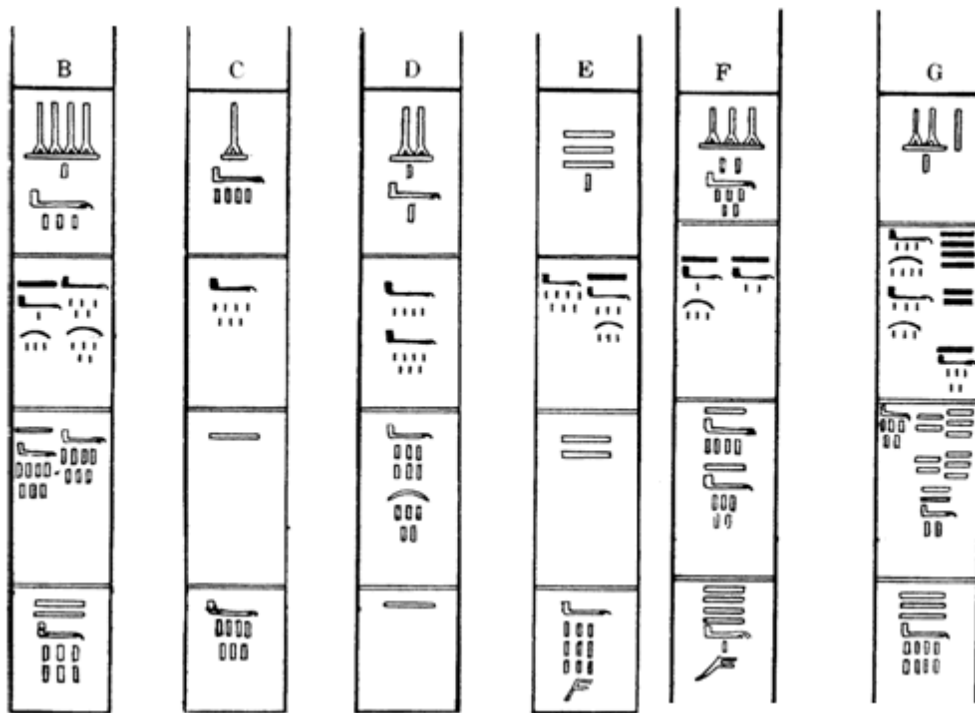


Figure 14. Linen calculations Quadrant and Marks (Posener-Kriéger, 1977a, pp. 86–96).

There is no way to distinguish in the Gebelein text or the Abusir papyri which cubit is meant (great, royal, or a third specialized cloth-weaver cubit) as only one cubit sign is used in the second, third, and fourth squares of Figure 14. In the first square, a very specific notation is used: one or several vertical strokes with one horizontal stroke as a base, suggesting, as Posener-Kriéger had already noted, more an area than a length. The width of a manufactured piece of cloth cannot exceed the width of the loom used to produce it. The possible widths of an ancient Egyptian ground loom, according to the figures mentioned above, are multiples of ten centimetres. Ten centimetres is the length of the palm in the great system (6 palms of 10 cm equal 60 cm, the length of great cubit) associating thus very closely the great system with the length of the cloth-weaver cubit and its divisions.

This third (red) square may represent the cubit system used by the second (black) square: the second (black) square is the quantity (area) delivered while the third (red) square is the description of this area i.e. the number of cubits with their respective widths and lengths. It is interesting to note that the number of cubit groups in the second [black] square corresponds to the number of red cubit (or palm) groups in the third [red] square: for example in the B entry 11

black cubits and three palms in the second [black] square correspond to 17 red cubits in the third [red] square, while five cubits and three black cubits correspond to 15 red cubits.

The problem is then to know which cubit and palm system is represented by the third (red [translation]) square; great or royal system. In accounts 50–52A of the Abusir papyri (Posener-Kriéger, 1976, pp. 361–62; 1977, p. 95 and note 34) 30 area-cubits (using the specialized linen signs) are translated as 30 delivered cubits, aligning the system of the 30 cubits ordered with the system of the 30 cubits delivered. In account 47A, 1 20 area-cubits (using the specialized signs for linen measures) are translated as 12 delivered cubits using the conventional cubit arm of the second [black] square of the Gebelein examples. The area of the 20 cubits of account 47A 1 corresponds to an area of 140 royal (area) palms or 54000 square cm, giving an area for each of the 12 cubits delivered of (54000 square cm divided by 12) 4500 square cm, or 12 pieces of cloth 60 cm long by 75 cm wide (within the range of the width a ground loom).

The calculations in Table 24 (based on the data of Figure 14 above) explore this hypothesis. The figures retained for the calculations are rounded figures¹⁰: a cubit area of 2700 cm² and an average palm area of 420 cm², the average between the area of a palm obtained by dividing by six the area (2700 cm²) given by a great cubit multiplied by a small cubit, and the area obtained by a squared royal cubit (2756.25 cm²) divided into seven palm divisions (the number of palm divisions in a royal cubit). The first column is the number of cubits delivered (the second [black] square) in the examples given by Posener-Kriéger; the second column is the calculation of the area delivered in square cm; the third column is the number of the type of cubits delivered (the third [red] square); the fourth column is the area of this type of cubit (the total area divided by the cubits shown in the third [red] square); the fifth column is the possible type of cubit delivered; the sixth column is the total discrepancy (in area palm) between the area delivered and the area obtained using the type of cubits delivered (great, royal, or shared).

Table 24. Gebelein Linen Calculations (Antoine Hirsch, 2012).

¹⁰ Following the “rounding off” principle of ancient Egyptian mathematics (Claggett, 1999, pp. 18-9).

Reference	Delivery	Area Delivered	Type Number	Area (cm2)	System Used	Discrepancy on Rounded Figures (cm2)
	Second (black) Group		Third (red) group	Per cubit type	Third (red) group.	on total area
A	6 (area) cubits	16200	9 Cubits	1800	Great.	0
					length 60 cm multiplied by width 30 cm.	
					1 great cubit multiplied by 1/2 great cubit.	
B	11 (area) cubits	29700.00	17			
	3 (area) palms	1260.00				
	Total	30960.00		1747	Great.	21
					length 60 cm multiplied by width 30 cm.	
					1 great cubit multiplied by 1/2 great cubit.	
	5 (area) cubits	13500.00	7 Cubits	2228.57	Great.	28.57
	5 (area) palms	2100.00			Length 75 cm multiplied by 30 cm.	
	Total	15600.00			1 great cubit and 1/4 multiplied by 1/2 great cubit.	
	Second (black) Group		Third (red) group	Per cubit type	Third (red) group.	on total area
C	7 (area) cubits	18900.00	10	1890.00	Shared.	150

					length 37.5 cm multiplied by width 50 cm.	Almost 1/4 (area) palm
					1 royal <i>remen</i> multiplied by one scribe's palette.	
D	4 (area) cubits	10800.00	6 Cubits	1800.00	Great.	
					length 60 cm multiplied by width 30 cm.	
					1 great cubit multiplied by 1/2 great cubit.	
	7 (area) cubits	18900.00	5 Palms (sic)	1890.00	Shared.	150
					length 37.5 cm multiplied by width 50 cm.	Almost 1/4 (area) palm
					1 royal <i>remen</i> multiplied by one scribe's palette.	
E	7 (area) cubits	18900.00	10	1890.00	Shared.	0
					length 37.5 cm multiplied by width 50 cm.	
					1 royal <i>remen</i> multiplied by one scribe's palette.	
	Second (black) Group		Third (red) group	Per cubit type	Third (red) group.	on total area
	13 (area) cubits	35100.00	10	3510.00	Great.	100
	3 (area) palms	1260.00			length 70 cm multiplied by width 50 cm.	Almost 1/4 (area) palm

	Total	36360.00			1 great cubit and one great palm multiplied by one scribe's palette.	
F	11 (area) cubits	29700.00	14	2211.00	Great.	1800
	3 (area) palms	1260.00			length 45 cm multiplied by width 50 cm.	4 (area) palms
	Total	30960.00			1 small cubit multiplied by one scribe's palette.	
	12 (area) cubits	32400.00	15	2160.00	Shared.	900
					length 52.5 cm multiplied by width 40 cm.	2 (area) palms
					1 royal cubit multiplied by four scribe's palette palms	
G	43 (area) cubits	116100.00	55	2130.50	Shared.	2280
	4 Palms	1680.00			length 52.5 cm multiplied by width 40 cm.	5 (area) palms
	Total	117780.00			1 royal cubit multiplied by four scribe's palette palms	
	23 Cubits	62100.00	50	1267.20	Great.	400
	Second (black) Group		Third (red) group	Per cubit type	Third (red) group.	on total area
			Cubits			
	3 Palms	1260.00			length 50 cm multiplied by width 25 cm.	1 (area) palm
	Total	63360.00		1245.38	1 scribe's palette multiplied by 1/2 scribe's palette.	

	15 Cubits	40500.00	22	1840.91	Great.	900
				1840.91	length 60 cm multiplied by width 30 cm.	2 (area) palms
					1 great cubit multiplied by 1/2 great cubit.	

If this interpretation is correct, then the *szf*, *h^c*, and *d3t* mentioned by Smith (1935, pp. 146–48), could be specialized division names of the cloth-weaver cubit. What length is meant by each division (10, 20, 30, or 40 cm) is impossible to determine. The only thing that can be said is that *szf*, *h^c*, and *d3t* must have been very well known units in the Old Kingdom, as seven (area) cubits in example D (second [black] square) correspond to five palm divisions in the type of cubit of the third [red] square, without any mention of any number of cubits. As the cloth-weaver cubit division of the Roman Period corresponds to the definition of the *remen* divisions (five palms), there is a strong possibility that the length of the cloth-weaver-cubit in the Old Kingdom was 50 cm (on par with the scribe’s palette) and that the system was part of the great system, with specialized cloth division names. In Table 24 the widths always belong to the great system and can be related very easily to the length of the palm of the great system, 10 cm, and the definition of the widths of the ground looms mentioned above.

8.1.2 Middle Kingdom

Linen lists and their cloth-weaver cubit association disappear after the Old Kingdom; it is therefore surprising to still find a reference to a cloth-weaver cubit in problem 18 of the Moscow Mathematical Papyrus. This may also suggest that the cloth-weaver cubit is aligned with the scribe’s palette or the great system. Problem 18, qualified as unclear (Gillings, 1982, p. 247) is difficult to understand following Struve’s transcription.

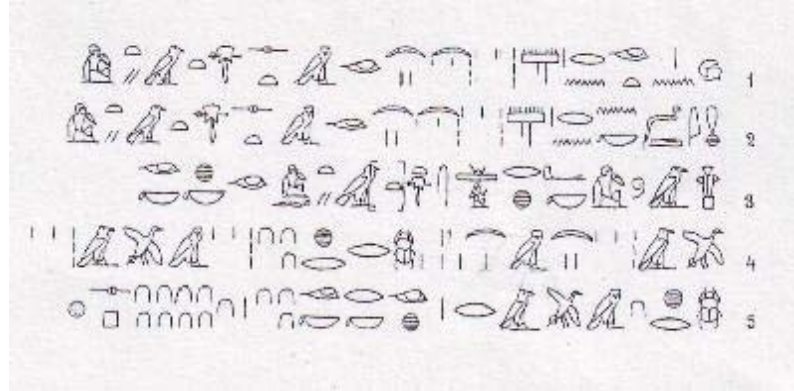




Figure 15. Moscow mathematical papyrus problem 18 (Struve, 1930, Table vii).

Problem 18 is the conversion into royal cubits of a length of five cloth-weaver cubits and five palms by a width of two palms into rectangular (*stwtj*) parts. The word *stwtj* is written in full on lines one, two, and three. It is written once with the specialized cursive sign () resembling the hieratic 30 mentioned by Peet in the Rhind Mathematical Papyrus (Peet, 1923, p. 25-Eisenlohr, 1877, Plate XVIII) on line four¹¹, which Struve translates as 30. Reading 30 instead of *stwtj* makes the text impossible to understand. Struve's hieroglyphic transcription of the five palms of line four (*m šsp 5*) is better understood as four palm divisions of the royal cubit, taking the stroke under the *šsp* sign not as part of the number five (five is written differently on line one, two, and four [three strokes, followed by two strokes] as Struve seems to indicate on page 122, notes h and i) but as the unit marker of the palm system into which the five cloth-weaver cubits by two palm divisions are translated: the five cloth-weaver cubits by two palm divisions are translated into royal cubit (area) palm divisions of four finger divisions.

The word *hbs* of line 1 and 2 can be read in two different ways: a piece of cloth or a cloth-weaver cubit. In the first case it is necessary to add, as Struve does in his translation (Struve, 1930, p.120), five after the *hbs* sign of line 1 to understand a piece of cloth of five cubits, five

¹¹ The word *stwtj* is used four times in the Rhind Mathematical Papyrus, written in full in problems 45 and 46 (Peet, 1923

Plates XII to XIV, lines three and five respectively) and in problems 53 and 54 with the cursive () [Peet, 1923, Plate P, lines two and four respectively, Eisenlohr, 1877, Plate XVII). The word always has a meaning of ten regardless whether it is associated with volumes in the proof of problems 45 and 46 or with rectangular areas (problems 45 and 46).

palms by two palms. Based on the Gebelein area calculation examples of the Old Kingdom, the word *hbs* can also be understood as *mḥ-hbs*, a cloth-weaver cubit, using the specialized *hbs* sign instead of the ordinary arm sign associated with cubits. The two palms represent the width of both the cloth-weaver cubit and the five palm divisions necessary to calculate the area of the *stwtj*.

Problem 18	Transliteration	Translation
Line 1	<i>tp-n-jrt r3 n hbs 5 šsp 5 šsp 2 jrj m stwtj</i>	Method of partitioning five cloth-weaver cubits, five palms by two palms, into a (rectangular) area
Line 2	<i>mj dd(.w) n=k r3 n hbs 5 šsp 5 šsp 2 jrj m stwtj</i>	If you are told: the fraction of five cloth-weaver cubits, 5 palms by two palms as a (rectangular) area
Line 3	<i>h3w dj=k rh=j stwtj=f jrj.h(r)=k jrj=k</i>	please let me know its (rectangular) area. Calculate
Line 4	<i>p3 5 šsp 2 m šsp 4 hpr-hr 35 m p3 5</i>	these five cloth-weaver cubits by two palms as 4 (royal) palms: result 35 (area) palms. These 5
Line 5	<i>hpr-hr 10 m p3 r 1 irj.hr=k jrj=k 30 1 10 80 sp</i>	(palms) become 10. With this one part calculate one <i>stwtj</i> . (Result) 10, 80 remaining over.

The rectangular adjective in the translation corresponds to the definition of the word *stwtj* mentioned in the Rhind Mathematical Papyrus. The word is also associated with the value ten (ten area palm divisions define a *stwtj* which is, in this particular case, a rectangle of five palm divisions by two palm divisions. The 80 are justified by 35 (royal) palm divisions (the five cloth-weaver-cubits translated into royal palm divisions) plus five palm divisions, in total a length of

40 palm divisions multiplied by two palm divisions in width or 80 (royal area) palm divisions. The area of one *stwtj* in this specific instance corresponds to ten squared royal area palm divisions of 7.5 cm or 562.5 square cm, or one royal palm division multiplied by two royal remens (7.5 cm multiplied by two royal remens of 37.5 cm equal to a double *remen* of 75 cm)¹².

The Moscow Mathematical Papyrus problem 18 may provide a translation example of the great system into the royal system. This does not imply that the notion of a cloth-weaver cubit disappears and is forgotten. The Ptolemaic (pCairo JE 89127–30, 89137–43) and Roman (pOxyrhynchus 669) Period texts still mention cloth-weaver cubits.

8.1.3 Ptolemaic Period

During the Ptolemaic and Roman Periods the cloth-weaver cubits are mentioned again. Translations between two systems are performed; which Ptolemaic cubits are involved is unclear.

The *mḥ-ḥbs* is not as new as Parker may have thought (Parker, 1972, p. 11). It is the descendant of the Old Kingdom cloth-weaver assimilated during the Middle Kingdom to the scribe's palette or the great cubit. The *mḥ-ḥbs* appears in problems 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, and 18 of pCairo 89127–30, 89137–43. Problem 8, according to Parker (Parker, 1972, p. 11), aligns the area of the *mḥ-ḥbs* with a cubit of 52.5 cm.

- Problem 8 (Parker, 1972, p. 20-lines 2 and 3) $w^c t t b ḥ j w = s j r m ḥ 7 t [s m ḥ 5 w s ḥ] r m ḥ - ḥ b s$ 35-A cloth-measure seven cubits high and five cubits wide] amounting to 35 {area} cloth-weaver cubits-calculates cloth areas from an unqualified *mḥ* and so does problem 9 (Parker, 1972, p. 21-line 11) $w^c j r - f^c j r m ḥ 6 t s [m ḥ 4 w s ḥ] r m ḥ - ḥ b s$ 24-A hair-cloth 6 cubits high and [4 cubits wide], amounting to 24 (area) cloth-weaver cubits.
- Problem 10 (Parker, 1972, p. 22-line 19) specifies a (length) cloth-weaver cubit: $w^c t m r j w = s j r m ḥ - ḥ b s 6 [t] s w [s] ḥ [m ḥ 1 1/2 r m ḥ - ḥ b s 9]$ -A band of six cloth-weaver cubits [high and [1 1/2 cubits [wide] amounting to 9 (area) cloth-weaver cubits].

¹²

A double *remen* is in fact the “true” *remen*. A length of 75 cm corresponds to the length of the hypotenuse of a right-angled triangle, the sides of which are one small cubit (45 cm) multiplied by one great cubit (60 cm).

- Problem 13 (Parker, 1972, p.24-line 1 uses an area $m\bar{H}$ -Hbs as a calculation result: $[kt \text{ } \bar{c}y \text{ } jw=s]jr \text{ } m\bar{h} \text{ } 3 \text{ } 1/2 \text{ } r \text{ } 2 \text{ } 1/2 \text{ } r \text{ } m\bar{h}\text{-}hbs \text{ } 8 \text{ } 2/3 \text{ } 1/12 \dots 11 \text{ } 1/4$ -[Another linen cloth which] is $3 \text{ } 1/2$ by $2/ \text{ } 1/2$ amounting to $8 \text{ } 2/3$ (area) cloth-weaver cubits $\dots 11 \text{ } 1/4$.

The mention of an unqualified length cubit ($m\bar{h}$) in cloth area calculation implies a translation from one system to another system; the question is which Ptolemaic cubit is meant in the conversion, the $m\bar{h} \text{ } Ntr$, the $M\bar{h} \text{ } D\bar{h}wtj$, the $m\bar{h}$, or the Alexandrian cubit.

8.2 The $st\bar{b}t$

The traditional definition of the $st\bar{b}t$ is an area of one hundred royal cubits multiplied by one hundred royal cubits.

“1 Elle = 52.5 cm, dann ist one Ellen Feld = 0.275 qm, 1 tA Feld wäre dann 100 Ellen Feld = 27.5 qm, 1 xA Feld wäre dann tausend Ellen Feld = 275 qm und one Arure wäre 2750 qm.” (Gödecken, 1976, p. 361)

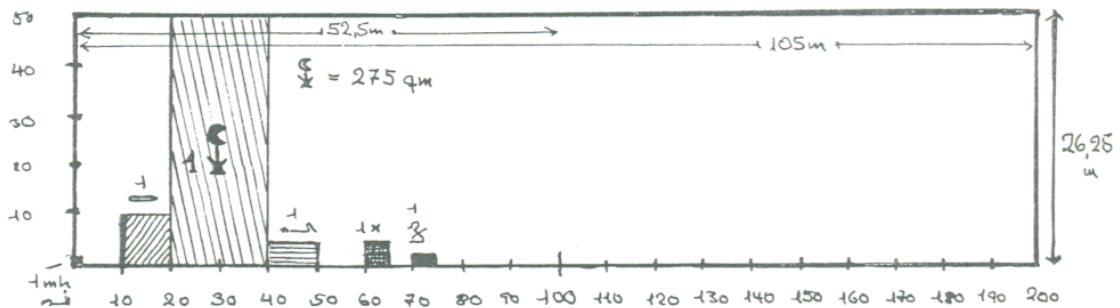


Figure 16. Gödecken's Rectangular Area Divisions (Gödecken, 1976, p. 361).

This “square” $st\bar{b}t$ calculation is purely geometrical. 1/100th of a $st\bar{b}t$, an Old Kingdom $t\bar{b}$, is a rectangular strip of land, 100 cubits long by one cubit wide. Adding 100 $t\bar{b}$ (the area of a $st\bar{b}t$) result in creating a square 100 cubits long by 100 cubits wide. On account of the rectangular definition of the $t\bar{b}$, the area of a $st\bar{b}t$ can be calculated in two different manners: multiplying 100 royal cubits by 100 royal cubits, or multiplying 100 great cubits by 100 small cubits. The difference in area is minimal, 56.25 square metres (around two per cent). Texts, mostly from the Middle Kingdom, seem to indicate two different area calculations.

8.2.1 Old Kingdom

Baer (1956, p. 116) suggests that the *stbt* of the Old Kingdom might be bigger than the *stbt* of the Middle and New Kingdoms. He gives the following values for units of area in the Old Kingdom: $1 \text{ stbt} = 3 \text{ hb} = 30 \text{ t3} = 60 \text{ rmn} = 120 \text{ hsb} = 240 \text{ z3} = 3000 \text{ mh}$, leading to the area of a *stbt* of 8.20 hectares (82,050 square metres). Baer does not contest the value of the *t3*, his figure is slightly smaller 27.35 square meters against 27.56 on a royal cubit of 52.5 cm long; according to him the *stbt* of the Old Kingdom is 30 times bigger than the *stbt* of the Middle Kingdom and following periods. There is no indication whether squares or rectangles are used in the Old Kingdom *stbt* calculations.

8.2.2 Middle Kingdom

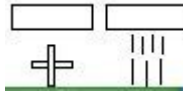
The Hekanakhte Papers indicate two different area calculations, but do not provide any indication on the cubits and system used to perform these calculations. The special hieratic numbers used with areas in the Hekanakhte Papers are the object of different interpretations by Allen, James, de Cénival, Goedicke, and Baer.

MEASURES OF AREA								
	Value	Hieroglyph	FORMS	Möller No.	Value	Hieroglyph	FORMS	Möller No.
A	1 aroura			683	$7\frac{1}{2}$ aroura	or		
B	$2\frac{1}{2}$ aroura				10 aroura			
C	5 aroura				5 aroura ?	?		

Figure 17. Hieratic Area Signs in the Hekanakhte Papers (James, 1962, Pla. 15).

Allen (2002, p.152–55) gives a value of one *dek^rrqur^c* (1 *hb-t3*) for the hieratic cross, following Baer (1962, p.34) and Goedicke (1984, p.43). On the other hand, James (1962, p.14) sees it as 2 $1/2$ *arouras*, a position followed by de Cénival (1963, p.141). These figures are the result of a

yield per *aroura* expressed in the Hekanakhte Papers (I, 9–13) where five sacks of barley per *aroura* of land yield 65 sacks for the (𓆎) sign. The three dots represent three *arouras* yielding 15 sacks, and the cross (𓆏) represents ten *arouras* yielding 50 sacks. The hieratic “cross” sign also matches a Middle Kingdom unpublished *ostrakon* (Allen, 2002, p.152) in possession of the



French Institute in Cairo. The combination of signs 𓆏 𓆎 represents 17 *arouras*. In the Hekanakhte Papers the *stbt* is always represented (when it is represented) as a single rectangle (Gardiner sign N 37). The regular sign for 10 (Gardiner V20) also appears in the Hekanakhte Papers; therefore the combination of the rectangle determinative for *stbt* and the (𓆏) sign must be a specialized notation for a rectangular area calculation in opposition to the more common squared royal cubit calculation. If squared royal cubits were meant, there would be no need for this particular combination of signs. If the cross means ten *arouras*, there is no problem following Allen (2002, p.153) and taking the other sign (𓆎) as 20. Regardless of the values given to the unusual hieratic signs in the Hekanakhte Papers, the important point is that they represent an area metrological system distinct from the royal system based on a squared royal cubit.

8.2.2.1 The White Chapel *Stbt* Inscriptions

Summary

The *stbt* inscriptions of the White Chapel of Sesostris I indicate which type of cubit is used in a nome. They are the first tangible indication that the royal cubit is an administrative converter and that the great system was currently used in ancient Egyptian nomes.

Zivie (1972, p. 185) thinks the calculations in the White Chapel *stbt* (*aroura*) reflect “le nombre de coudées et de palmes qu’il faut soustraire de 100 coudées pour obtenir la longueur de la *stbt* utilisée dans le nome.” Graefe (1973, pp. 72–6) suggests the existence of regional cubits with different lengths for each nome (denied by Vleeming [1985, p.209]). Girndt (1996, pp. 56–9) suggests an increased length of the royal cubit starting with the Twelfth Dynasty. All three suggestions imply that there is a specific cubit for each nome; the question is then which system does each specific cubit belong to, great or royal? Before performing calculations it is useful to

remember that 1/100th of a *stbt*, a *t3* in the Old Kingdom or a *mḥ t3* in the Middle Kingdom (Clagett, 1999, pp. 12-3), is a rectangular strip of land, 100 cubits long by one cubit wide. Adding 100 *mḥ t3* (the area of a *stbt*) creates a square 100 cubits long by 100 cubits wide. Calculating the area of a *stbt* can be performed in two different manners: multiplying 100 royal cubits by 100 royal cubits, or multiplying 100 great cubits by 100 small cubits. The difference in area is minimal, (2756.25 [royal cubit] versus 2700 [great cubit {0.60 cm} multiplied by small cubit {0.45 cm}]) 56.25 square metres (around two per cent). The area of the *mḥ t3* is constant since the Old Kingdom; the need of a centralized state is to have an area measure, the *stbt*, common to all nomes for tax assessments. How this common measure is reached is indifferent, provided it is reached. The text at the bottom of Column 17 of the North Side (Figure 18 below [Schlott-Schwab, 1981, Table VII]) shows the calculation.

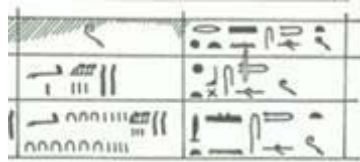


Figure 18. Sample *stbt* Calculations on the 14th Nome of Lower Egypt (Schlott-Schwab, 1981, Table VII).

<i>100</i>	100	<i>rḥt n stbt</i>	Amount of <i>aroura</i>
<i>mḥ 1 šsp 3 dbꜥ 2</i>	1 Cubit three Palms 2 Fingers	<i>ḥbt stbt</i>	Deduction (of-from) <i>aroura</i>
<i>mḥ 98 šsp 3 dbꜥ 2</i>	98 Cubits three Palms 2 Fingers	<i>d3t n stbt</i>	Remainder of <i>aroura</i>

In the first line, in the 14th nome of Lower Egypt, one rectangular strip of land the area of which corresponds to 100 cubits, 300 palms, and 200 fingers multiplied by one cubit, 3 palms 2 fingers is deducted from 100 strips of land, each 100 cubits long by one cubit wide. This area, based on a royal cubit of 52.5 cm long, equals 29.31 square metres. In the second line, in the 14th nome of Lower Egypt, the remainder of the *aroura*, using a royal cubit of 52.5 cm, covers an area of 98

cubits, 3 palms, 2 fingers or 2727.58 square meters. This calculation provides three pieces of information:

- The total area based on royal cubits of 52.5 cm is 29.31 square metres plus 2727.58 square metres which equals 2756.89 square metres, the *aroura* calculated by squaring the royal cubit.
- The “administrative” cubit used in the nome to perform the calculation is the royal cubit: adding one cubit, 3 palms, two fingers to 98 cubits, 3 palms, two fingers can only give 100 royal cubits of seven palms of four fingers: 98 cubits plus 1 cubit equal 99 cubits; three palms plus three palms equal six palms; two fingers plus two fingers equal one (royal) palm; in turn six palms plus 1 palm equal 1 (royal) cubit, 99 cubits + 1 cubit equal 100 cubits.
- The local cubit used in the 14th Nome of Lower Egypt is also the royal cubit. 100 squared royal cubits corresponds to an area of 2756.25 square metres, the area of the *st3t* in the nome.

A similar calculation can be performed for the 8th Nome of Upper Egypt (Abydos) in Figure 19.

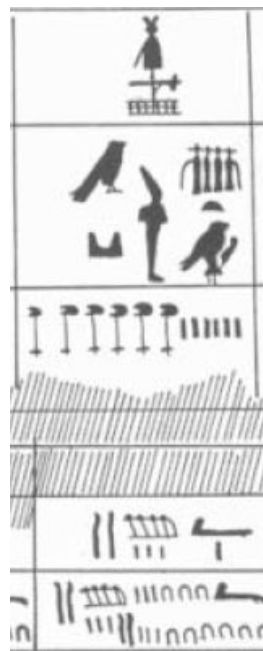


Figure 19. Sample *st3t* Calculations on the 8th Nome of Upper Egypt (Abydos) (Schlott-Schwab, 1981, Table II).

In the sixth line, in the 8th nome of Upper Egypt, one rectangular strip of land the area of which corresponds to 100 cubits, 300 palms, and 200 fingers multiplied by one cubit, 3 palms 2 fingers is deducted from 100 strips of land, each 100 cubits long by one cubit wide. This area, based on a royal cubit of 52.5 cm long, equals 29.31 square metres. In the seventh line, in the 8th nome of Upper Egypt, the remainder of the *aroura*, using a royal cubit of 52.5 cm, covers an area of 96 cubits, 3 palms, 4 fingers or 2646 square metres. The results are slightly different from the 14th Nome of Upper Egypt, even if the number of cubit deducted is the same.

- The total area based on royal cubits of 52.5 cm is 29.31 square metres plus 2646 square metres which equals 2675.31 square metres, the *aroura* calculated by multiplying a great cubit by a small cubit with a discrepancy of 24.69 square metres, less than 1 per cent.
- The “administrative” cubit used in the nome to perform the calculation is the great cubit of eight palms of three fingers. C16 on the Amenemope cubit artifact. The total does not add up to 100 cubits but to 98 cubits; adding one cubit, three palms, two fingers to 96 cubits, three palms, four fingers only gives 98 cubits: 96 cubits plus 1 cubit equal 97 cubits; three palms plus three palms equal six palms; four fingers plus two fingers equal two palms; two palms plus six palms equal 1 cubit of eight palms, 97 cubits + 1 cubit equal 98 cubits.
- The local cubit used in the 8th Nome of Upper Egypt is the great cubit. 98 squared royal cubits equal 2701 square metres, the area of the *stbt* in the nome. 2701 square metres equal 100 great cubits of 60 cm multiplied by 100 small cubits of 45 cm.
- The use of the great cubit at Abydos is documented in the database Excel Spreadsheet *database_5*. It is, by far, the largest percentage.

The calculation for the 4th Nome of Upper Egypt is interesting because the remaining quantity of line 3 is greater than 100 cubits.

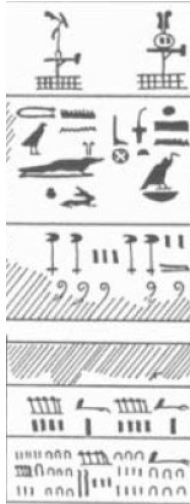


Figure 20. Sample *stbt* calculations on the 4th Nome of Upper Egypt (Schlott-Schwab, 1981, Table I).

- The “administrative” cubit used in the nome to perform the calculation is the royal cubit of seven palms of four fingers of line five.
- As 106 cubits, three palms and five fingers exceed 100, the cubit used in the nome must be smaller than the royal cubit. Adding the “deduction” of one cubit and four palms to 106 cubits and three palms gives a total of 108 cubits. 2701 square metres (the area resulting from the great system) divided by 108 gives a square “*mḥ t3*” of 25 square metres. The square root of 25 metres gives 50 cm for the 4th nome cubit.
- The local cubit used in the 4th Nome of Upper Egypt is the scribe’s palette, the physical representation of the great system.

The White Chapel *stbt* inscriptions are an acknowledgement of the parallel existence of the great and the royal systems.

8.3 The *Remen*

Summary

The “royal” *remen* of 37.5 cm is related to the great system. It is exactly the length of half the hypotenuse of a great cubit multiplied by a small cubit.

Triangular area calculations lead to rectangular and square calculations in the Moscow Mathematical Papyrus (First Intermediate Period). Problems seven and 17 (Gunn and Peet, 1929,

pp. 171–76, plates XXXV and XXXVI) use squares to calculate the area of triangles. The area of each triangle in both problems is multiplied by two to obtain the area of a rectangle twice the size of the original triangle. The rectangles are then converted into squares allowing square root calculations (*kḥbt*-Plate XXXV, Problem 7, Line 5; Problem 17, line 6) to obtain the length of the side of each rectangle. The known ratio between breadth and length (two and a half for problem seven, and two thirds for problem 17) is then applied to find the length of the two missing sides of each triangle.

Gillings (1982, pp. 208–9), using a *mḥ t3* (100 royal cubits multiplied by one royal cubit), gives the length of the double *remen* as 29.135 inches (74 cm). A squared *remen* of 37 cm, according to him, is equal to half a squared (royal) cubit: 37 cm multiplied by 37 cm give 1369 square cm against the 1378.125 square cm expected. Furthermore, the length of the royal *remen* is 37.5 cm, yielding 1406.25 square cm for the (area) *remen*. The discrepancy of almost 100 square cm resulting from a squared (length) *remen* strongly suggests a different origin.

To a square cubit of 0.275 square metre corresponds the area of a rectangle of one great cubit of 60 cm multiplied by one small cubit of 45 cm or 0.27 square metre. One does not need to be aware of the Pythagorean theorem (awareness contested by Gillings [1982, p.242] but accepted by Bibé [1987, pp. 19–22]) to divide this rectangle into two triangles. The hypotenuse of such a triangle is exactly 0.75 cm. The royal (length) *remen* of 37.5 cm (half a double *remen*) is therefore derived from the great system rectangular area construction. As Galán (1990, pp.161–4) remarks the *remen* sign appears after a rectangle in the Rhind Mathematical Papyrus problems 51 and 52, with the meaning of half a rectangle, in opposition to *gs* referring to the “abstract” calculated half of the same rectangle.

8.4 The Ptolemaic Period

The Ptolemies maintain the legacy area units and add the Nilometric cubit to define the *mH-jtn*. The Romans change nothing to the Ptolemaic legacy. Only Ptolemaic and Roman texts provide information regarding the cubits used for areas. Not surprisingly, the area units are aligned with the length of the reformed cubit of 52.5 cm and on the length of the Nilometric cubit of 54/55 cm. Ptolemaic texts describe area units while Roman texts tend to enumerate the units of lengths that are used for area calculations.

8.4.1 *Mh-hty*

In problems 32 to 38 of pCairo 89127–30, 89137–43 the *mH-xty* is mentioned (Parker, 1972, pp. 10–11) corresponding to the *mḥ ht* of Erichsen (1954, p.173) and the *mH* of pBritish Museum 10520 also published by Parker (1972, pp. 10–11). This unit of area is the traditional strip of 100 *mḥ* or *mḥ Ntr* of 52.5 cm by one *mḥ* or *mḥ Ntr* cubit of 52.5 cm.

8.4.2 *Mh-jtn*

The *mḥ-jtn* is also linked to areas. Griffith takes it as a cubit of land of one cubit wide multiplied by 100 cubits long (Griffith, 1892, p. 419). The same definition is given by the Lexikon der Ägyptologie (Helck et al., 1972–1922, p.1210, col. 2). A better and more logical definition of the *mḥ-jtn* is given by Segrè (Segrè, 1928, p. 44, note 5): the *mḥ-jtn* corresponds to 1/96 of a *stbt* :

- One *mḥ-jtn* = 1/96 of a *stbt* (2756.25 square metres divided by 96) or 28.71 square metres. The square root of 28.71 is 5.358; in other words a *mḥ-jtn* corresponds to the area of ten square Nilometric cubits. It is the equivalent of the Old Kingdom *t3* or New Kingdom *mḥ t3* with a change of unit. The *mḥ-jtn* uses a Nilometric cubit instead of ten great cubits multiplied by ten small cubits or ten squared royal cubits.

8.5 Roman Period

From the pOxyrhynchus 669 (Grenfell and Hunt, 1904, Part IV, p. 117, col. i, lines 1–4), it is possible to identify the types of cubits used for area calculations before and during the Roman Period. Two *schoenia* define areas: a *schoenium* of 100 cubits and a schoenium of 96 cubits.

Col. i.

[ἔχει τὸ σχοινίον] τὸ γεωμετρικὸν ὄγδοα η,
[τὸ δὲ ὄγδοον ἔχει] πῆχισ ιβ, ὥστε ἔχειν τὸ
[σχοινίον τὸ γεω]μετρ[ικ]ὸν πηχῶν ρϛ.
[τὸ δὲ]κὸν ἐστὶν πηχῶν ρ.

Figure 21. pOxyrhynchus 669-Col. i-lines 1 to 4 (Grenfell and Hunt, 1904, Part IV, p. 117).

The *schoenium* used in land-survey has eight eighths, and the eighth has 12 cubits, so that the *schoenium* used in land-survey has 96 cubits, while the [roy]¹³ al *schoenium* has 100 cubits

8.5.1 *Schoenium* of 100 Cubits

“The [roy]al *schoenium* has 100 cubits,” this definition of the *schoenium* corresponds to the *ht* of 100 of the former royal cubit of 52.5 cm. It establishes the length of the *schoenium* during the Roman Period: 52.5 m. This *schoenium* of 100 cubits is on par with the *mḥ-hty* area calculations of the Ptolemaic Period mentioned above.

8.5.2 Land-Survey *Schoenium* (96 Cubits)

“The *schoenium* used in land-survey has eight eighths, and the eighth has 12 cubits, so that the *schoenium* used in land-survey has 96 cubits.” Land-survey is the translation of the Greek *gewmetrikon*. The length of the royal *schoenium* is 52.5 metres and is constant regardless of the system used. 52.5 metres divided by 96 give a cubit length of 54.68 cm, matching the length of a Nilometric cubit, on par with the *mH-jtn* area calculations of the Ptolemaic Period.

9 Volume Related Lengths

Two new volume related lengths confirm the administrative role of the royal system. One may question the need for a *nbt* and a *dnj* bag during the Eighteenth Dynasty; sandals and *h3r* sacks could have been used instead, particularly as water is associated to *h3r* sacks during the late New Kingdom, stressing the role of the *h3r* sack as an accounting unit. These new units strengthen the role of a royal system as an administrative system.

9.1 *Tbt* and *Nbj*

Summary

The sandal (*tbt*) and *nbt* are associated with construction excavations. The length of the *tbt* equals seven royal palms or half a great span. The word *nbt* does not appear before the New

¹³ Restoring *basil i* before the *kon* of line 4.

Kingdom. It is a multiple of the sandal or the royal hand: five sandals equal one *nj*; seven royal hands equal one *nj*.

9.1.1 Old Kingdom

9.1.1.1 The Ptahshepses Inscription

In the *mastaba* of Ptahshepses the following graffito can be found (Verner, 1981, pp. 479 ff.):

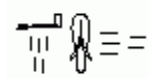


Figure 22. Verner's sandal inscription. (Verner, 1981, p. 479).

The graffito is best read as a volume calculation (*mh 5 tbwt 5*). The cubits express an area. Expressing volumes as an area multiplied by a height is common in the Middle Kingdom (see the Rhind Mathematical Papyrus problem 44 above, where the volumes of granaries are obtained by multiplying an area by a height). The *tbwt* of the graffito corresponds therefore to a height or depth. Verner (1981, pp. 479 ff.) gives a length of about 10 cm for the *tbwt*, while Roik (1993, pp. 62–3) sees it as a division of the *nj*, with a length of 8.125 cm.

9.1.2 Middle Kingdom

9.1.2.1 Papyrus Reisner I

The Papyrus Reisner I is a Twelfth Dynasty analysis of the volume of earth and rubble that needs to be excavated for the construction of a temple. The results are used to determine the manpower and rations necessary to complete the project. The papyrus is divided into sections by Simpson. Section K is the summary of three sections (G, H, and J) to which is added the total number of man-days necessary for the project. The word *tbwt* appears in two sections: G and H. Simpson gives the following tentative definition for the *Tbwt* calculations encountered in the papyrus (Section G, Plate 13A, lines 16–8 and Section H, Plate 14A lines, lines 32–4):

“The building term in English, ‘footing’ used in the foundations for walls, etc., is offered with reservation as the most acceptable translation.” (1963, p.79, col. 1)

In other words, Simpson considers the *Tbwt* as architectural elements of the temple.



Figure 23. Papyrus Reisner I-Plate 13A (Extract-Simpson, 1963)



Figure 24. Papyrus Reisner I - Plate 14A (Extract-Simpson, 1963).

Plate 13A	Columns						
	f	e	d	c	b	a	
line 1	[<i>jr m ḥsb</i>]	<i>sty</i>		[<i>mdwt</i>]	<i>wsh</i>	<i>3w</i>	Not included in Figure 23.
line 16	1/10	16	1	2	2	4	
Plate 13A							
line 17	6, 1/4, 1/10, 1/20	64	2	2	4	4	[...] <i>rdyt</i> <i>n=f</i> [...] <i>tbwt</i> [...]

Plate 13A	Columns						
	f	e	d	c	b	a	
line 18	3, ½, 1/10	36	2	2	3	3	
Translation							
line 1	Man-days	Product	[Times]	Depth Thickness	Width	Length	Not included in Figure 24.

line 16	1/10	16	1	2	2	4	
line 17	6, 1/4, 1/10,1/20	64	2	2	4	4	[. . .] given to him [. . .] Sandals [. . .]
line 18	3, ½, 1/10	36	2	3	3	3	

Three elements suggest a different interpretation.

- In Plate 13A, a sign resembling the Roman letter “Z” precedes the amount in column A and follows the amount in column D (place which is used by the times sign, *r*, in Plate 14A). Simpson (1963, p.61, col. 1) considers the sign as a clerical inscription separating calculations between columns, which he translates as a colon (:). Plate 13A has six columns. It seems rather strange that only two columns should be separated by a special marker. Column d of Plate 13A does not have any heading title. It represents the number of times that the product of length, width, and thickness is to be multiplied to reach a volume (Simpson, 1963, p.53, col. 2). Plate 14A does not show the Roman letter “Z.” In column D the mouth sign (Gardiner D 21) with the meaning “times” (Gardiner, 1988, p.126 -163–5) appears. Plate 13A and 14A show two different units: cubits and sandals. The meaning of the Roman letter “Z” in column A and D of Plate 13A could very well be the cubit sign (forearm with palm turned down [Möller sign 679]), rejected by Simpson. The meaning corresponds then to a translation of *tbwt* into cubits for columns A, B, and C, of Plate 14A, while column d of Plate 13A is a multiple on par with Plate 14A column d: *r* (times). As the translation is done from one unit (*tbwt*) to another (*mḥ*) the conclusion (and the justification of a *tbwt* as metrological division in this thesis) is that the *tbwt* is a division of the royal cubit, representing through lack of alternatives (four, five, six, and eight fingers making up respectively a palm, a hand, a fist, and a double palm), seven fingers or a quarter of a royal cubit.
- In the Rhind Mathematical Papyrus, the word *tbwt* is used in the pyramid problems 56, 57, 58, and 59 with the word *wh3*. A *wh3-tbwt* is a ground measurement understood by Peet

(1923, p.97) as the diagonal or the base of a pyramid. The *tbwt* of Plate 14A, line 33 (the *tbwt* of Plate 13A line 17 are part of a line with lost signs) could represent, according to Simpson, architectural “footings.” Simpson’s explanation (1963, p.79 col. 1–2 is not very convincing. He suggests that the *tbwt* could be the sand footing of the pillars in the construction, but admits that no pillars are mentioned in the project. To consider the *tbwt* as a unit of measurement is more compatible with the Rhind Mathematical Papyrus’ expression *wh3-tbt*.

- Finally, the inscription in the *mastaba* of Ptahshepses does not fit Simpson’s architectural hypothesis. The inscription reads five cubits, five sandals. The five sandals correspond, the way they are written, to the place usually occupied by the more common division of the cubit, the palm. If the sandals were architectural units, the Ptahshepses inscription would read *tbwt 5 mh 5*, five “footings” of five cubits.

9.1.3 New Kingdom

Summary

With the New Kingdom appears the *dnj* (a volume unit equal to one cubed royal cubit [Cerný, 1973, pp.20–2]). The *dnj* is regularly associated with the word *nbj*. This association relates the *nbj* to the royal system, particularly the sandal and the hand, the *nbj* being a multiple of both divisions.

9.1.3.1 *Nbj*

The *nbj* is one unit of length used in the context of earth and rubble excavations. The term *nbj* appears for the first time in the ostraka found in the Tomb of Sen-Mut (Hayes, 1942, pp. 37 ff.). These ostraka are related to the actual construction of the tomb. The *nbj* mentioned in the dimensions is associated with royal cubits, as a length, in *qstr^ckqn* 62 and with volumetric (*dnj* sacks) in *ostrakon* 69: the volumes expressed in *nbj* are collected in *dnj* sacks.

Table 25. Ostrakon 62 - Plate XIII

End of Line 2	<i>hty(w)-Ntr 11</i>	11 masons,
Line 3	<i>jr n nbj 1 m mdwt r nbj 6 m wsh</i>	amounting to: 1 <i>njb</i> in depth by six <i>najs</i> in width.
Line 4	<i>r-gs mh 1m rkr Nw</i>	besides one cubit towards the interior.

Table 26. Ostrakon 69 - Plate XIV

Line 1	<i>3bd fqur 3t [sw] rht p3 b'sks n p3 b'wd n n'eb n sdmw-š</i>	Month four of Inundation [day] report of the corvée of the servants
Line 2	<i>jj hr p3 hm ntr m hrw pn nbj 11 hrty-ntr Hpy[hr]</i>	who came with the hem-netjer priest on this day: 11 <i>najs</i> . The mason Hepy[her]-

Table 27 Ostrakon 69 - Plate XIV

Line 3	<i>s3=f dn(j) hrty-ntr Bš3w dnj qne tw3w Snwj nfr 3'y [dn]</i>	Saf one dnj sack. The mason Bashaw one dnj sack. The sho Snwjnefer smoothed 1 [dn]-.
Line 4	<i>j 1 mh 6</i>	j (sack) six cubits.

This association is an indication that the *njb* belongs to the royal system. Its length is defined as the length of a stonemason's pick of 65 cm (Hayes, 1941, page 36) confirming the length definition given by Roik (1993, p. 5). The unit previously used in the same context (excavations) during the Old and Middle Kingdoms was the sandal (*tbt*) of 13.125 cm, or one quarter of a royal cubit. A *njb* of 65.625 cm represents five sandals (*tbwt*). The close multiple similarity between the *tbt* and the royal cubit (4 *tbwt* equal one royal cubit) on one hand, and the *tbt* and the *njb* (5 *Tbwt* equal on *njb*) indicates that the *njb* belongs to the royal system.

9.1.3.2 Examples

9.1.3.2.1 The Tomb of Sen-Mut

As Sen-Mut “coined,” at least according to the available records (the *ostraka* found in his tomb), the word *njb* as a metrological unit (65.625 cm), it is interesting to check whether this new unit appears in the tomb itself. Dorman (1991, p. 30) in his summary of the cubit system used in Sen-Mut's tomb (Tomb 71) finds only two discrepancies among royal cubit calculations: 115 cm (the jambs of the main entrance) and 35 cm (the outermost niche surfaces between the windows): both lengths belong to the great system.

9.1.3.2.2 The Cenotaph of Seti I

The inscriptions in the Cenotaph of Seti I (Frankfort, 1933, pp. 93–4, Plate XC, 1) also associate the words *nbj*, *dnj*, and cubits very closely.

Line 7	<i>jryt m dnj m p3 dnjt nty hr rsj 3h Jmn R° nh wd° snb n Wsjr</i> <i>jryt m m3wt</i>	Made in dnj (bags) in the dike which is south of Akh-Amen-Ra-Ankh-Wudja-Seneb-n-Wusir. Renewed:
Line 8	<i>nbj 25 wsh̄t n mh̄ 3 mdwt n mh̄ 2 jrj.n=f m jrr s3tw nbjw 25</i> <i>wsh̄t n mh̄ 3 mdwt n mh̄ 1 gs</i>	<i>nbjs</i> 25, width 3 cubits, depth two cubits. What it did in the canal: ground: soil 25 <i>nbjs</i> , width of three cubits, depth of one and a half cubit.

The first part of the inscription (lines 1 to 6) is an inventory of block stones quarried and transported necessary to repair the dikes of a canal (*Jrr-* 𓆎𓅓 canal, a loan word from ancient Egyptian (Brown 1979, p.384) Coptic ⲉⲓⲟⲡ [Westendorf, 1965, p. 51]). The volumes are expressed in *nbj* for length and royal cubits for width and height and handled in *dnj* bags, confirming more precisely the close relationship between the *nbj*, *dnj* and royal cubit already appearing in the *ostraka* from the Tomb of Sen-Mut.

9.1.3.2.3 Panastasi I

The Papyrus Anastasi I is a challenge between two scribes on how to provide answers to difficult problems, the construction of a ramp being one of them. In this context, it is interesting to note, particularly from people flaunting their knowledge, that the word *nbj* never appears in a construction project far bigger (even if it is theoretical) than Sen-Mut’s tomb or the Cenotaph inscriptions of Seti I.

All the cubit dimensions mentioned (5, 55, 30, 60, and 730) translate into even numbers of *nbjw* when divided by 65.625 cm (the length of a *nbj*):

- 5 cubits = four *nbjw*
- 30 cubits = 24 *nbjw*
- 55 cubits = 44 *nbjw*
- 60 cubits = 48 *nbjw*
- 730 cubits = 584 *nbjw*

The fact that the word *nbj* is not mentioned in pAnastasi I is an indication that the *nbj* as a unit of measurement is not as universal as Roik suggests: it is merely a specialized length division of the royal system used in excavation projects and easily replaced by royal cubits: one *nbj* equals one royal cubit and a quarter.

9.2 Ptolemaic Period

Nbj and **naubion** are related but not similar. The **naubion** is a volumetric unit derived from the word *nbj* as the suffix **on** in Greek usually denotes an origin.

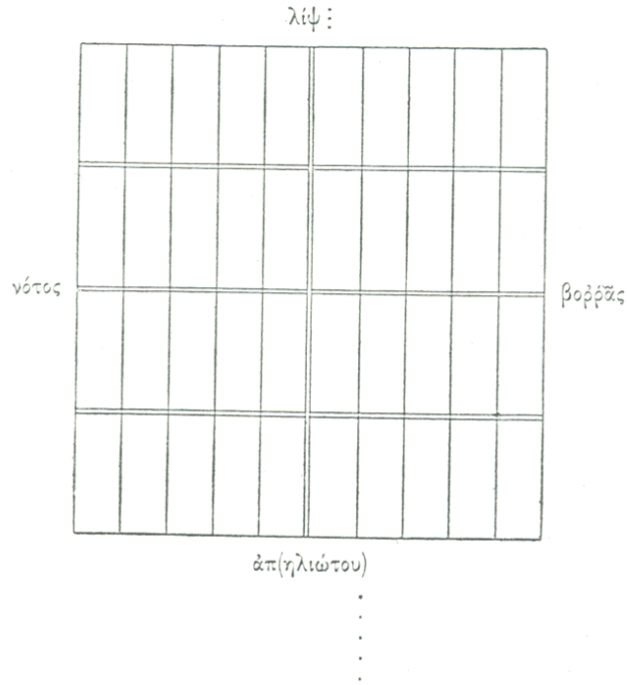
9.2.1 The Ptolemaic **naubion** and **awiPion**

When the word *nbj* is used in Ptolemaic texts, it does not carry any specific metrological meaning and is associated with the ceremony of the stretching of the cord (Zába, 1953, p. 59, Plate 1, line aA; Wilson, 1997, pp. 501–2). By Ptolemaic times, there are two units of volumes used for earth excavations, the **naubion** and the **awiPion**. Liddell and Scott give the following definitions:

- **Naubion**: “cubic measure in Egypt, with side = two royal cubits (in Roman times, three cubits) used with application to earth as a unit. In singular, tax paid by landholders in lieu of digging so many **naubia**” (Liddell and Scott, 1968, p. 1161).
- **awiPion**: “Egyptian measure of capacity = 2 cubic cubits” (Liddell and Scott, 1968, p. 299).

The Papyrus Lille I (Jouguet, 1928, pp. 13–25) is an estimate of the price needed to repair the dikes and canals within 10.000 arouras. The papyrus is dated to the 27th year of Ptolemy II, or 258–59 B.C. The recto of the papyrus, the measurement part of the estimate, clarifies the definition of the **naubion**.

Στοθεῖη ἀντιγραφ(εὺς) Ἀπολλωνίωι. L κζ κ...υπ...ν..
 τὸ αὐτὸ μηνὸς Φαῶφι...
 ἀντιγραφομένου Διοδώρου.



Ἔστιν ἡ περίμετρος τῶν μυρίων ἀρουρῶν σχοινία υ, χῶματα δὲ δ,
 ἀπὸ νότου εἰς βορρᾶν
 ; καὶ ἐν μέσῳι χῶματα γ ἀπέχον^α ἀπ' ἀλλήλων σχοινία κε,
 καὶ ἄλλοι διαπλευρισμοὶ ἀπὸ ἀηλιώτου εἰς λίθα^θ, ἀπέχον
 ἀπ' ἀλλήλων σχοινία δέκα, ἐν ταῖς M ἀρούραις περιγῶματα μ ἀν(ἄ) (ἀρούρας)
 ὡς ἡ μέτροσις κε ἐπὶ ι, ὡς διαγέγραπται ἐν τῷ πλινθείωι / χῶματα
 ἀνὰ σχοι(νία) ρ / Ἄχ, ὧν δεῖ τὴν ἀνασκαφὴν γενέσθαι· πλάτος μὲν τοῦ

Figure 25. Recto pLille I – p. 16 (Jouquet, 1928 , p. 16).

ὀρύγματος πήχεις δ, βάρους δὲ β, δ ὑποτιθέμεθα ἐκ τοσούτου ἄν
 10 ὀρύγματος γενέσθαι τὰ ὑποκείμενα μέτρα τῶν χωμάτων· γίνεται οὖν εἰς τὸ
 σχοινίον ναύβια πς, εἰς δὲ τὰ Ἄχ ναυβίων $\overset{\ddot{M}}{\overset{\circ}{Z}}\chi$, [εἰς γ τῶν δ $\overset{\circ}{\Gamma}$]
 [/ $\overset{\circ}{\Lambda}$ α $\overset{\circ}{\Gamma}$ $\overset{\circ}{\Xi}\eta$]· καὶ ὑδραγωγῶν δ' ὧν δεῖ γενέσθαι πρὸς ταῖς ὑπαρχούσας
 δ ἄλλας δ, ἀνά σχοινία ρ / σχοινία·υ ἀν(ἀ) πς / ναύβια $\overset{\ddot{M}}{\overset{\circ}{\Delta}}\upsilon$ / $\overset{\ddot{M}}{\overset{\circ}{B}}$.
 Ἐὰν μὲν κατὰ χειμῶνα συντελῆται τὰ ἔργα, τίθεμεν ἕσεσθαι
 15 εἰς ο τοῦ στατηῆρος, ὥστε εἶναι $\overset{\circ}{\Lambda}$ α $\overset{\circ}{\Gamma}$ $\overset{\circ}{\omega}\lambda\delta$ · ἐπιβάλλει τῆι (ἀρούραι)
 $\overset{\circ}{\Gamma}$ α $\overset{\circ}{\beta}\lambda\acute{\alpha}\beta\omicron\varsigma$,

Figure 26. Recto pLille I – p. 17 (Jouquet, 1928, p. 17).

“The perimeter of the 10,000 arouras is 400 schoenia; there are four dikes; there are three dikes in the middle going from South to North, separated from each other by 25 schoenia, furthermore there are nine crosswise levies going from West to East, separated from one another by ten schoenia. There are therefore in the 10,000 arouras 40 divisions of 250 arouras (25 by 10) each, as shown on the plan. The total number of dikes is 16, each 100 schoenia long, or 1600 schoenia that need to be excavated; the width of the trench is four cubits, its depth two cubits as we anticipate that a trench of this dimension will produce the dimensions planned for the dikes, namely a total of 86 naubia per schoenium and 137,600 naubia for the 1,600 schoenia. Four new aqueducts need to be added to the previous four, 100 schoenia each, a total of 400 schoenia of 86 naubia each, 34,400 schoenia in total. Naubia [grand] total: 172,000. Provided work starts during the winter, the estimate is 70 naubia per stater: total price one talent 3,834 drachmae plus one drachma per aroura for incidentals.”

[Jouquet’s translation revised and translated into English by A. Hirsch]

86 **naubia** equal a volume of one *schoenium* in length multiplied by four cubits in width multiplied by two cubits in depth. The *schoenium* of pLille I must match the definition given by the later pOxyrhynchus 669 (Figure 27-pOxyrhynchus 669-Col. i-lines 1 to 4) as there is no indication of a change in its length:

“The *schoenium* used in land-survey has eight eighths, and the eighth has 12 cubits, so that the *schoenium* used in land-survey has 96 cubits, while the royal *schoenium* has 100 cubits.”

The length of the *schoenium* is a constant: 52.5 metres. The value of one eighth of a *schoenium* equals 6.565 metres or ten *nbjw*, that is to say that 80 *nbjw* equal one *schoenium*. The rest of the calculation becomes straightforward; it is performed from area to volume:

- Area: 86 **naubia** are equivalent to one *schoenium* (80 *nbjw* multiplied by 65.625 cm [the length of a *nbj*] equal 52.5 metres), multiplied by four Nilometric cubits (54 cm multiplied by four equal 2.16 metres), or 112.66 square metres.
- Volume: This area (112.66 square metres) multiplied by two Nilometric cubits (54 cm multiplied by two equal 1.08 metres) give a volume of 121.60 cubic metres.
- **Naubion**: This volume (121.60 cubic metres) divided by 86 gives the volume of a **naubion** (1.41 cubic metres), the cubic side of which equals 2 Nilometric cubits (1.12 metres) and not the length of the two royal cubits expected (1.05 metres) according to Liddell and Scott’s definition.

Sample calculations of the **awil~~ion~~** are given in Papyrus Petrie III. Segrè, came to the conclusion that the basis for the calculation of the **awil~~ion~~** was the royal cubit and that **awil~~ion~~** and **naubion** are the same thing. This is perhaps not completely accurate. Page 345 of the appendix of Papyrus Petrie III, quoted by Segrè (Segrè, 1928, p.25, note 4) shows an **awil~~ion~~** calculation, expressed in double cubits for width and height: 2 *schoenia*, multiplied by a width of two double cubits, multiplied by a height of one double cubit give 43 **awil~~ia~~**.

The principles used in the **naubion** calculation above apply for the **awil~~ion~~** calculation: the value of the *schoenium* is constant (52.5 metres) and the volume calculation is based on a rectangular area multiplied by a height. The only difference with the **naubion** calculation is that the cubit used is not the Nilometric cubit but the Alexandrian cubit (Public or Carpenter’s cubit) of six palms and 24 fingers with a length of 46/47 cm.

- Area: 43 **αwiPia** are equivalent to two *schoenia* (160 *njw* [of 65.625 cm each] equal 1.05 metres), multiplied by four Alexandrian cubits of 46 cm each equal 193.2 square metres.
- Volume: This area (193.2 metres) multiplied by two Alexandrian cubits (0.92 metres) gives a volume of 177.74 cubic metres.
- **ΑwiPion**: This volume (177.74 cubic metres) divided by 43 gives the volume of an **αwiPion** (4.13 cubic metres), the cubic side of which (1.60 metres) equals three Nilometric cubits of 53.5 cm each¹⁴.

9.3 The Roman Period

9.3.1 The Roman **xuʁon**

The word *njw* as a metrological length unit is replaced by the word **xuʁon** in pOxyrhynchus 669. The **xuʁon** is used to define the **naubion** (and the **αwiPion**). Liddel and Scott give the following definition of the **xuʁon** (Liddel and Scott, 1968, pp. 1191–2):

*“II. in sg., piece of wood, log, beam, post; V. a measure of length, = 3 (also 2 2/3 cubits) the side of the **naubion**.”*

The definition of the **xuʁon** corresponds exactly to the definition of the *njw*: a piece of wood used (from the Eighteenth Dynasty) as a metrological unit of length which it replaces in Greek texts.

¹⁴ Based on Public or Carpenter cubit of 47 cm, the length of the Nilometric cubit is 54 cm in the **αwiPion** calculation, illustrating the ambiguity when making the difference between 7 palms of the Alexandrian system (circa 53 cm) and the Nilometric cubit (circa 54 centimeters).

	[τῶ δὲ ξύλῳ καταμ]ετρί[τα]ι τὰ ναύβια· τὸ μὲν βα-	
	[σιλικόν ἐστι π]ηχῶν	γ,
	[παλαιστῶν]	ιη,
	[δακτύλων]	οβ.
15	[τὸ δὲ] ἐστὶν πηχῶν	ββ',
	[παλαιστῶν]	ις,
	[δακτύλων]	ξδ.
	[ὥστ' ἔχειν τὸ σχοινίον] τὸ γεωμετρικόν	
	[ξύλα βασιλικά]	λβ,
20	[ξύλα]	λς.

Figure 27. pOxyrhynchus 669-col i-lines 11 to 20 (After Grenfell and Hunt, 1904, Part IV, p. 117).

“*Naubia* are measured by the *xulon*; the royal *xulon* contains three cubits, 18 palms, 72 fingers, while the . . . *xulon* contains 2 2/3 cubits, 16 palms and 64 fingers; so that the *schoenium* used in land-survey contains 32 royal *xula* and 36 . . . *xula*.”

- The length of the royal **xulon** is equal to one *schoenium* (52.5 metres) divided by 32 or 1.64 metres equal to three Nilometric cubits of 54 cm. The length of this **xulon** is aligned with the **naubion** calculations of pLille I above.
- The undefined **xul on**, 36 of which make a *schoenium* of 52.5 metres, has a length of 1.45 metres. These 1.45 metres divided by the length of an Alexandrian cubit of 46/47 cm equal three Alexandrian cubits, used for the **awiPion** calculations of pPetrie III above.

9.4 Conclusions Derived From the Ancient Egyptian Closed Metrological System

Weight and volume related lengths linked to emmer, barley, and wheat have illustrated the agricultural origin and evolution of the majority of ancient Egyptian linear measurements from the Old Kingdom down to the Ptolemaic and Roman Periods. The volumetric capacity defining the great and royal system has been estimated, in this thesis, to 450 *hins* during the Old Kingdom. This estimation is linked the volumetric capacity of 100 *dw.t* containers equal to 200/250 litres, rounded to 450 *hins*. This estimation in *hins* fits the ancient Egyptian method: the

Ptolemaic pBritish Museum 10399 problems 42 and 43 (Parker, 1972, p. 57) relates, in a genuinely ancient Egyptian manner, length of cubits with *hins*. It also fits the 2/3 principle of ancient Egyptian mathematics: 2/3 of 450 *hins* give the volumetric capacity of a royal cubit, 300 *hins*. Applying the specific weight of emmer and barley to the 450 *hins* define the great and small *debens*. The great *deben* is related to barley and to the royal system, the small one to emmer and the great system. The organization of the treasury in the Third Dynasty corresponds to the importance of barley over during the Old Kingdom. It is fitting that the royal system, based on barley, becomes the official metrological system to match the relative importance of barley over emmer.

From area related lengths, the parallel use of the great and the royal systems appear in garment and field calculations. They are difficult to quantify in pMoscow 18, the Gebelein linen inscriptions, the Abusir papyri, and the Hekanakhte Papers. The *stbt* inscriptions of the White Chapel of Sesostris, however, indicates the legacy administrative conversion role associated with the royal cubit and confirms the existence of the great system by providing an indication of the cubit and system used in each ancient Egyptian nome.

Volume related lengths confirm the administrative role of the royal system. The *tbt* appears during the Old Kingdom in one inscription in the *mastaba* of Ptahshepses and during the Middle Kingdom in Papyrus Reisner I. During the New Kingdom, two new metrological units are introduced. The *nbt* (seven royal hands or five *tbwt*) is associated with a *dnj* in the ostraka found in the Tomb of Sen-Mut and the inscriptions of the Cenotaph of Seti I. The volume of a *dnj* equals the volume of a cubed royal cubit, linking the *nbt* to the royal system. These new units replace the *tbt* and *h3r* sack in construction projects. Their introduction matches the reversal of the importance of two commodities, barley and emmer, already traceable in the change of the volumetric capacity of the New Kingdom (*h3r*) sacks and confirms the conversion role played by the royal cubit evidenced by the black and white inscriptions of the Amenemope cubit artifact.

Chapter 5 - Conclusion

Before the Ptolemies and the Roman Period, ancient Egyptian linear metrology can be defined as an agriculture-based, dual, parallel, and backward-compatible closed metrological system using barley and emmer as reference commodities. This dual parallel system is materialized by the use of two measuring devices, the royal cubit and the scribe's palette. A key $1/3$ ratio explains ancient Egyptian linear measurements and their origin. Ancient Egyptian linear measurements are linked to agriculture. From the Rhind Mathematical Papyrus, the volume of a *h3r* sack equals the volume of $2/3$ of a cubed royal cubit. For tax purposes, *h3r* sacks contain either emmer or barley. Emmer is $1/3$ lighter than barley, consequently, for an equal weight, a container filled with emmer will be $1/3$ greater than a container filled with barley. The lengths derived from both containers share the same $1/3$ ratio: for a common length of a royal or great palm division, before the reformed cubit and the Ptolemaic and Roman Periods, the great (3) finger division is $1/3$ longer than the royal finger division. The great (3) system is therefore an independent linear system linked to the cultivation of emmer while the royal system is linked to the cultivation of barley. The Ptolemies, followed by the Romans, inherited and adapted these two legacy systems. They added their own measurements following the introduction of wheat (*triticum durum*) and water as reference commodities.

The role of the royal cubit is limited to “official” architectural projects and to administrative conversion between the great and royal systems. This limited legacy role can be dated to the final organization of the treasury responsible for grain taxation during the Third Dynasty (Sekhemkhet) which matches, in time, the first mention of a *hin* associating linear divisions and volumes, and the brutal introduction of fraction of finger divisions in the Palermo Stone at the end of the Second or at the beginning of the Third Dynasty. The reversal of importance of emmer over barley during the New Kingdom did not warrant a change in linear metrology. The royal system, based on barley, remained the official administrative system even when the great system was overwhelmingly used in everyday measurements during all periods prior to the Ptolemaic Period.

10 Glossary of Key Metrological Terms

10.1 Accounting

- **Grain** (*sšr*) - The Rhind Mathematical Papyrus does not use the word *jt* (barley) or *bdt* (emmer), but *sšr* (grain) when volume calculations are involved (problems 41 to 46). This implies that *sšr* must be the accounting reference commodity for grain with a base of one. The weight and volume ratios of emmer and barley, as we know them in modern grams and litres (using water as a reference commodity), are not affected: emmer still weighs 44% of the weight of one volume of *sšr* while barley will weigh 66% of the weight of the same volume of *sšr*. These ratios reflect the physical properties of both emmer and barley (see Excursus A) regardless of the base-one reference commodity used.
- **Hekat** (*ḥkꜣt*) – A multiple of the *hin* (see *hin* below) and a division of the *ḥꜣr* sack. There are ten *hins* in one *hekat*.
- **Sack** (*ḥꜣr*) – A multiple of the *hin*; defined as 20 *hekats* in the Middle Kingdom and 16 *hekats* in the New Kingdom (200 and 160 *hins* respectively).

10.2 Commodities

- **Barley** (*jt*) – Linked to the royal system.
- **Emmer** (*bdt*) – Linked to the ʿ3 (great) system.
- **Water** – Linked to the Nilometric system.
- **Wheat** (*swt*) – *triticum durum* Linked to the Alexandrian system.

10.3 Systems

- **Alexandrian** – Based on wheat (*swt*) – *triticum durum*. The Ptolemaic innovation, based on wheat, carried over during the Roman Period.
- **Digital** – From the Latin *digitus* - finger. A royal or reformed cubit divided into finger divisions.
- **Great** (ʿ3) – Based on emmer (*bdt*).
- **Nilometric** – Based on water.
- **Royal** (*nsw*) – Based on barley (*jt*).

- **Uncial** – From the Latin *uncia* – the twelfth part. A reformed uncial *djeser* corresponds to 18 digital finger divisions divided by 12.

10.4 Specific Weights

The specific weight of a commodity is the weight of a commodity for a given unit of volume. Using water as a reference commodity, the specific weights below are expressed in grams per litre.

- **Barley:** 660 grams per litre (royal [*nsw*] system).
- **Emmer:** 440 grams per litre (great [ʕ] system).
- **Water:** 1000 grams per litre (Nilometric system).
- **Wheat:** 800 grams per litre (Alexandrian system).

10.5 Area

- **Aroura** (*stbt*) – Equal to 100 *mḥ ʕ* multiplied by 100 *mḥ šrj* (Old and Middle Kingdoms – great [ʕ] system), or 100 *mḥ nsw* multiplied by 100 *mḥ nsw* (Middle Kingdom, New Kingdom, Late Period – royal [*nsw*] system), or 100 Ptolemaic *mḥ-hty*, or 96 squared Ptolemaic *mḥ Dḥwtj* (Alexandrian system), or 96 squared Roman Nilometric cubits (Nilometric system). The known divisions of the *stbt* are: the *rmn* ($1/2$ *stbt*), the *ḥsb* ($1/4$ *stbt*), the *s3* ($1/8$ *stbt*), the *sw* ($1/16$ *stbt*), the *rm3* ($1/32$ *stbt*).
- **Cloth-weaver Cubit** (*Mḥ-ḥbs*) – Both a unit of area and length linked to textiles, disappears at the end of the Old Kingdom, mentioned again during the Ptolemaic and Roman Periods. Part of the ʕ (great system); known divisions are the *szf*, *ḥʕ*, and *d3t* (lengths unknown). As unit of length, see Cloth-weaver Cubit in the Length section below.
- **Mḥ-jtn** – Ptolemaic, equal to $1/96$ of a *stbt* (*aroura*) or the area of 100 *mḥ Dḥwtj* of $53/54$ cm by one *mḥ Dḥwtj* of $53/54$ cm.
- **Mḥ-hty** – Ptolemaic, equal to 100 *mḥ Ntr* of 52.5 cm by one *mḥ Ntr* of 52.5 cm, equal to $1/100$ of a *stbt* (*aroura*).
- **Remen** (*Rmn*) – A right-angled triangle area equivalent to half a *stbt* obtained by multiplying 100 small cubits (*mḥ šrj*) by 100 great cubits (*mḥ ʕ*) which may explain the

“tongue of land” determinative (Gardiner sign N 20 [8]) in certain geographical or land-related terms, particularly *ꜣḥt* (field). Half the length of the hypotenuse of a rectangle the side of which corresponds to one great cubit (60 cm) multiplied by one small cubit (45 cm) corresponds to the length of a royal *remen* (37.5 cm).

10.6 Cubits

- **Alexandrian Cubit** – Ptolemaic and Roman – with a length of circa 47 cm given by a foot and a half (the cubic root plus half the cubic root) of the volume of a talent of 30.4/30.88 kilograms obtained by multiplying 80 *hins* (the volume of a Ptolemaic *artaba*) by the specific weight of wheat (800 grams per litre).
- **Cubit** (*Mḥ*) – See Reformed Cubit.
- **Cubit of Thoth** (*Mḥ Dḥwtj*) – Ptolemaic – Its length (53/54 cm) is very close to the Nilometric cubit (see Nilometric Cubit), leaving doubt about the system it belongs to: seven Alexandrian palms (Alexandrian system based on wheat) or the precursor of the Nilometric system (using the volume of a talent of water as a reference commodity, the usual Greek and Hellenistic way of deriving lengths from volumes).
- **God’s Cubit** (*mḥ Ntr*) – Ptolemaic – The old royal cubit (*mḥ nsw*) of 52.5 cm.
- **Great Cubit** (*Mḥ ʿ3*) - The cubic root of 450 *hins* (100 Old Kingdom *ḏwt* - 217/220 litres) equal to a length of circa 60 cm. Shown in white on the Amenemope cubit artifact.
- *Mḥ* – See reformed Cubit.
- *Mḥ ʿ3* – See Great Cubit.
- *Mḥ Nsw* – See Royal Cubit.
- *Mḥ Ntr* – See God’s Cubit.
- *Mḥ Dḥwtj* – See Cubit of Thoth.
- **Nilometric Cubit** (**Phḥꜣw NilometrikoV**) – Roman – 54 to 55 cm in length derived from the cubic root of 343 *hins* of water. Equivalent to the Ptolemaic *mḥ Dḥwtj*. It illustrates the fundamental difference between Roman (and Ptolemaic) and ancient Egyptian calculation principles. Roman and Ptolemaic cubit calculations are based on a foot (the cubic root of the volume of water of given talent) where a cubit equals a foot and a half,

while purely Egyptian cubit calculations are based on the cubic root of the volume of the weight of a given commodity: 343 *hins* of water weigh (343 multiplied by 482/485 grams) 166.35 kilograms. 166.35 kilograms of water have a volume of 166.35 litres. The cubic root of 166.35 litres gives the length of the Nilometric or *mḥ Dḥwtj* cubit: between 54/55 cm.

- **Phéw Nilometrikov** – See Nilometric Cubit.
- **Reformed Cubit** - Twenty-sixth Dynasty – The length of the royal cubit (*mḥ nsw*) divided into six palms instead of seven; also known as the Ptolemaic *mḥ*. The reformed cubit is *uncial* (it can be divided by 12) by opposition to *digital*, the royal cubit, which can only be divided into finger divisions.
- **Royal Cubit** (*mḥ nsw*) – Old Kingdom to Ptolemaic - The cubic root of 300 *hins* (144 litres) equal to a length of circa 52.5 cm.; shown in black on the Amenemope cubit artifact.
- **Second Cubit** – Expression used by Lepsius; in this thesis, equivalent to the great cubit (*Mḥ ʕ3*).
- **Scribe’s Palette** (*Gstj*) – All periods - Five palms of the great (*ʕ3*) system or 50 cm; the only physical evidence of the great system; assimilated to a cubit (*Mḥ*) in the Edfu temple inscriptions.

10.7 Cubit Divisions

- **Bema** (*Bḥmā*) – See *Remen* below.
- **Cloth-weaver Cubit** (*Mḥ-ḥbs*) – The Ptolemaic and Roman length equivalent of a royal (*nsw*) *remen*, known as a **Phéw LinoufikoĒ** (see Area, Cloth-weaver Cubit above).
- **Djeser** (*Dsr*) – All periods – The “sacred” conversion division par excellence with a length of 30 cm. Two royal *djesers* equal one great cubit (*mḥ ʕ3*). In pOxyrhynchus 669, the *djeser* corresponds to the Egyptian Foot. It is shown in black on the Amenemope cubit artifact as a true royal (*nsw*) division. It could have been shown in white as half a great cubit (*mḥ ʕ3*) to reflect its origin (the great [*ʕ3*] system).

- **Double Palm** – Half a *djeser* – All periods – 15 cm - Written in black on the Amenemope cubit artifact probably for the same reasons why the *djeser* is written in black on the artifact (see *djeser*). Becomes the Ptolemaic and Roman *Lichas*.
- **Egyptian Foot** – See *djeser*.
- **Finger** (*Db^ϙ*) – All periods - In the royal system 1.875 cm long, in the great system 2.5 cm long. Three great fingers equal one palm of the great system written in white on the Amenemope cubit artifact. The Ptolemaic and Roman **δαυτιον**.
- **Fist** (*3mm*) – Old Kingdom to Ptolemaic - Twice three great (*ϙ*) fingers or 15 cm; the equivalent of a double palm in the royal (*nsw*) system or half a *dscr*.
- **Foot** (**Που^ϙ Αιγυπτιον**) – Ptolemaic and Roman – The cubic root of the volume of water of a talent. In Greek and Hellenistic metrology, one foot and a half equal a cubit.
- **Great Span** (*Pd^ϙ*) – Old Kingdom to Ptolemaic - A division of the royal cubit with the length of half a royal cubit (26.25 cm) shown in black on the Amenemope cubit artifact.
- **Hand** (*Drt*) – Old Kingdom to Ptolemaic – A quarter of the length of a royal *remen*. A division of the *nbj* of 65.625 cm; seven hands equal one *nbj*.
- **Lichas** (**Λικα^ϙ**) – See Double Palm.
- **Nbj** (*nbj*) – New Kingdom to Ptolemaic - A multiple of the Old and Middle Kingdoms *tbw* appearing only during the New Kingdom with a length of 65.625 cm equal to five *tbwt* of 13.125 cm, or seven royal (*nsw*) hands (*drt*), or a royal cubit and a quarter implying (contrary to Roik's conclusions) that the *nbj* belongs to the royal (*nsw*) system and is not an independent length unit; used in volumetric calculations.
- **Palm** (*Šsp*) – All periods - The basic division of the royal (*mḥ nsw*) and great (*mḥ ϙ*) cubits. In both systems one *Dsr* equals 4 palms; equivalent to the Ptolemaic and Roman **Παλαισθη**
- **Pal aisthv** - See Palm.
- **Ph^ϙ Δημοσιον τεκτονικον** – See Small Cubit (*Mh šrj*).
- **Ph^ϙ Λινοφικον** – See Cloth-weaver Cubit.
- **Pou^ϙ** – See Foot.

- **Πουῦ Αἰγυπτίου** – See Egyptian Foot.
- **Public or Carpenter’s Cubit (Phεῦ Dhmosioῦ keÉTektonikoῦ)** – See Small Cubit (*Mh šrj*).
- **Remen (Rmn)** – Old Kingdom to Ptolemaic - Both a unit of length and of area (see Area above). As a length division, the great (ϙ) *remen* is the length (75 cm) of the hypotenuse of a right-angled triangle the sides of which are a small (*mh šrj*) and a great cubit (*mh ϙ*); also known in the literature as a double *remen* or in pOxyrhynchus 669 as a **Bhḥā**.
- **Sandal (Tbt)** – Old Kingdom to Eighteenth Dynasty - The precursor of the *nbt*. Appears in texts and inscriptions of the Old and Middle Kingdoms and is replaced by its multiple (the *nbt*) during the New Kingdom. Belongs to the royal system: seven royal fingers (13.125 cm) equal a *tbt*.
- **Schoenium (Scoino)** – A Ptolemaic and Roman unit of length used for area calculations. Equivalent to the length of 100 *mh Ntr* (see *mh-hṯy* above) or 96 Nilometric cubits (see *mh-jtn* above).
- **Small Cubit (Mh šrj)** – Disappears during the Twenty-sixth Dynasty - A division of the great (ϙ) cubit; 45 cm. The small cubit becomes the Ptolemaic and Roman Public or Carpenter’s cubit (the length of an Egyptian foot and a half [**Phεῦ Dhmosioῦ keÉ Tektonikoῦ**]).
- **Small Span (Pd šrj)** – All periods - A division of the great cubit (*mh ϙ*) equal to half the length of a small cubit (*mh šrj*) or 22.5 cm; shown in white on the Amenemope cubit artifact. The small span becomes the Ptolemaic and Roman *Spithame (Spiqamh)*.
- **Spiqamh**- see small span.
- **Xulon (Xulon)** – The Ptolemaic and Roman equivalent of a *nbt*. Used in volume calculations.

10.8 Volume

- **Artaba (Artabh)** – Ptolemaic and Roman with a Persian origin – The *artaba* has a volume of 80 *hins*.

- **Aoilion (AwiPion)** – Ptolemaic and Roman – The volume (4.13 cubic metres) of a cube with a side length of three Nilometric cubits.
- **Djw.t (Dw.t)** – Old Kingdom – a volume unit of approximately four hins (2/2.5 litres). The volume of 100 *djw.t* (200 to 250 litres – rounded to 450 *hins* in this thesis) is not an unknown entity in the ancient Near East; it corresponds to the volume of a *kurru* used in Ugarit during the New Kingdom and associated with barley.
- **Hin (Hnw)** – All Periods - 0.485 litre - Probably the basis of all ancient Egyptian metrology. Grain volumes and their divisions are multiples of the *hin*: 1 *hekat* equals 10 *hins*, one Middle Kingdom sack (*h3r*) equals 200 *hins*, and one New Kingdom sack (*h3r*) equals 160 *hins*. It is from these numbers of *hins* and the specific weights of emmer, barley, wheat, or water that all ancient Egyptian linear metrology is defined. The fact that there is no mention of a sack (*h3r*) in the Old Kingdom can be explained by 450 *hins* (the volume of 100 Old Kingdom *Dw.t*). The cubic root of the 450 *hins* gives the length of a great [ꜥ] cubit (circa 60 cm). 2/3 of 450 *hins* equal 300 *hins* or 144 litres, the cubic root of which gives the length of a royal cubit, 52.5 cm.
- **Naubion (Naubion)** – Ptolemaic and Roman – The volume (1.41 cubic metres) of a cube with a side length of two Nilometric cubits (see *Aoilion*).

10.9 Weights

- **Great Deben (Dbn ꜥ3)** – Old and Middle Kingdom to Eighteenth Dynasty – 140 grams. The weight of 450 *hins* of barley, weighing approximately 140 kilograms, divided by 140 grams (the weight of a great *deben*) give 1000 great *debens*.
- **Small Deben (Dbn šrj)** – Eighteenth Dynasty to the Ptolemaic Period – 95 grams. The weight of 450 *hins* emmer, weighing approximately 95 kilograms, divided by 95 grams (the weight of a small *deben*) give 1000 small *debens*
- **Talent (Tavanton)** – A Greek and Hellenistic measure of weight equivalent to the weight of water of a Greek or Hellenistic *amphora* (25 to 30 litres) filled with water or 25 to 30 kilograms.

EXCURSUS A - CLOSED METROLOGICAL SYSTEMS AND COMMODITIES

11 Closed Metrological Systems

A closed metrological system is a system where units of length, volume, and weight are related to each other usually using one reference commodity. The study of metrological systems, present and past, provides useful information on the principles used to define these systems. These principles, in turn, help understand the origin of ancient Egyptian linear systems. In a closed metrological system, units of length, weight, and volume are related to each other using a reference commodity. The metric system is the best illustration of this principle.

11.1 The Metric System

The *Décret relatif aux poids et aux mesures* of the French Republic (8 Germinal An 3 [April 7 1795]) states:

- Article 5-The new measures will be differentiated now by their republican names; their names have been permanently defined as follows:
- *Mètre*, the length equal to the ten millionth part of the arc of the meridian of the earth between the North Pole and the Equator.
- *Are*, the area length, for land, equal to a square the side of which equals ten metres.
- *Litre*, the measure of capacity for liquids and dry commodities alike, the capacity of which will be the cube of a tenth of a metre.
- *Gramme*, the absolute weight of the volume of pure water equal to a cube equal to the hundredth part of a metre, at the temperature of melting ice.

To “close” the metric system one goes from length to volume, and from volume to weight, using water as a commodity: the gram is defined by a cube (volume) the side of which (length) equals the hundredth part of a metre, filled with water (commodity-related weight). There is a second method of “closing” a metrological system: going from weight to volume and from volume to length. The Athenian system is a good example of this model.

11.2 The Athenian System

Solon's Athenian (Bailly, 1950, p. 2197) reform replaces the old Athenian talent weight-unit with two standard *talents*: one from Echinae, of Phoenician origin, weighing 37.11 kilograms, and the other from Euboea, of Persian origin, weighing 25.92 kilograms. Solon used for weight, currency, and length the *talent* from Euboea; a slightly modified *talent* from Echinae is used for commercial transactions. To a *talent* of 25.92 kilograms (Euboea), corresponds a *mina* of 0.432 kilograms (60 *minas* in a talent), a *drachma* of 4.32 grams (a *mina* divided in 100 instead of the traditional Near Eastern 60).

The weight of water is 1000 grams (1 kilogram) per litre. 25.92 kilograms correspond therefore to 25.92 litres (25920 grams divided by 1000 grams per litre give 25.92 litres). In classical metrology, a common cubit is one foot and a half long. The Athenian foot corresponds to the side of a cube of 25.92 litres: 25.92 litres are contained in a cube, the side of which measures 29.59 cm which is the length of an Athenian foot; one foot and a half (29.59 plus 14.79) equal 44.38 cm, which is the length of the Athenian cubit.

To close the Athenian system, and differently from the metric system, one goes from weight (*talent*), to volume using water as a reference commodity (a cube of water representing the weight of a *talent*), and from volume to length, the foot being the side of the cube of water of water weighing 25.92 kilograms¹⁵.

11.3 Pommerening's Example-Specific Weights

There is a third method to close a metrological system: using weight relationships.

There are two *Djas* mentioned in the Middle Kingdom (¹*Dja* of 75 cubic cm and ²*Dja* of 150 cubic cm) used for water, oily products, and powdery substances including myrrh. Pommerening gives a weight between 0.42 and 0.62 grams per cubic cm of myrrh (Pommerening, 2005, p. 236). Water weighs one gram per cubic cm (the specific weight of water), and if one retains the

¹⁵ The Athenian system is in fact a partially closed composite system. There is no mathematical relationship between dry commodity units and liquid commodity units as Solon modified the talent from Echinae (38.88 kilograms instead of the original 37.11 kilograms) to define the Athenian *metrthth* for liquids.

average between 0.42 and 0.62 grams per cubic cm, or 0.50 gram per cubic cm of myrrh (the specific weight of myrrh), one ¹*Dja* of water (one gram multiplied by 75 cubic cm) equals one ²*Dja* of Myrrh (0.50 gram multiplied by 150 cubic cm) or 75 grams. It is possible to consider that the ²*Dja* is the double of ¹*Dja*. This however would be confusing in the definition of rations or exchanges, if one were to ask for one *Dja*, without mentioning the commodity (water or myrrh), what would one get: 75 cubic cm or 150 cubic cm of a given commodity?

Weight becomes the link between volumes. Based on weight, the difference in volume between the two *Djas* (one ¹*Dja* of 75 cubic filled with water, and one ²*Dja* of 150 cm filled with myrrh) disappears: both *Djas*, filled with their respective commodities, weigh 75 grams. Using weight as the link between volumes, it is possible to define lengths reflecting the weight of commodities: From the example above, one litre of water weighs one kilogram, while one litre of myrrh weighs on average 500 grams. In order to match the weight of one litre of water, two litres of myrrh are necessary. The implication for lengths is as follows:

- The side of a cube of one litre is ten cm.
- The side of a cube of 2 litres is 12.6 cm.

The only relationship between the two lengths above is weight, based on the specific weight of the commodities used: a cube having a side of 12.6 cm and filled with myrrh weighs one kilogram which is the weight of a cube having a side of 10 cm filled with water.

In this ancient Egyptian model, lengths are related to the volumes necessary to match a common weight of different commodities. The association based on volumetric weight has one advantage, it makes the use of scales redundant in specific exchange operations: in our theoretical example, and not taking into consideration the “market” value of the commodities involved, one litre of water weighing one kilogram could be exchanged for one kilogram of myrrh equivalent in volume to two litres. In other words, and differently from the metric and Athenian systems using a single reference commodity (water), the ancient Egyptian system is closed using several reference commodities, The closing factor in this case is weight and the practical vehicles are containers with different volumes adapted to the commodities they contain in order to match a defined exchange weight.

12 Commodities And Specific Weights.

In this thesis the specific weight of emmer is 440 grams per litre. The specific weight of barley is 660 grams per litre.

12.1 Commodities

There are four cereals appearing in the Rhind Mathematical Papyrus: emmer, *bdt* (Peet, 1923, Problem 79, p. 122, Plate W), barley, *jt* (Peet, 1923, Problem 74, pp. 119–120, Plate V), naked¹⁶ wheat *swt* (Peet, 1923, Problem 82, pp. 124–25, Plate X), and the more problematic *bš3* (Peet, 1923, Problem 71, p. 117, Plate U)

The ancient Egyptian word for emmer (*bdt*) appears in the First Dynasty (Brewer, Redford and Redford, 1994, p. 27). There are several words for emmer, each reflecting the colour of the commodity that can vary from white, yellow to red, and purple turning to black: *bdt ḥdt* (white emmer), *bdt dšrt* (red emmer), *bdt kmt* (black emmer). There are also other categories of emmer: *bdt hrnt* (hernut emmer), *bdt kt* (ketch emmer), *bdt Ptḥ* (Ptah emmer), more difficult to identify with precision (Hannig, 2001, p. 266; Brewer, Redford and Redford, 1994, p. 27). Emmer prevails in the fifth millennium b.c. in the Fayum (Kom K site); at Merimde (Beni Salâme, western Nile Delta) 4800-4400 b.c.; at Nagada (3850–3650 b.c.) Site KH3 and South Town (c. 3400 b.c.); at Kom el-Hisn in the Old Kingdom deposits, at Saqqara in the Djoser pyramid (Third Dynasty) and in Queen Icheti's tomb of the Sixth Dynasty (Zohary, 2000, pp. 219–20).

The ancient Egyptian word for barley (*jt*) also appears in the First Dynasty (Brewer, Redford and Redford, 1994, p. 28). The words used are: *jt* (barley), *jt mhj* (Lower Egyptian barley), *jt šm^c* (Upper Egyptian barley), and *jt m jt* (real barley [barley as barley] in Dynasty XX). Several adjectives qualify barley: *jt w3d* (fresh barley), *jt nd.w* (ground barley [as in flour]), *jt sk.w* (ground barley), *jt šhm.w* (crushed barley), *jt ʿwg.w* (parched, roasted barley), *jt psj.w* (cooked barley), and *jt snwh.w* (boiled barley). (Hannig, 2001, p. 111; Brewer, Redford, Redford, 1994, p. 27). Barley, in importance, is second to emmer in quantity during the fifth millennium B.C. in the Fayum (Kom K site); at Merimde (Beni Salâme, western Nile Delta) 4800-4400 B.C.; at

¹⁶ Peet's interpretation. Naked wheat (*triticum durum* or *aestivum*) only appears extensively in the Ptolemaic period.

Nagada (3850–3650 B.C.) Site KH3 and South Town (c. 3400 B.C.); at Kom el-Hisn in the Old Kingdom deposits, at Saqqara in the Djoser pyramid (Third Dynasty) and in Queen Icheti's tomb of the Sixth Dynasty (Zohary, 2000, pp. 219–20).

Wheat (*triticum dicoccum*) appears during the Naqada II Period (Krzyzaniak, 1977 pp. 139–40) and plays a limited role in ancient Egyptian diet (Murray, 2000, pp. 511–13). Wheat (*triticum durum*) becomes preeminent during the Ptolemaic and Roman Periods, replacing emmer, as a major Egyptian export commodity (Smith, 2003, p. 22).

bš3 is usually assimilated with barley, to such an extent that some authors consider *bš3* as another word for barley (Delwen, 1993, p. 278). When it is associated with beer, it is usually translated as malt (Nims, 1958, p. 56–65).

12.2 Specific Weights

Emmer, barley, and wheat have distinct weight characteristics. These different weights mean that for a given volume, the heavier the cereal, the less space it will occupy. In other words if emmer is lighter than barley, to match an equal weight of emmer and barley, the volume of emmer will be greater than the volume of barley. Using similar sacks as units of volume, the implication is that more sacks of emmer are required to match an equivalent weight of barley.

This thesis utilizes the following weights per litre (see note three below for sources):

- Emmer 440 grams per litre, or 0.44 gram per cubic cm (1 litre of water equals 1000 cubic cm and weighs 1000 grams; 1000 cubic cm weighing 440 grams give 0.44 gram per cubic cm)
- Barley 660 grams per litre, or 0.66 gram per cubic cm (1 litre of water equals 1000 cubic cm and weighs 1000 grams; 1000 cubic cm weighing 660 grams give 0.66 gram per cubic cm)
- Wheat 800 grams per litre, equal or 0.80 gram per cubic cm (1 litre of water equals 1000 cubic cm and weighs 1000 grams; 1000 cubic cm weighing 800 grams give 0.80 gram per cubic cm)

Malt is not a commodity-associated with the payment of taxes. One litre of malt weighs between 336 and 460 grams, or 0.46 gram per cubic cm (1 litre of water equals 1000 cubic cm and weighs 1000 grams; 1000 cubic cm weighing 460 grams give 0.46 gram per cubic cm), very close to emmer. The association of *bš3* with barley and malt is however difficult due to the specific weight of *bš3* that can be deducted from pBerlin 10078 of the Middle Kingdom (Spalinger, 1988, 271 ff.): *bš3 ḥk3t 81 jr m jt šmᶜ ḥk3t 121 1/2*: 81 *hekᶜts* of *bš3* correspond to 121 1/2 *hekats* of barley of Upper Egypt. The equivalence of pBerlin 10078 gives a specific weight for *bš3* of 908 grams per litre, one third heavier than barley. Considering the specific weight of *bš3*, Posener-Krieger's suggestions (Posener-Krieger, 1977b, pp. 67–71) makes perfect sense: *bš3* corresponds to any ground grain. The word is as generic as the word *sšrw* used from the Middle Kingdom onwards. Faltings gives several options for the definition of *bš3* (Faltings, 1995, pp.38–9): malt, bulgur, and parboiled emmer. It is difficult to associate *bš3* with flour, as the specific weight of (wheat) flour is 593 grams per litre¹⁷.

I am grateful to Professor Larry Pavlish for checking the specific weight figures used in this thesis. Using United States Department of Agriculture data based on humidity per pound in a

¹⁷ The specific weight figures retained in this thesis are derived from historical and modern data. Historical data: *Traité pratique des poids et mesures des peuples anciens et arabes* by J. Decourdemanche who gives the following information: barley, two thirds of the weight of water, and wheat 80% of the weight of water (Decourdemanche, 1909, pp. 6–8); *Metrologia e circolazione monetari degli antichi* by A. Segrè with the following information (based on Pliny) for the specific weight of wheat 778 grams per litre (Segrè, 1928, p.30, note 2) rounded to 800 (Segrè, 1928, p.38); *Anciens systèmes de poids et mesures en Occident* by Jean- Claude Hocquet. Hocquet, quoting Pliny and a German report published by Soetbeer in 1815, gives the following figures: Wheat 707 to 809 grams per litre, barley 685 to 788 grams per litre, emmer with husk 406 to 468 grams per litre. (Hocquet, 1992, pp. 215–38). Modern data: Wheat and Emmer: G.F. Stallknecht, K.M. Gilbertson, and J.E. Ranney, *Alternative wheat cereals as food grains: einkorn, emmer, spelt, kamut, and triticale* in J. Janick (ed.), pp. 156–70. Progress in new crops, ASHS Press, Alexandria, VA. 1996. Web sites: Emmer- <http://www.ag.montana.edu/carc/Cereals/1998/SpringEmmer.htm> for the publication of a paper called 1999 Evaluation of Spring Emmer, a study made by the University of Montana. Emmer, Barley, and Wheat- <http://www1.agric.gov.ab.ca> the information centre of the Government of Alberta. Barley-<http://www.hgca.com> Home Grown Cereals Authority of the UK and <http://www.reggen.vic.gov.au> the Office of the Regulator General of Melbourne Australia. Specific weights in general: <http://www.simetric.co.uk>. The ranges of weight coming from historical and modern data are as follows:

- Emmer: 360 to 440 grams per litre.
- Barley: 600 to 700 grams per litre.
- Wheat: 700 to 800 grams per litre.

Winchester bushel of 32.5 litres, he came up with the specific weights at 0% humidity of the commodities listed below:

- Wheat: 660 grams per litre.
- Barley: 520 grams per litre.
- Emmer: 465 grams per litre.

Professor Pavlish also compared his results with anthropological data coming from the latest research data in Ethiopia (D'Andrea and Mitiku, 2002, pp. 179–217; Lyons and D'Andrea 2003, pp. 515–30) and made the following remarks: the life-time storage of grain does not exceed 150 days if humidity is superior to 0%; the presence or absence of husks affects the weight of cereals. These figures show important differences with the ranges given by ancient and modern data.

12.2.1 Emmer

There is no major discrepancy between Professor Pavlish's emmer figures and those utilized for this thesis: 465 gram per litre, against the historical and modern data 440 grams per litre.

12.2.2 Barley

There is a major discrepancy between 520 grams per litre provided by Professor Pavlish and the range (600 to 700 grams per litre) given by historical and modern data. 660 grams per litre have been retained for the following reason: the Hekanakhte Papers (James, 1962, p.55; Allen, 2002, p.143) are clear: the ratio between emmer and barley is two thirds. Taking emmer at a specific weight of 440 grams per litre, the resulting figure for the specific weight of barley is 660 grams per litre, well within the range of historical and modern data. Based on this evidence, it seems more logical to maintain the theoretical specific weight of barley at 660 grams per litre.

12.2.3 Wheat

The discrepancy for wheat between Professor Pavlish's figures and the historical and modern data is very important: 660 grams per litre against 800 grams per litre. This thesis retains 800 grams per litre based on Pliny's *Natural History*, XVIII, 65–66 (Plinius Secundus, 1995, pp. 49–50):

66-Nunc ex his generibus¹⁸, quae Romam invehuntur¹⁹, levissimum est Gallicum atque Chersoneso advectum, quippe non excedunt modii vicenas libras, si quis granum ipsum ponderet. Sardum adicit selibram, Alexandrinum et trientem-hoc et Siculi pondus-Baeticum totam libram addit, Africum et dodrantem.

Now amongst the kinds brought to Rome, the lightest is from Gaul and the one transported from the Chersonese, as they do not exceed 20 pounds to the modius, if one weighs the grain by itself. The Sardinian adds half a pound, the Alexandrian a third of a pound more (et)-this is also the weight of the Sicilian-the Spanish (Baeticum) adds a full pound, the African a pound and three quarters more (et).

Taking the Roman modius at 8.78 litres and the Roman pound at 327 grams, the specific weight of the Alexandrian *triticum* is: 20 5/6 pounds multiplied by 327 grams equal 6.82 kilograms, divided by 8.78 litres or 780 grams per litre, and rounded to 800 grams per litre. This figure is further confirmed by the Kellis Agricultural Account Book (4th Century A.D., Dakhleh Oasis [Bagnall, 1997, p.47]: one artaba of 38.78 litres weighs 30 kilograms, giving a specific weight of 770 grams per litre.

12.3 Husks, Humidity and Storage

12.3.1 Husks

Husks do not influence ancient Egyptian metrological calculations. The current literature, based on archaeological evidence, seems to indicate that ancient Egyptian emmer and barley were stored as emmer spikelets or hulled barley respectively, (Delwen, 1993, p. 278–9), in accordance with Pliny's Natural History XVIII, 86–93 (Plinius Secundus, 1995, pp. 49–50).

¹⁸ *Generibus* - kinds. This refers to the previous paragraph (65): *Tritici genera plura, quae fecere gentes*. There are several kinds of wheat produced by nations. The *triticum* in question, as far as Alexandria is concerned, is according to the literature (Murray, 2000, p. 513) *triticum durum* and/or *triticum aestivum*.

¹⁹ *Quae Romam invehuntur* (brought to Rome) and *advectum* (transported) imply that the grain must have had 0% humidity to be safely transported.

12.3.2 Humidity and Storage

The life-time storage of any type of grain does not exceed 150 days if the grain is humid. Emmer and barley collected for tax purposes did not stay long at the points of collection. According to Murray a crop of emmer and/or barley takes six months to grow (Murray, 2000, p.520) Pliny gives six months for barley and seven months for emmer (Plinius Secundus, 1995, p. 46).

Murray goes into all the phases of a crop from tilling to harvesting (Murray, 2000, pp. 515–28) and also mentions that harvesting is best done when the crop is moist (evening or early morning) in order to minimize grain loss (Murray, 2000, p.520). Pliny states, Natural History XVIII, 169–70, that harvesting takes a maximum of two months (Plinius Secundus, 1995, p. 106).

Transportation to the granary appears to be quite swift, judging by an example calculated from pTurin 1895+2006 (Gardiner, 1941, pp.30–1): 8 days by boat. The optimistic time frame from field to granary, based on the same example, is therefore, including transportation, 68 days. The truth probably lies between these 68 and 150 days. There is one comment that can be added to the necessity of 0% humidity for storage: storage is the last stage of harvesting. There is no indication that the collected grain was put into granaries immediately. The word *ḥꜥ*, heap, associated with granaries, particularly during the New Kingdom, may suggest a “drying period” prior to physical storage into granaries.

12.4 Water

12.4.1 pTurin 1880

With the year 29 of Ramesses III comes the first association of a sack (*ḥꜣr*) with water (pTurin 1880-Gentet and Sweydan, 1992, pp. 177–85). The authors provide three hypotheses for the association of a sack (*ḥꜣr*) with water:

- The *ḥꜣr* sack is used for the payment of water (Gentet and Sweydan, 1992, pp. 180–81).
- The *ḥꜣr* sack is used for the payment of the transportation of water (Gentet and Sweydan, 1992, pp. 181–82).
- The *ḥꜣr* sack is used as an accounting unit to standardize the administrative costs associated with delivering water. This third hypothesis is illustrated by the authors, quoting pTurin 1880 (RAD 51, Gardiner, 1995, p. 51, lines 6, 6–10).

pTurin 1880	Transliteration	Translation
Line 6,6	$\epsilon^3 n js.t H^c w mw \underline{h}3r 17 \frac{1}{2} m \underline{h}3w$	The foreman Khaw, water, 17 and a half (<i>h3r</i>) sacks in attribution.
Line 6,7	$P3b3s3 \underline{h}3r 8 \frac{3}{4}$	Pabasa eight and three quarters (<i>h3r</i>) sacks.
Line 6,8	$H^c y-m-wj3 \underline{h}3r 8 \frac{3}{4} dmd \underline{h}3r 35$	Khamwja eight and three quarters (<i>h3r</i>) sacks, total 35 (<i>h3r</i>) sacks.
Line 6,9	$s\check{s} Jmn-n\check{h}t \underline{h}3r 8 \frac{3}{4} w\check{h}m \underline{h}3r 2 \frac{1}{2} dmd \underline{h}3r 11 \frac{1}{2} dmd wd3.w.t mw \underline{h}3r 47$	The scribe Amennakht 8 and 3/4 (<i>h3r</i>) sacks, renewal two and a half (<i>h3r</i>) sacks, total 11 and a half (<i>h3r</i>) sacks, remainder total water 47 (<i>h3r</i>) sacks.
Line 6,10	$H^c y-m-W3st \underline{h}3r 12 \frac{1}{2} Mry-R^c \underline{h}3r 12 \frac{1}{2} Ms.w \underline{h}3r 17 \frac{1}{2} Nfr-htp \underline{h}3r 8$	Khaemwaset 12 and a half (<i>h3r</i>) sacks, Meryra 12 and a half (<i>h3r</i>) sacks, Mesw 17 and a half (<i>h3r</i>) sacks, Neferhotep 8 (<i>h3r</i>) sacks.

The association of *h3r* sacks with water provides new insights in volume definitions. As Gentet and Sweydan have already noted, volumes in the Rhind Mathematical Papyrus are also expressed in *hins* in problems 80 and 81 (Gentet and Sweydan, 1992, pp. 179) with a ratio of ten *hins* to the *hekat*. They also appear in the pKahun LV, 4 (Gillings, 1982, pp. 162–65) with the same ratio of *hins* per *hekat*. As it is impossible to transport water in a canvas *h3r* sack, the Middle Kingdom *h3r* can be considered as a purely Middle Kingdom multiple of 200 *hins* (200 *hins* of 0.48 litre equal 96 litres, the volume of a Middle Kingdom *h3r* sack). This in turn would explain why the word *h3r* does not appear during the Old Kingdom: the taxation unit of the Old Kingdom could very well be the *hin* of circa 0.48 litre; 450 *hins* being the volume of an Old Kingdom “sack” leading to the length definition of the great cubit of circa 60 cm and its physical representation, the scribe’s palette. This possibility does not go against the written record: the *hin* appears in the Third Dynasty (Kahl, 2003, p. 279) and the relationship between *hins* and sacks (*h3r*) is common from the Middle Kingdom onwards (Schack-Schackenburg, 1900, pp. 135–40; 1902, pp. 65–6; Eyre, 1980, pp. 108–19; Spalinger, 1992, pp. 87–94). The *hin* as the basic unit for volumes also

explains the close relationship between *n bj*, *dnj*, and royal cubits. A *dnj* equals 300 *hins*, the volume of a royal cubit (144.7 litres); a *n bj* equals one royal cubit and a quarter.

In the Ptolemaic pBritish Museum 10399, problems 42 to 43, the *mḥ Dhwtj* is associated with *hins* of water in the volume calculations of a mast:

12.4.1.1 Problem 42 (Parker, 1972, p. 55)

Problem 42	Translation	Transliteration
Line 10	<i>jw=k rḥ p3 r3 šsp nty šm r p3 mḥ Dhwtj r 7</i>	You know the palm divisions that will go in the cubit of Thoth amounting to seven
Line 11	<i>jw=k jr 7 r sp 7 r 49</i>	you will make 7 seven times: 49
Line 12	<i>jw=k jr 49 r sp 7 r 343</i>	you will make 49 times seven: 343
Line 13	<i>jw=k dd p3 hn šsp 1 ḥr- ḥt 1 mty=f 1</i>	you shall say; “the hin is one palm by one, its depth 1”
Line 14	<i>jw=k dd ḥr šm hn [343 r mḥ 1 ḥr- ḥt 1 mty=f 1]</i>	you shall say: [343 hins usually go [into a cubed cubit, 1 by 1, its depth 1]

12.4.1.2 Problem 43 (Parker, 1972, p. 57)

Problem 43	Translation	Transliteration
Line 25	<i>p3 gy n gm p3 r3 hn nty šm r p3 mḥ 1 ḥr-ḥt 1 mty=f 1</i>	The method of finding the number (parts) of hin(s) that will go in the cubit, 1 by 1, its depth 1.
Line 26j	<i>jw=k rḥ p3 r3 šsp nty šm r p3 mḥ Dhwtj r 7 jw=k jr 7 r sp 7 r 49</i>	You know the palm divisions that will go in the cubit of Thoth amounting to seven; you will make 7 seven times: 49

Line 27	<i>jw=k jr 49 r sp 7 r 343 jw=k [d]d p3 hn šsp 1 hr-ht 1 mty=f 1</i>	you will make 49 times seven: 343; you shall [sa]y; “the hin is one palm by one, its depth 1”.
Line 28	<i>jw=k dd hr šm hn [343 r mḥ 1 hr-ht 1 mty=f 1]</i>	you shall say: [343 hins usually go into a cubed cubit, 1 by 1, its depth 1].

Parker gives 7.833 cm as the length of one *mḥ Dḥwtj* palm on the basis a *hin* of 0.4805 litres. The length of the palm corresponds to the cubic root of the *hin*, the length of which is 7.833 cm. Seven of these palms give the length of a *mḥ Dḥwtj*: 54.831 cm. This cubit is known during the Roman Period (pOxyrhynchus 669) as the Nilometric cubit.

This calculation, based on the cubic root of one *hin* or water to define a palm or 343 *hins* of water to define a cubed cubit, goes against the principles of Greek and Hellenistic metrology where a cubit is defined as a foot and a half, the foot being the cubic root of a *talent* of water. The cubit construction of pBritish Museum 10399 problems 42 and 43 is therefore genuinely Egyptian. 343 *hins* to define the volume of a cubit is not an easy number. As indicated, the traditional definition of the length of a royal or reformed cubit is 52.5 cm, corresponding to 300 *hins* or 144.7 litres. 144.7 litres divided by 343 *hins* give a volume of circa 0.42 litre for a *hin*, which does not go against Pommerening’s or Schmitt’s records (Pommerening, 2005, p. 207; Schmitt, 2005, p. 63). It is difficult to reconcile 343 *hins* with the traditional 300 *hins* defining a royal or reformed cubit. There is one possibility: 300 *hins* of 0.42 litre each give 126 litres. 126 litres of water weigh 126 kilograms. 126 kilograms is the weight of two Middle Kingdom sacks (XAr) of 96.5 litres each filled with barley.

13 Weights, Sacks, and Cubits

The easiest way to pay taxes or give rations is to use scales. Taxes, rations, and wages, paid in kind are easily expressed as so many units of weight of a certain commodity (emmer, barley, or wheat) representing one day’s, week’s, or month’s wages of any commodity. Instead of using scales for grain transactions, ancient Egypt prefers to use containers. Taxes are expressed in a number of sacks (*h3r*) or *artabas* during the Ptolemaic and Roman Periods. These containers

serve a triple purpose: taxation, ration, and transportation units. As scales are not used for grain transactions, the volume of these containers must have originally reflected the weight of the commodity they contained. In order for any exchange to be fair, one would trade a given weight of a commodity against an equivalent weight of another commodity. If scales are not used, the volume of standardized and commodity-associated containers replacing scales must match the weight of the commodities exchanged: for example, barley being one third heavier than emmer, using a standardized sack, one sack of barley corresponds in weight to one and a half sack of emmer. From the weight and volumetric differences, lengths (cubits) are defined.

EXCURSUS B - DIVISION EVOLUTION

Summary

Four cubit artifacts, *seqed* calculations, and the disappearance of traditional names, particularly in *aroura* calculations, indicate an evolution and standardization in ancient Egyptian metrology: traditional division names are replaced by $\frac{1}{2}$, $\frac{1}{4}$ (and so forth) fraction divisions. The names of the divisions: hand, fist, double palm, small span, great span, *djeser*, and small cubit partially present on the Palermo Stone, fade away, and are replaced, starting with the *seqed* calculations of the Rhind Mathematical Papyrus, by royal cubits, palms, fingers, and fractions of finger. The original number of palms (seven) of the royal cubit is officially replaced by six palms in the Twenty-sixth Dynasty reformed cubit.

14 Group 1 and Group 2 Cubits

The comparison of four cubit artifacts shows a trend in the evolution of the divisions of the cubit. The traditional division names of the cubit (for example: palm, double palm) are replaced by fraction markers in accordance with the Denderah dedicatory texts published by Cauville. Adelheid Schlott's doctoral thesis, *Die Ausmaßse Ägyptens nach altägyptischen Texten*, (Schlott, 1969) classifies cubit artifacts into three groups:

- Group 1: operative - carrying only metrological divisions;
- Group 2: cubits carrying an offering formula;
- Group 3: votive - carrying various inscriptions (nome names, Nile heights, etc.) but no offering formula.

14.1 Similarities and Differences between Three Almost Contemporary Artifacts – The Aper-El, Amenemope, and Maya Cubit Artifacts

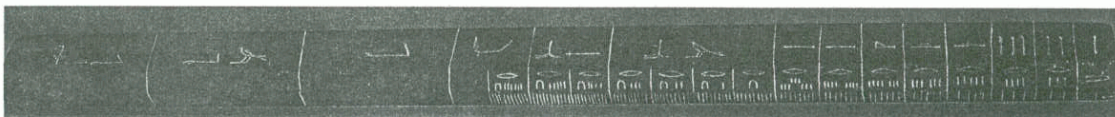


Figure 28. Aper-El Cubit (Zivie,1990, p.136) - Scale 1/10).

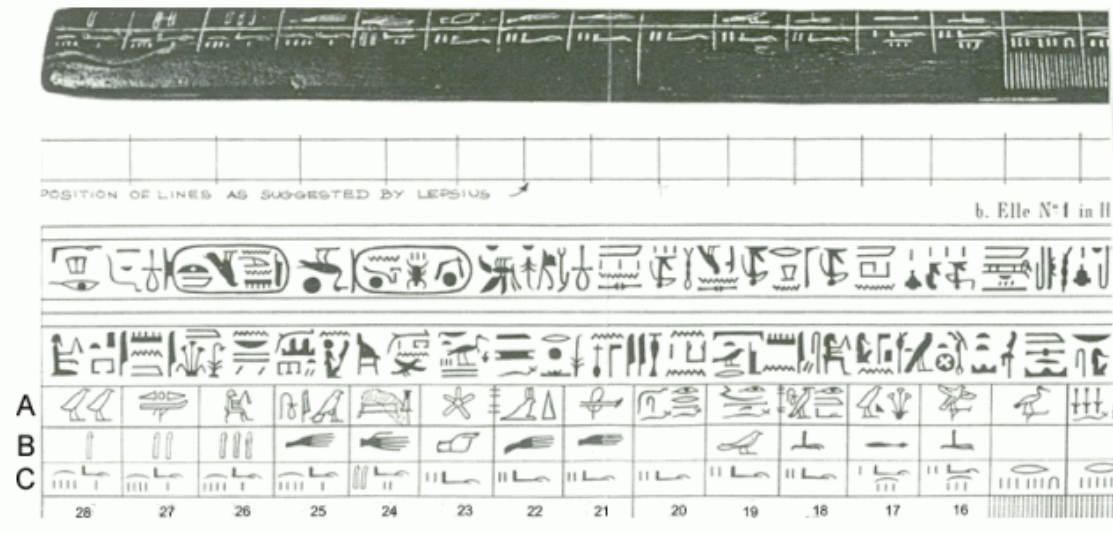


Figure 29. Amenemope division markers (Turin museum #6347 – Lorenzen, 1966, Plate I) – Scale 1/1 - see Appendix A.

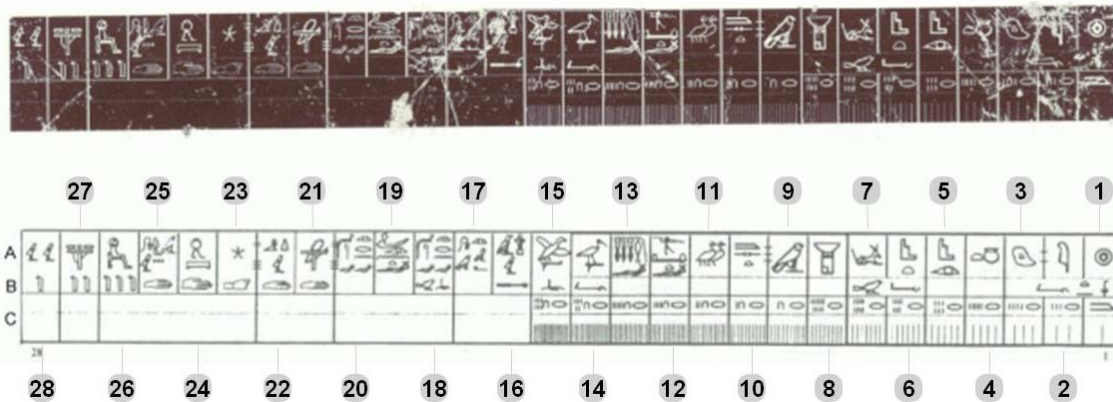


Figure 30. Maya division markers Louvre museum N 1538 (Louvre museum N 1538 - Scale 28/100-St. John, 2000, pp. 1–2).

The Aper-El cubit, found by Alain Zivie, dates from the Eighteenth Dynasty (the reigns of Amenophis III and Akhenaten). It is almost contemporary to the Amenemope and Maya cubit artifacts. It lacks the information provided by the Amenemope and Maya cubit artifacts.

The Amenemope cubit artifact (Turin museum #6347), thanks to its dedicatory text, can be dated to the end of the Eighteenth Dynasty (Horemheb). It is contemporary with the Maya cubit artifact (Louvre museum: N 1538). Maya was the “Overseer of the Treasury of the Lord of the Two Lands” under Tutankhamen and Horemheb, while Amenemope was “Overseer of the Granaries”

under Horemheb. The artifacts show similarities and differences, they are not major. Comparing both artifacts, are they are contemporary, provides answers to divisions which are not clearly marked on a single artifact. The black and white colour scheme, only present on the Amenemope cubit artifact, was essential to isolate the great system from the royal system. The Maya cubit artifact shows its palm divisions more clearly than the Amenemope cubit artifact to such an extent that Lepsius, in his drawing of the Amenemope artifact, added a palm marker which is



only present on the Maya artifact between B22 and B23 (). This is an important marker as it clearly separates the artifact into a palm division of five finger division (the hand) and more importantly into a sandal of seven finger divisions difficult to locate on the Amenemope cubit artifact.

14.1.1 The Aper-El Cubit Artifact

The Aper-El cubit artifact is very basic in comparison to the Amenemope and Maya cubit artifacts. It is divided into seven royal palm divisions but only shows a double palm divided into eight finger divisions, a four finger palm division under the small span division, two two-finger palm divisions with the great span division and the *djeser* division above a three finger palm division materialized by three divisions of three fingers. Without the visual clues provided by the Amenemope and the Maya cubit artifacts, it would have been very difficult to suggest that the royal cubit is a converter.

14.1.2 The Amenemope Cubit Artifact

The Amenemope shows its palm divisions in a different manner on lines B and C of Figure 29. It uses different colours, white and black, to show:

- A black royal palm division on B25.
- A black palm division of five finger division (the hand of B24).
- A white palm of six finger divisions (the fist) on B 23.
- A double royal palm division on B21 and B22.

The Amenemope cubit artifact does not show directly a palm of seven finger divisions. Seven finger divisions appear using lines A and C: between C15-16 and A22-23.

14.2 Similarities between The Amenemope and Maya Cubit Artifacts.

Both artifacts show, with some discrepancy in the number of finger divisions, a small span (with a white quail chick on the Amenemope cubit artifact), a great span (black on the Amenemope cubit artifact), a *remen* (black on the Amenemope cubit artifact), a small cubit (with a white quail chick on the Amenemope cubit artifact), and a royal cubit (black on the Amenemope cubit artifact). On both the Amenemope and Maya cubit artifacts, the small span division has a length of ten or 11 finger divisions (B18-19, with the glyphs on B18 and B19) and between C17-18 and C27-28 with the small span glyphs on B18 on the Maya artifact, with both number of finger divisions different from the expected 12 finger divisions of the royal system.

14.3 The Maya Cubit Artifact

The Maya cubit artifact shows on Figure 30:

- An emphasis on palm divisions of two fingers (the number of finger divisions of half a royal palm division) shown three times between C15-16 and C17-18, C20-21 and C22-23, and C26-27 and C28 (the left edge of the Maya cubit artifact). The two-finger palm division also appears on B.27.
- A palm of three finger divisions between C17-18 and C20-21, also appearing on B.26.
- A royal palm division of four finger divisions between C22-23 and C 26-27, appearing also on B.25.
- A palm division of five finger divisions between C15-16 and C20-21; the hand of B24.
- A palm of six finger divisions between C22-23 and C28 (the left edge of Maya cubit artifact), the fist of B23.
- A palm of seven finger divisions (two finger divisions plus three finger divisions plus two finger divisions) between C15-16 and C22-23.

- A royal double palm division of eight finger divisions between C20-21 and C28 (the left edge of the Maya cubit artifact), the double palm division marker of B21 and B20.

14.4 The Denderah Cubit Artifacts

Legrain (1916, pp. 149-152) has published three cubits 53 to 53.5 cm long. He gives a date of 200 B.C. based on calligraphy. They match exactly the lengths given by Cauville (1990, p. 110).

What is questionable is whether these three cubits are royal.

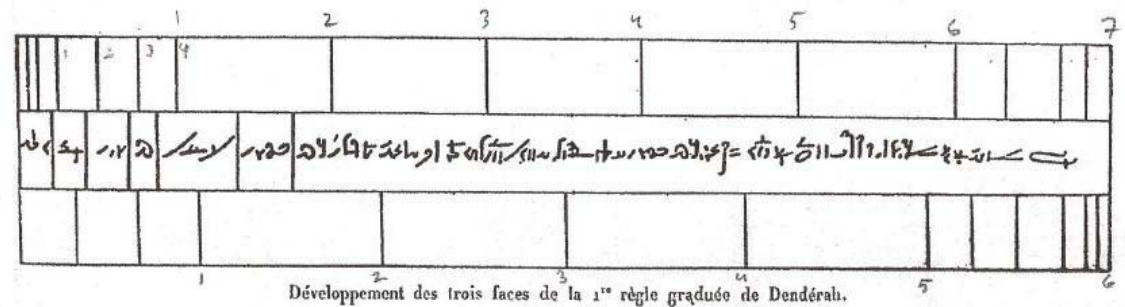


Figure 31. Denderah Cubit Artifact – Scale 1/7 – (Legrain, 1913, p.150).

The Denderah cubit artifact is probably a converter between three cubits displayed on three lines. During the reign of Ptolemy V Euergetes there were three cubits in use: the *Mh Ntr* with a palm of 7.5 cm, the Alexandrian cubit with a palm of 7.6 cm, and the *Mh* with a palm of 8.75. The different length of the palms suggest that line one is the *Mh Ntr*. Line two, the middle line, with a length of 53 cm represents seven palm divisions of the Alexandrian cubit of 53 cm.

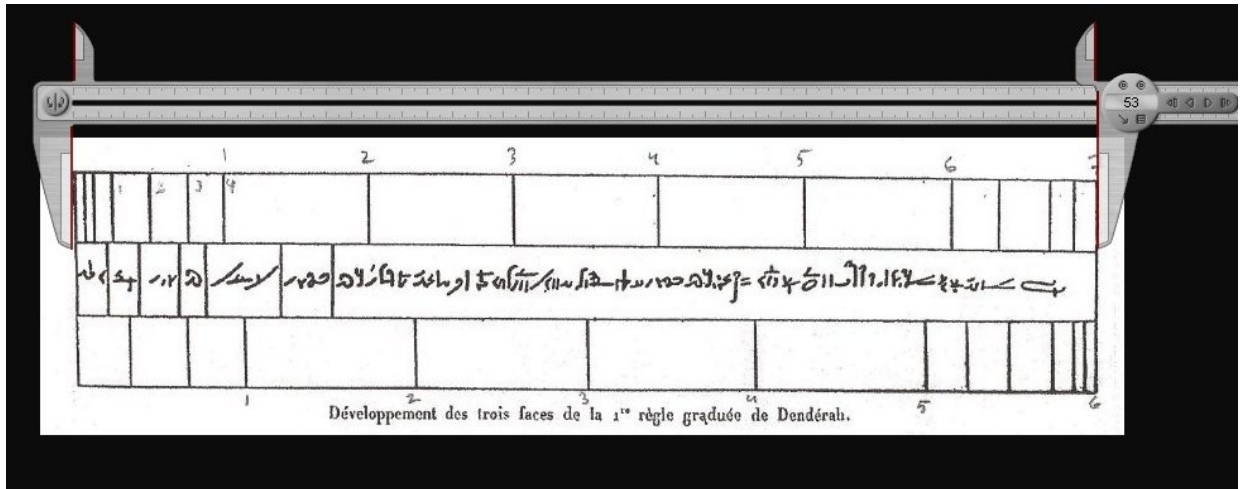


Figure 32. The Length of Seven Alexandrian Palm Divisions

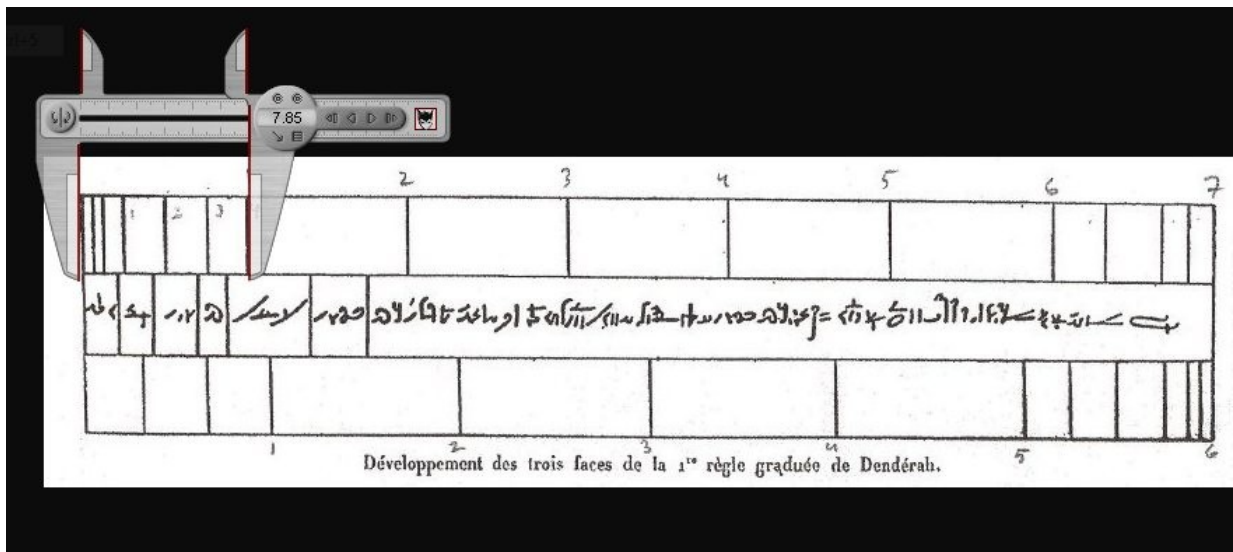


Figure 33. The *Mh Ntr* palm division of four finger divisions.

The first (top lines) is divided into seven palms of circa 7.5 cm corresponding to the *Mh Ntr*, the former royal cubit. The palm divisions are divided into palm divisions of either three or four finger divisions each. The number of finger divisions in a palm division matches the colour scheme found on the Amenemope royal cubit artifact: three white or four black finger divisions in a palm division.

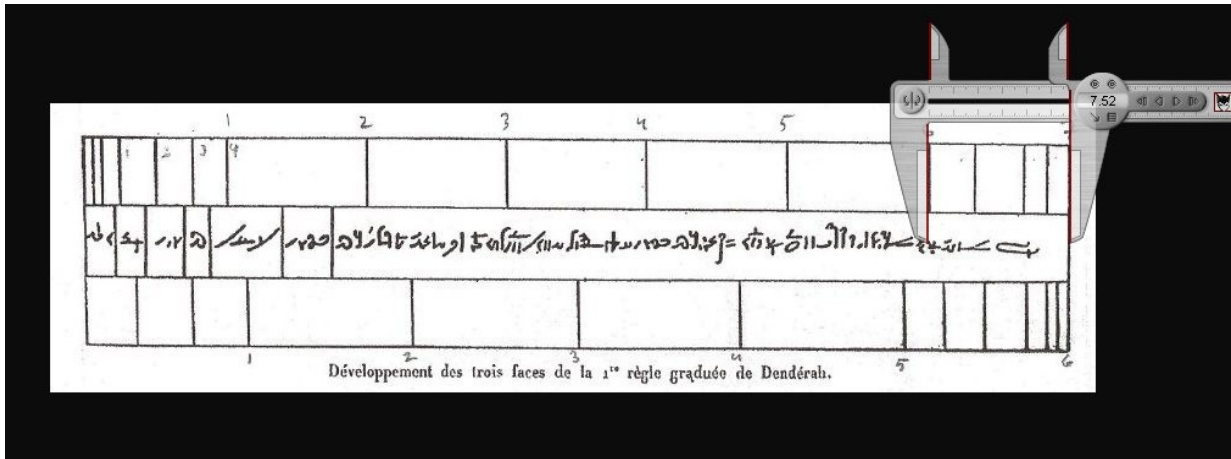


Figure 34. The *Mh Ntr* palm division of three finger divisions.

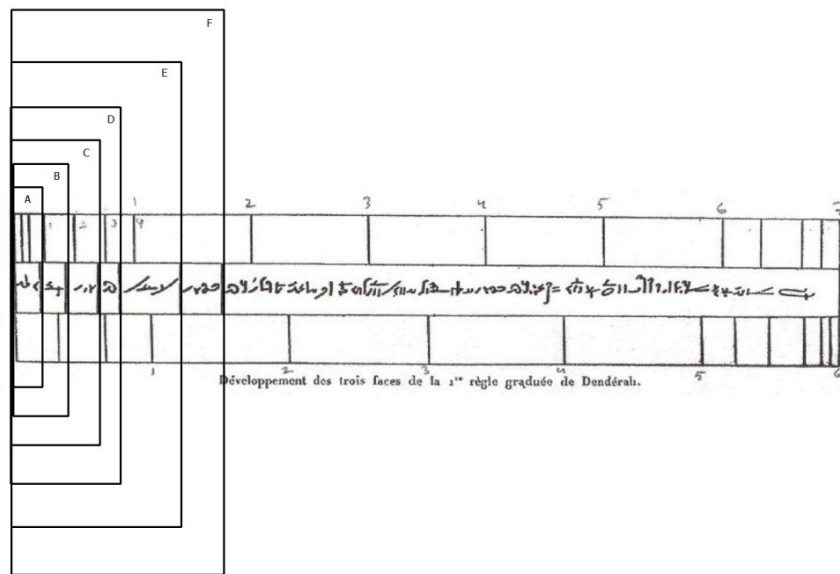


Figure 35. The divisions of the Alexandrian cubit.

- Seven Alexandrian palm divisions give the length of the artifact (53 cm) which is 0.5 cm longer than the royal cubit, the *Mh Ntr* or the *Mh* (the reformed cubit of the Twenty-sixth Dynasty). The long sides of the A, B, C, D, E, and F rectangles of Figure 35 indicate that the Alexandrian cubit can be divided into $1/32$ (A), $1/16$ (B), $1/10$ (C), $1/8$ (D), $1/5$ (E), and $1/4$ (F).

The third line (bottom row), with its six palm divisions of three or four finger divisions of circa 8.8 cm matches the definition of the reformed cubit (Mh) and is probably a rare representation of this cubit. It indicates that the reformed cubit, as the $Mh Ntr$, the Ptolemaic equivalent of the royal cubit can be divided into palm divisions of three or four finger divisions each.

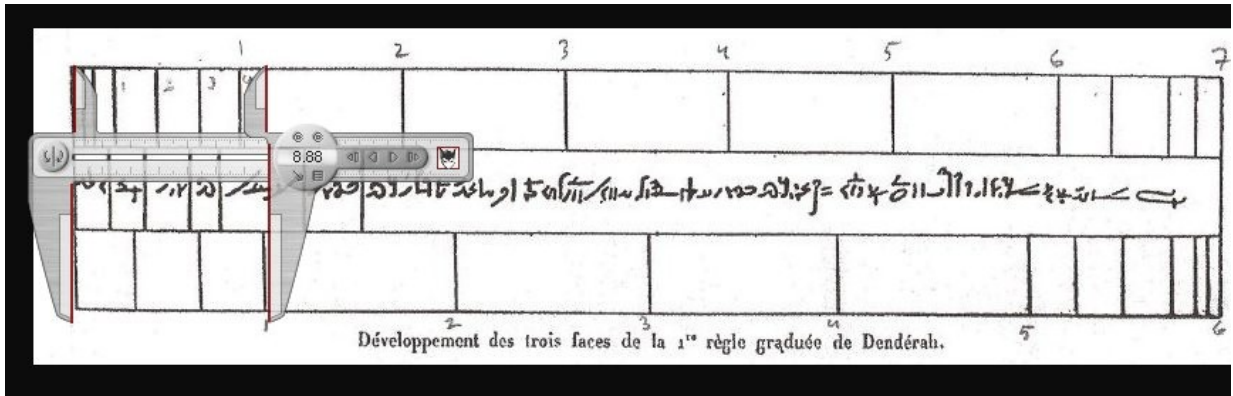


Figure 36. The Reformed Cubit (Mh) palm division of three finger divisions each.

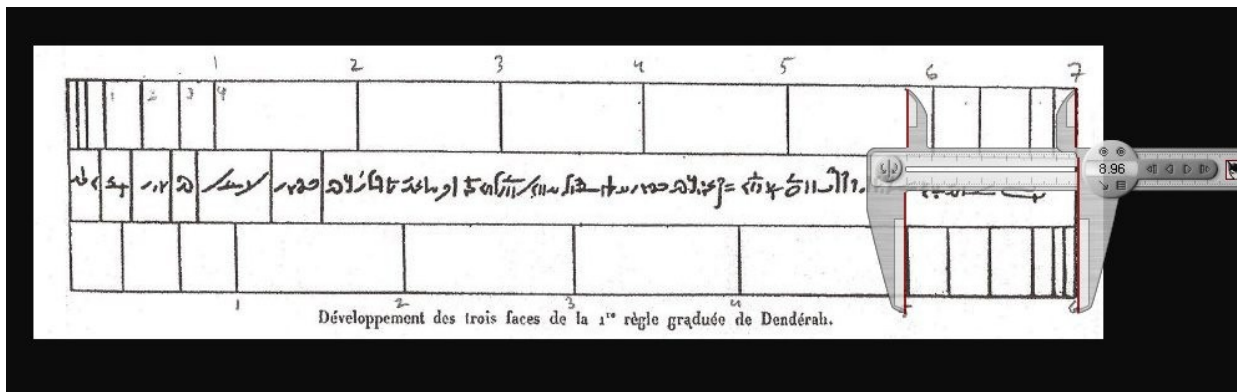


Figure 37. The traditional Reformed Cubit (Mh) palm division of four finger divisions each.

The screen calipers appearing on all the Figures above have been calibrated as 818 screen pixels for 53 cm. The slight measurement discrepancies (1/10 or 2/10 of a millimetre) are due to the figure reproductions, not to the calipers: some vertical lines are not absolutely straight.

14.5 Miscellaneous Group 1 Artifacts and Nilometre Inscriptions

The list of operative artifacts and nilometre inscriptions taken from Roik (1993, pp. 47–51), and Simon-Boidet (1993, pp. 157–77). None of them shows true multiples of the royal cubit.

Table 28. Operative Cubits and Nilometres Divisions (Roik, 1993, pp. 47-51; Simon-Boidot, 1993, pp. 157-77).

Name and Date	Period	Length (cm)	Major Divisions	System
First Kahun Rod	Middle Kingdom	65.20	7	<i>Nbj</i> as 7 hands
Second Kahun Rod	Middle Kingdom	67.30	7	Great system
Deshasheh Rod	18th Dynasty	66.42	8 (doubtful)	Great system
Lisht Rod	Unknown	69.85	7	Great system
Kom el Gizeh Nile Scale II	Ptolemaic	66–67	6 1/2	Great system
Kom el Gizeh Nile Scale IV	Ptolemaic	65.6	5	<i>Nbj</i>
ABC Rods (Simon-Boidot)	Unknown	66–70	7	Great system
Science Museum Slab 1935–462	Unknown	68.1	7	Great system
Aswan Obelisk Marks	Unknown	NA	60	Great Cubit

What is unexpected in Table 28 is that according to the lengths and number of divisions given, the *nbj* is traceable in Ptolemaic times (Kom el Gizeh Nile scales).

15 A Change In System - The *Seqed*

In the Rhind Mathematical Papyrus, the *seqed* calculations are performed using royal cubits. It is the only place in the papyrus where cubits of seven palms are mentioned. The *seqed* calculations, using a royal cubit, give uneven values (cubits and divisions of cubits). Using different units royal, particularly sandals, or great gives more practical even values.

The *seqed* is a simple method used to define the inclination of faces of any inclined wall. The established relationship between the *seqed* and the royal cubit dates from the Hyksos Period (the date of the Rhind Mathematical Papyrus). From the Rhind Mathematical Papyrus (problems 56 to 60 [Peet, 1923, pp. 97–102, plates Q and R]), it corresponds to the relation between the horizontal length and the vertical rise of one cubit, corresponding to the inclination angle or slope of a monument (Arnold, 1991, pp. 11–12) or the lateral displacement in palms for a drop of seven palms or one royal cubit (Robins, 2000, p. 1810, col. 1) and is illustrated in Figure 38 below.

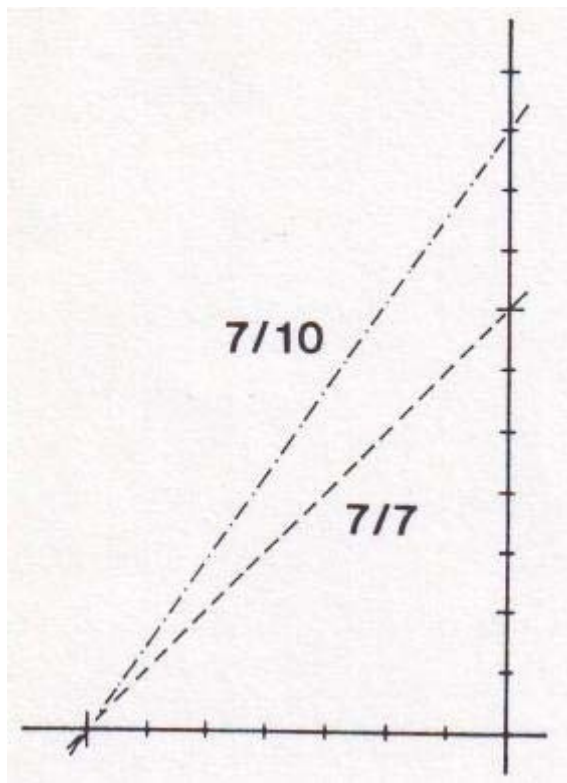


Figure 38. *Seqed* – (Arnold, 1991, p.12, Fig. 1.7).

The *seqed* was used for pyramid slope angles, but also for *mastabas*. The figures below are taken from Petrie (1892, Plate VIII [Figure 39]; Arnold, 1991, p.12 [Figure 40]).

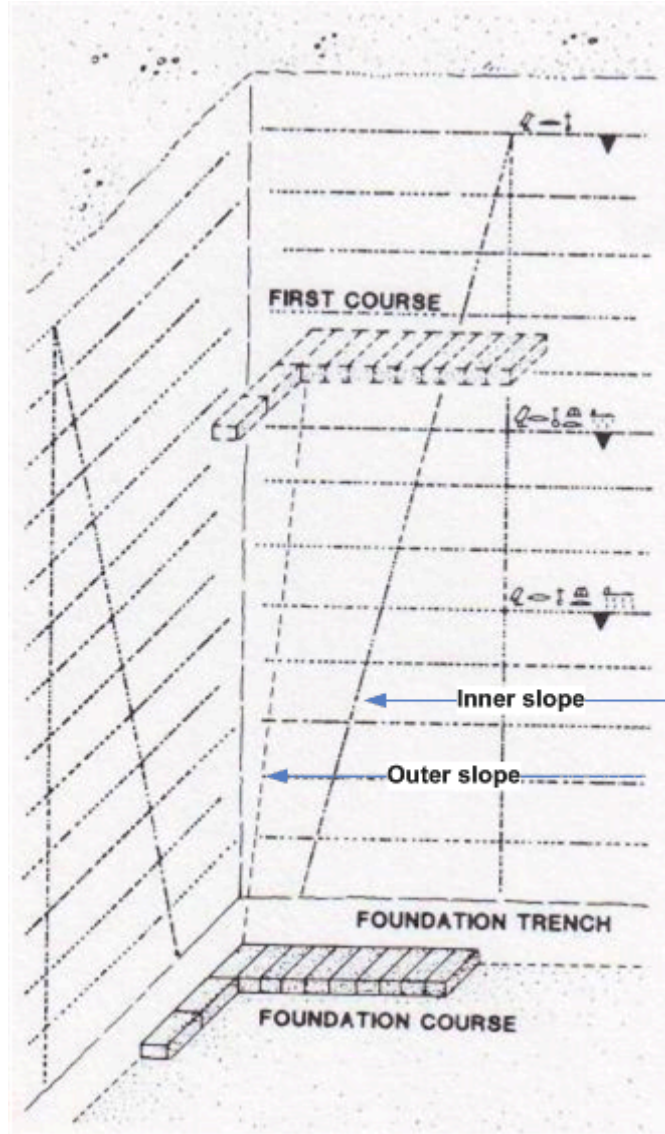


Figure 39. Meidum - Arnold (Arnold 1991, p. 12, Fig. 1.8).

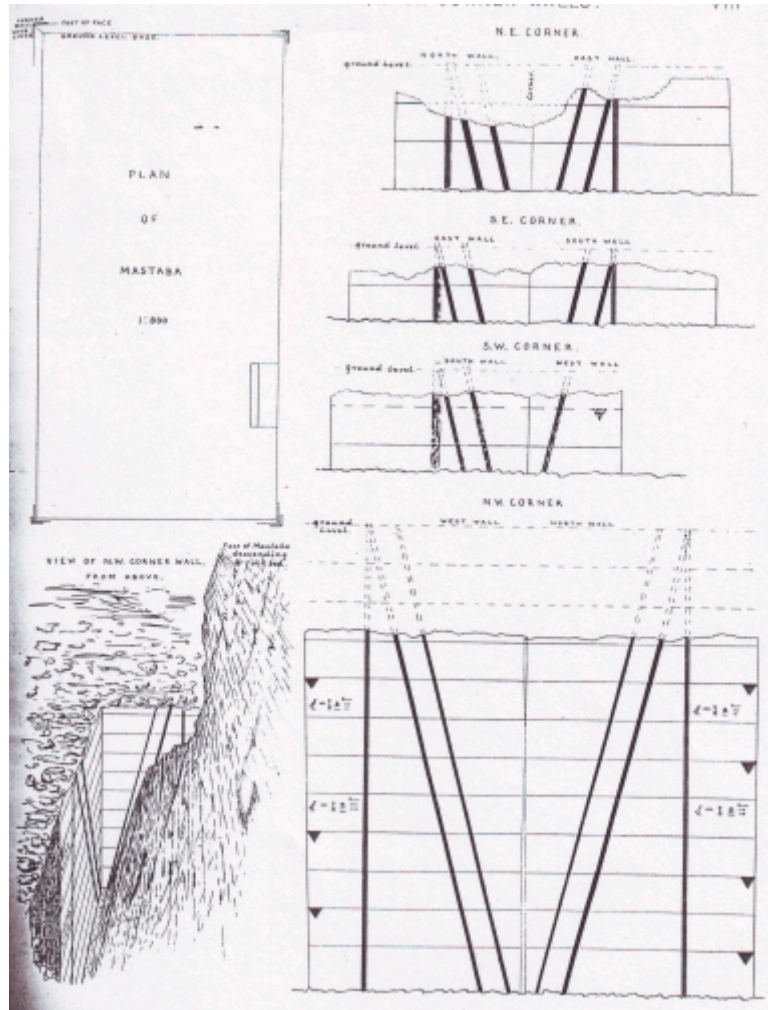


Figure 40. Meidum - Petrie – (Petrie 1892, Plate VIII).

Both figures show two lines of inclination lines starting at the base of the corner of the *mastaba*: the first original (inner) slope line shows a ratio of one to four cubits. The second (outer) slope line corresponds with the enlargement of the *mastaba* by one cubit, giving a ratio of one to five cubits.

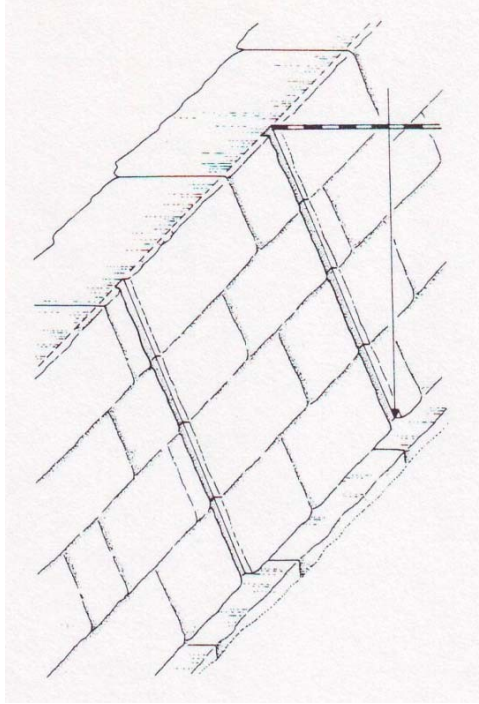


Figure 41. Grooves in an Undressed Wall Prepared for Seqed Measurements (Arnold, 1991, p.13, Fig. 1.9).

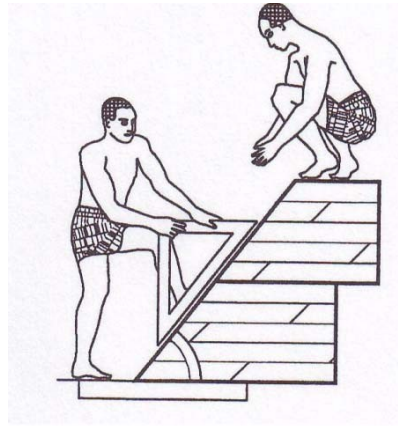


Figure 42. Tentative "Practical" Seqed Operation (Figure 24, Butler, 1998, p.78).

Table 29. Old Kingdom Pyramid Seqeds Expressed in Royal Cubit Divisions (Antoine Hirsch 2012 - Based on Butler 1998, p.78).

Pyramid	Palms	Fingers	length (cm)	Alternative
Neferikare	5		37.5	1 <i>remen</i>

Pyramid	Palms	Fingers	length (cm)	Alternative
Khentkawes	5		37.5	1 <i>remen</i>
Khafre	5	1	39.375	3 sandals
Userkaf	5	1	39.375	3 sandals
Djedkare-Isesy	5	1	39.375	3 sandals
Tety	5	1	39.375	3 sandals
Pepy I	5	1	39.375	3 sandals
Merenre	5	1	39.375	3 sandals
Pepy II	5	1	39.375	3 sandals
Khufu	5	1	39.375	3 sandals
Neuserre	5	1	39.375	3 sandals
Sahure	5	1	39.375	3 sandals
Red Pyramid	7	1	54.375	NA
Isesy sub. Pyramids	3	2	26.25	2 sandals

Table 30. Alternative Divisions in the Rhind Mathematical Papyrus (Antoine Hirsch 2012 – Based on Peet, 1923, pp. 97–102, plates Q and R).

<i>Seqed</i>	Cubit	Palm	Fingers	Length (cm)	Alternative
Problem 56		5	1	39.375	3 <i>tbwt</i>
Problem 57		5	1	39.375	3 <i>tbwt</i>
Problem 58		5	1	39.375	3 <i>tbwt</i>

<i>Seqed</i>	Cubit	Palm	Fingers	Length (cm)	Alternative
Problem 59b		5	1	39.375	3 <i>tbwt</i>
Problem 60	one quarter			13.125	1 <i>tbwt</i>

The examples of Table 30 show the close relationship between royal cubit and *tbwt*: the *tbwt* equals one quarter of a royal cubit (13.125 cm). The preference for the royal cubit is even more conspicuous in the *seqeds* of some of the Middle Kingdom pyramids published by Arnold (1991, p.12): eight palm divisions, ten palm divisions, and ten palm and a half divisions cannot easily and naturally accommodate a royal cubit of seven palm divisions of four finger divisions. The alternatives provided in Table 31 below are more consistent with actual divisions of the royal and great systems.

Table 31. Alternative Divisions in Arnold's Seqed Calculations (Antoine Hirsch, 2012 - Based on Arnold, 1991, p.12).

Pyramid	<i>Seqed</i> (Royal Cubit)	Palm (cm)	Finger (cm)	Length (cm)	Alternative
Amenemhat I					
	7 palms	52.5		52.5	1 Royal Cubit
	5 palms	37.5		37.5	1 <i>Remen</i>
	7 palms	52.5		52.5	1 Royal cubit
	8 palms	60		60	1 Great Cubit
Senowsret II					
	7 palms	52.5		52.5	1 Royal Cubit
	10 palms and a half	75	3.75	78.75	6 <i>tbwt</i>
Amenemhat II					
Dashur					
	7 palms	52.5		52.5	1 Royal Cubit

Pyramid	<i>Seqed</i> (Royal Cubit)	Palm (cm)	Finger (cm)	Length (cm)	Alternative
	10 palms	75		75	2 <i>Remens</i>
Hawara					
	7 palms	52.5		52.5	1 Royal Cubit
	8 palms	60		60	1 Great Cubit

===== Table end =====

16 The Tomb Of Ramesses Iv – The Disappearance of Division Names

In the tomb of Ramesses IV published by Carter and Gardiner (1917, pp. 130–57) and recently by Demichelis (2004, pp. 114–33), some dimensions published in royal cubits and fractions thereof can be easily named, indicating that traditional division names are gradually disappearing. Table 32 (based on Carter’s data) illustrates the artificial standardization of the royal cubit divisions expressed in cubits and palms, as in most cases, an even number of royal divisions (*nbj*, royal cubit, small cubit, *remen*, *djeser*, great span, small span, double palms, sandal, fist, and hand) can be substituted for the recorded palms and fingers. The table also shows the relationship between *tbt* and *nbj*: the *nbj* is a multiple of the *tbt* (sandal) and belongs to the royal system. Its length (65.625 cm) is part of the royal system as a multiple of the *tbt* used since the Old Kingdom: one *nbj* equals 5 *tbt*, or a royal cubit and a quarter.

Table 32. Ramses IV Tomb Dimensions – (Antoine Hirsch, 2012 - Based on Carter and Gardiner, 1917, pp. 149–56).

Description of the Tomb	Actual Lengths (metres)	Papyrus Recto	Papyrus Verso	Divisions on Actual Length
First Corridor				
Length	15.693		30 cubits	30 cubits
Breadth	3.662		7 cubits	7 cubits

Description of the Tomb	Actual Lengths (metres)	Papyrus Recto	Papyrus Verso	Divisions on Actual Length
Height	7.846		15 cubits	12 <i>nbjs</i>
Door-jamb (Thickness)	1.009		1 cubit three palms	9 fists
Doorway (Breadth)	2.69		5 cubits 1 palm 2 fingers	6 small cubits
Door-jamb(Height)	3.848		7 cubits 1 palm	29 <i>tbwt</i>
Lintel	0.374		2 cubits six palms	1 <i>remen</i>
Total Height of Corridor	4.222		10 cubits	8 cubits

Third Corridor

Height	4.185		7 cubits	14 <i>djesers</i>
Door-jamb (Thickness)	1.046		1 cubit three palms	2 cubits
Door-jamb (Projection)	0.187		3 palms	3 palms
Doorway (Breadth)	2.746		5 cubits 2 palms 1 finger	10 and a half great spans
Lintel (Height)	0.374		6 palms	1 <i>remen</i>
Door-jamb (Height)	3.811		6 cubits 3 palms 2 fingers	8 and a half small cubits
Door-jamb(Total height)	4.185		7 cubits 3 palms 2 fingers	8 cubits

Fourth Corridor

Length	13.624	25 cubits	15 cubits	26 cubits
Breadth	3.157	6 cubits	9 cubits	6 cubits
Height	5.007	9 cubits 4 palms	7 cubits	9 and a half cubits

Description of the Tomb	Actual Lengths (metres)	Papyrus Recto	Papyrus Verso	Divisions on Actual Length
Door-jamb (Thickness)	1.074		1 cubit three palms 1 finger	2 cubits
Door-jamb (Projection)	0.187		2 palms two fingers	$\frac{1}{2}$ <i>remen</i>
Doorway (Breadth)	2.746		5 cubits 1 palm	10 and a half great spans
Lintel (Height)	0.374		5 palms	1 <i>remen</i>
Door-jamb (Height)	3.961		6 cubits 2 palms	6 <i>nbjs</i>

Sarcophagus--Slide

Length	10.537	20 cubits		20 cubits
Breadth	2.746	5 cubits 1 palm		10 and a half great spans

Niche

Height	1.046		2 cubits	2 cubits
Breadth	0.673	1 cubit two palms		1 <i>nbj</i>
Depth	0.673	1 cubit two palms		1 <i>nbj</i>

Hall of Waiting

Length	4.708	9 cubits	8 cubits 4 palms	9 cubits
Breadth	4.203	8 cubits	8 cubits	8 cubits
Height	4.185	8 cubits	7 cubits	8 cubits
Door-jamb(Thickness)	0.598		1 cubit three palms	2 <i>djesers</i>
Door-jamb (Near Face)	0.224		3 palms	1 small span
Doorway (Breadth)	2.728		5 cubits 1 palm 2 fingers	6 small cubits

Description of the Tomb	Actual Lengths (metres)	Papyrus Recto	Papyrus Verso	Divisions on Actual Length
Lintel (Height)	0.43		5 palms	1 small cubit
Door-jamb (Height)	3.755		6 cubits 1 palm	10 <i>remens</i>

Sarcophagus Chamber

Length	7.398	16 cubits	13 cubits six palms	14 cubits
Breadth	8.398	16 cubits	16 cubits	16 cubits
Height	5.231	10 cubits	7 cubits	10 cubits

Inner Corridor

Length	7.66	14 cubits 3 palms		17 small cubits
Breadth	2.615	5 cubits		5 cubits
Height	3.456	6 cubits 3 palms 1 finger		11 and a half <i>djesers</i>
Recess				
Length	2.391	4 cubits 4 palms		8 <i>djesers</i>
Height	0.897	1 cubit five palms		2 small cubits
Depth	0.785	1 cubit three palms 2 fingers		3 great spans

Left Hand Chamber of the Shawabti Figures

Length	4.017	10 cubits		6 <i>nbjs</i>
Breadth	1.569	3 cubits		3 cubits
Height	1.793	3 cubits 3 palms		4 small cubits

End Room

Description of the Tomb	Actual Lengths (metres)	Papyrus Recto	Papyrus Verso	Divisions on Actual Length
Length	2.877	10 cubits		11 great spans
Breadth	2.26	3 cubits 3 palms		5 small cubits
Height	2.092	4 cubits		4 cubits
Length of Tomb from the First Corridor to the Sarcophagus Chamber	71.049	136 cubits 2 palms		189 and a half <i>remens</i>
to the End Room	10.537	24 cubits 3 palms		20 cubits
Total Length of Tomb	81.586	160 cubits 5 palms		272 <i>djesers</i>

17 A Change in System-*Aroura* Calculations

Aroura calculations confirm a change from the great system to the royal system. With a slight discrepancy, an *aroura* in the great system corresponds to 100 great cubits of 60 cm multiplied by 100 small cubits of 45 cm. It is replaced by squaring 100 royal cubits. In the process, the traditional division names of the *aroura* disappear.

An *aroura* is made of 100 cubits of land (*mH tA*) each 100 cubits in length with a depth of one cubit. 100 royal cubits in length multiplied by one cubit in depth give an area of 27.565 square metres or an area for each squared royal cubit of 0.2756 square metres. The great cubit of 60 cm multiplied by the small cubit of 45 cm, belonging both to the great system, gives an area of 0.27 square metres, very close to the result obtained by a squaring a royal cubit. The hypotenuse of such rectangle equals 75 cm, the length of a double *remen*. In the royal system, the *remen* equals 37.5 cm, half of 0.75 cm, respecting the halving principle of ancient Egyptian mathematics.

Using the royal system, Reineke (1963, p. 161), gives the name of the divisions of the *stbt* with their values expressed as one half, one quarter, one eighth etc. of the *stbt*. Reineke’s definitions are based on the Rhind Mathematical Papyrus. The definition of new “rectangles” based on royal cubits (Table 33) instead of the more natural rectangular construction derived from the great system corresponds to the shift from the great system to the royal system: the basic unit of the *stbt* is not a great cubit multiplied by a small cubit but one squared royal cubit. In order to avoid the introduction of fractions of cubits to define the *s3*, *sw*, and *rm3*. $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$ of an *aroura* are introduced. Once the traditional divisions are abandoned, the shift from rectangles to squares is greatly facilitated: a *stbt* corresponds to 100 squared royal cubits, half a *stbt* or *remen* becomes, for calculation purposes, one half of that square, slowly but surely starting a process at the end of which the notion that an *aroura* originates from 100 rectangular strips of land (*t3* or *mḥ t3*) is gradually lost. As Galán (1990, pp.161–4) remarks the sign *remen* appears after a rectangle in the Rhind Mathematical Papyrus problems 51 and 52, with the meaning of half a rectangle, in opposition to *gs* referring to the “abstract” calculated half of the same rectangle.

Table 33. Reineke’s *Stbt* Divisions (Reineke, 1963, p. 161).

Division Name	Ratio Values	Great System Divisions	Royal System Divisions	Area (Square Metres)
<i>stbt</i>	1	100 great cubits by 100 small cubits	100 royal cubits multiplied by 100 royal cubits	2756.5 square metres
<i>rmn</i>	1/2	100 <i>djesers</i> by 100 small cubits	100 royal cubits multiplied by 50 royal cubits	1378.25 square metres
<i>ḥsb</i>	1/4	100 <i>djesers</i> by 100 small spans	100 royal cubits multiplied by 25 royal cubits	689.125 square metres
<i>s3</i>	1/8	100 <i>djesers</i> by 100 fists	100 royal cubits multiplied by 12 1/2 royal cubits	344.563 square metres
<i>sw</i>	1/16	100 <i>djesers</i> by 50 fists	100 royal cubits multiplied by 6 1/4 royal cubits	172.282 metres

Division Name	Ratio Values	Great System Divisions	Royal System Divisions	Area (Square Metres)
<i>rm3</i>	1/32	100 <i>djesers</i> by 25 fists	100 royal cubits multiplied by 3 1/8 roya cubits	86.141 square metres
Unknown	1/64	100 <i>djesers</i> by 12.5 fists ?	100 royal cubits multiplied by 1 9/16 royal cubits	43.071 square metres

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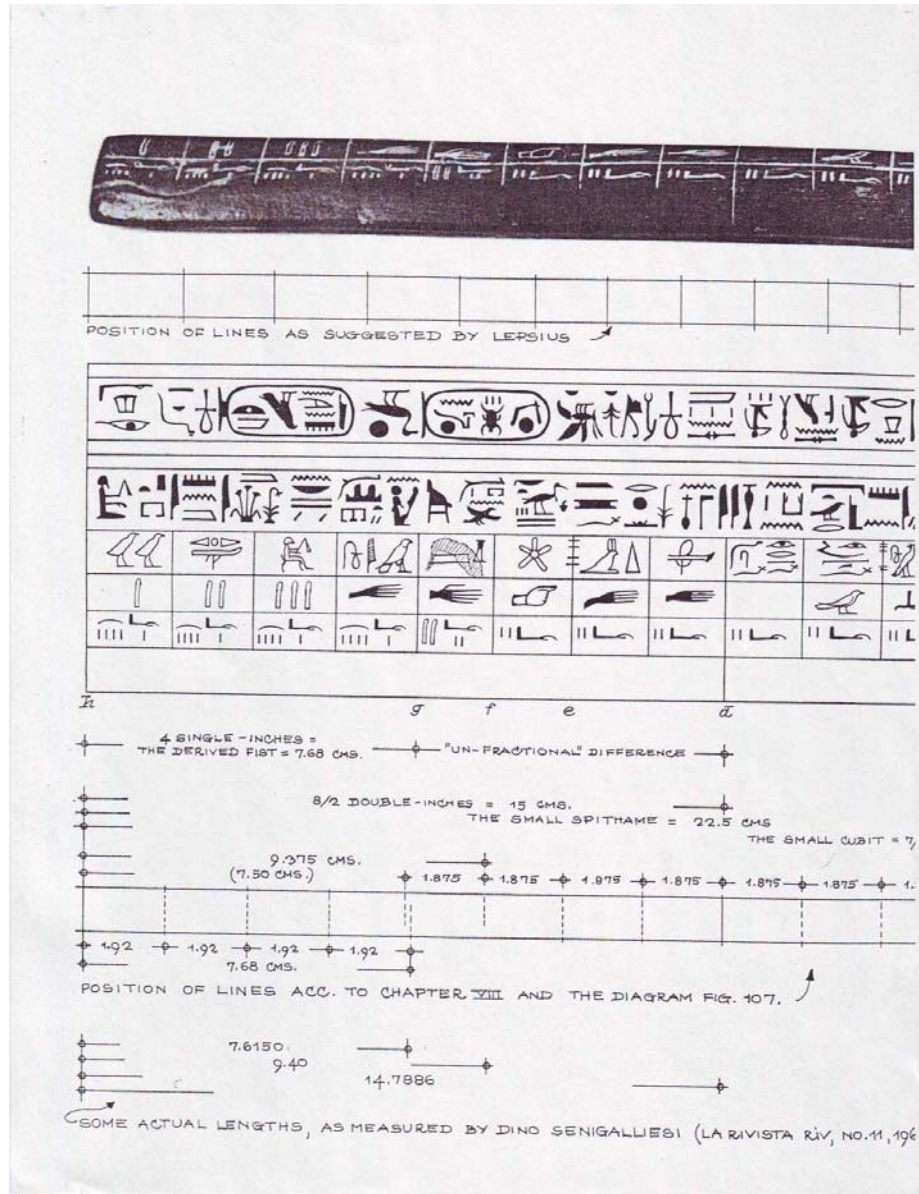
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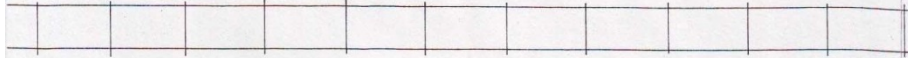
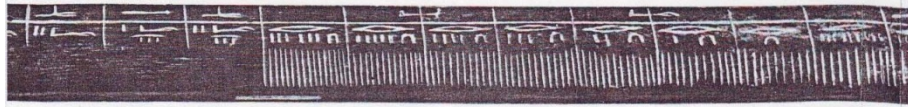
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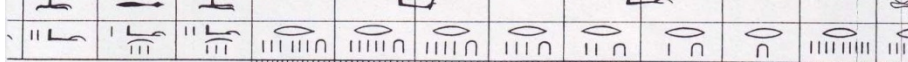
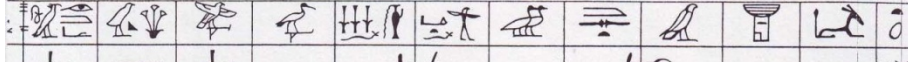
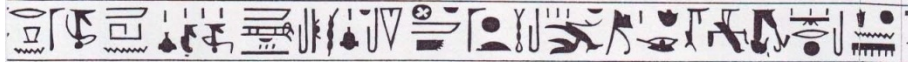
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Appendix A - Amenemope Full Scale Cubit Artifact





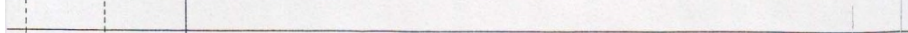
b. Elle N°1 in Holz. Turin.



c b

THE FOOT = 16 1/2 DOUBLE-INCHES = 1/2 ROYAL CUBIT OF SCALE A1
 1 FT = 7/2 DOUBLE-FISTS = 28" (IN SCALE A1 = 52.50 CMS.)

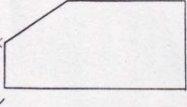
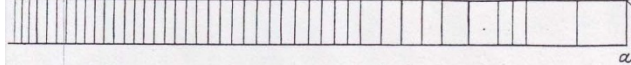
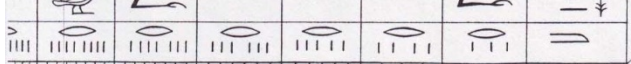
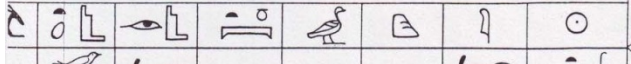
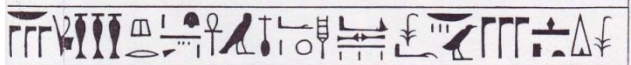
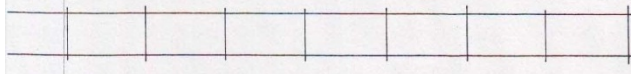
1.875 1.875 1.875 16" = 30 CMS. EACH = 1.875 CMS.



52.36 CMS. (LEPSIUS AND THIS WORK = 52.50 CMS.)

11, 1961 - SEE THE COVER) - SEE FURTHER NOTE B4 OF THIS WORK, ABOUT SHRINKAGE OF WOOD

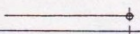
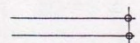
PLATE



SCALE A1 = .30 CMS. (= 14 SINGLE-INCHES)

.175 CMS.

OF WOOD.



Appendix B - Iversen's Divisions of The Small, Royal, and Reformed Cubits

Fig. p. 138

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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Fig. p. 139

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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Database

The methodology followed in the creation of the databases is inspired by G Grasshoff's (1998, pp. 97-148) article Normal Star Observations in Late Babylonian Astronomical Diaries and includes:

(1) The *database_1* spreadsheet containing 3866 entries with a total of 4905 measurements (heights, lengths, widths, and depths) of measurable objects from museums in the world (mostly from Cairo). This is a statistical database including published measurements of unbroken objects from museum catalogues. Its purpose is to verify whether both the royal and great systems measurement patterns are common occurrences, regardless of the type of object measured. All objects and measurements are referenced by their catalogue number, catalogue name, volume, page, period, and when available, location, sub-location, and dynasty. All objects and measurements are referenced by their catalogue number, catalogue name, volume, page, period, and when available, location, sub-location, and dynasty. The objects have been classified by periods from Predynastic to Roman and have been labeled Pre to 3 (inclusive) covering objects from the Predynastic to the Third Dynasty, Old Kingdom, First Intermediate Period, Early Middle Kingdom, Middle Kingdom (early?), Middle Kingdom, Second Intermediate Period, New Kingdom, Third Intermediate Period, Bubastite to Saite, Late Period, Ptolemaic, Late Ptolemaic-Early Roman, and Roman Period²⁰

²⁰ The statistical object categories included in the Database_1.xlsx spreadsheet table are: Alabaster block, Arrow point, Arrow-head, Axe, Axe head, Base of Bowl, Base of statue, Beads, Blade of adze, Blade of dagger, Blade of hoe, Block of stone, Boat, Bowl, Bracelet, Bull's hind leg, Cartonnage, Cartouche, Chest, Chisel, Cist, Clay-cone, Clay-seal, Coffin, Coffin inscription, Comb, Cone, Copper tool, Crescent, Cubit type used, Cubit type used in room, Cylinder, Dish, False door or part of, Figure, kneeling, Fish, model of, Flat dish, Flat disk, Flint, Flint chisel, Flint dagger, Flint flake, Flint knife, Flint scraper, Foot of bull, Foot of chair, Foot of chair leg, Foot of offering table, Foot of statuette, Fore limb of an articulated statuette, Forefoot of bull (model), Foundation Stone, Funerary inscription or part of, Grindstone, Hair pin, Harpoon, Hawk's head, Head, Head of statuette, Head rest, Hind leg of bull (model), Hippopotamus (model), Inner coffin, Inscription, Ivory object, Ivory plaque, Jar with sealing, Jar-sealing, Knife, Knob, Lid of jar, Lion, Mace-head, Mask, Mastaba or part of, Obelisk, Offering, Offering dish, Offering table, Offering trough, Outer coffin, Pad, Painting, Palette, Part of Statue, Part of cartouche, Part of coffin, Part of false door, Part of offering list, Part of sarcophagus, Part of temple wall, Part of tomb, Part of wall, Pebble, Pendant, Pin, Painting, Plaque, Polisher, Pot, Rattle, Relief, Relief or part of, Rod, Rod of faience, Rod of ivory, Round offering table, Rubber, Sarcophagus, Sarcophagus/Coffin, Statue, Saucer, Schist plaque, Seal, Seal of a jar, Sealing, Shell shaped pendant, Sign, Slag of pottery, Slate, Slate palette, Small chisel, Small tablet, Spinning-whorl, Spoon, Stand, Stand of offering table, Statue, Statuette, Stela, Stela/Inscription, Stone block, Strip of copper, Strip of thin copper, Thin plate, Three armed tool, Tile, Tool, Triangular plate, Trough, Tube, Vase, and Vase-stand.

. The majority of the entries are from the Predynastic to the Middle Kingdom Periods as the use of a royal cubit as a converter, according to textual references in the fifth chapter of this thesis, corresponds to this time period.

(2) The *database_6* spreadsheet contains 56 entries representing the lengths of 1 to 28 finger divisions of the royal cubit by increments of half a royal finger division up to 52.5 cm.

(3) The *database_4* spreadsheet contains the lengths of 1 to 40 royal finger divisions necessary to reach the length of a double *remen* (75 cm). It also contains the 120 lengths corresponding to the length of the great cubit (60 cm) of one palm division of five finger divisions (C24) divided into half finger divisions of 0.5 cm each. The lengths they contain have been applied to *database_2*, *database_3*, and *database_5*.

(4) The *database_2* spreadsheet contains the percentage of 3651 measurements²¹ matching finger divisions or divisions (without any delta adjustment) of the scribe's palette, the royal cubit, the great cubit, or the double *remen*.

(5) The *database_3* spreadsheet contains the percentage of royal cubits used for "elite" objects in the database.

(6) The *database_5* spreadsheet contains the percentage of royal cubits used in the database at Abydos.

Two addenda (*Addendum 1* and *Addendum 2*) palliate the statistical effects of *database_1*. *Addendum 1* performs an analysis of the heights and widths of stelae in the British Museum contrasting the results given by the measurements of stela and inscriptions not clearly demarcated by register lines in the elite "Stela/Inscription" category in *database_1*. *Addendum 2*

²¹ 1244 entries in Database_1 do not match great, royal, or double *remen* measurements for several reasons. Measurements from broken artifacts or uncertain measurements were not entered in DATABASE_1, but their references were to maintain the integrity of the catalogues used. Multiples of the scribe's palette, royal cubit, great cubit, and double *remen* have been discarded as trends are based on the maximum length of a double *remen* (75 cm) which includes the length of the scribe's palette, the royal cubit, the great cubit, the *nbi*, and the *remen*. Finally, the rest of the entries (582) would require a "subjective" delta calculation between the royal and the great systems to match them to either system; they represent 12% of the 4905 total measurements.

performs an analysis of the clearly defined items (51) of the "False door or part of" category of *database_1*. Both addenda show the combined use of both the royal and great systems, labelled as "mixed," in single objects in both "elite" sub-categories.

Addendum 1

Purpose

The "Stela/Inscription" category in the thesis database can be considered "statistical." It includes complete measurements of inscriptions not clearly demarcated by register lines. The purpose of this addendum is to remove the statistical entries from the previous "Stela/Inscription" study and perform a new analysis of the heights and widths of stelae. This addendum includes two museum collections. The stelae from the British Museum (British_Museum_Collection spreadsheet on the CD) include items dated from the 4th Dynasty to the 19th Dynasty, almost half of which are dated to the 12th Dynasty. The stelae from the Cairo Museum, all from the 12th Dynasty, and, on purpose, all from Abydos, are included in the *Abydos_12th_Dynasty_Collection* spreadsheet on the CD.

The items in both collections have been categorized as *Oben abgerundeter Grabstein* when they have a rounded top and *Rechteckiger Grabstein* when they are rectangular. All item references in both spreadsheets include their respective bibliographic sources and hyperlinks to their respective plates (British Museum) or descriptions (Cairo Museum). The lengths and widths in both collections are metric; the Imperial foot of the British Museum collection corresponds to 0.3048 metres; the Imperial Inch equals 0.0254 metres. There are 111 items in the British Museum collection (47 of which belong to the 12th Dynasty) and 489 items in the Cairo Museum collection. The classification repartition details for each collection appear in Table 1 and Table 2.

Methodology

In order to determine the percentage of royal and great measurements and eventual regional differences (Abydos) in the items of both collections the following methodology has been applied.

(1) The British Museum collection items have been used as a general and 12th Dynasty reference benchmark against the items contained in the Abydos collection.

(2) Each collection spreadsheet contains a worksheet (*Ordered Multiple Cubit Lengths*) including referenced and sorted divisions of the royal cubit and the great cubit over the length of three metres, corresponding to the length of 5 royal and great cubits. These 261 divisions are incremented by the length of a royal finger division (0.01875 metre) in the royal system and by the length of a great finger division (0.025 metre) in the great system. Division lengths shared by both systems are marked "Shared".

(3) Each item height and width in each collection is compared to the cubit divisions of the *Ordered Multiple Cubit Lengths* worksheet. The computer assigns for each item height and width a lower and upper threshold length and a system reference (royal or great) based on whether the item height or width is greater or equal to a cubit division (lower threshold) and smaller than the next cubit division(s) (upper threshold).

(4) The system assigned to the item height or width is then adjusted. The adjusted system corresponds to the lower or upper threshold system the length difference of which, when compared with the item height or width, is the smaller.

(5) The overall length system (royal or great) of the item is assigned on the following basis:

i. Shared divisions by both the royal and great systems are treated as royal divisions.

ii. When both (height and width) adjusted system definitions are "royal", the item is considered to follow the royal system.

iii. When one (height and width) system definition is adjusted to the royal system while the other is adjusted to the great system, the item is declared "mixed."

iv. When both (height and width) adjusted system definitions are "great", the item is considered to follow the "great" system.

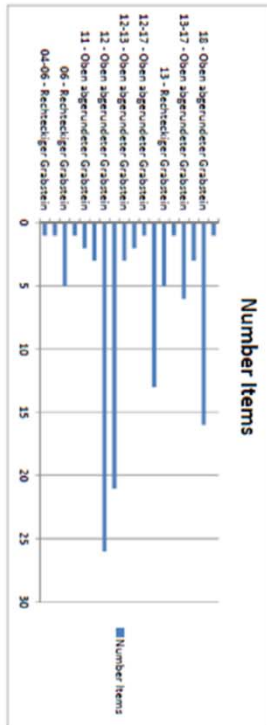
Interpretation

From Table 3, the number of items following the royal system is lower (less than 25%) than expected in an "elite" category. Following traditional belief, one would have expected to find a majority of items following the royal system in this category. The increased number of great

system items in the 12th Dynasty Abydos series may suggest the use of a local great cubit in this nome.

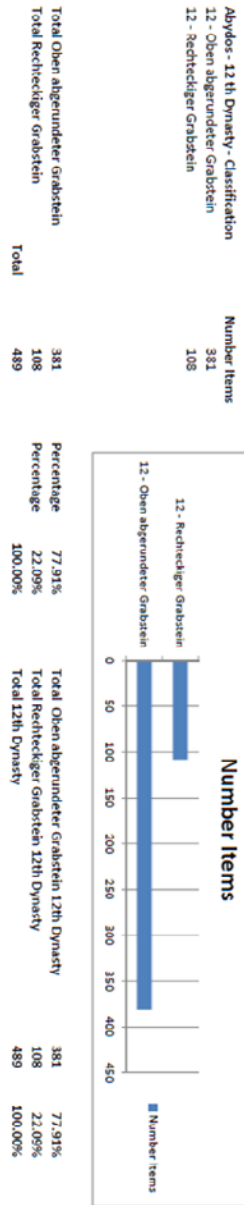
Addendum 1. British Museum Collection Item Distribution

BM - Dynasty - Classification	Number Items
04-06 - Rechteckiger Grabstein	1
05 - Rechteckiger Grabstein	1
06 - Rechteckiger Grabstein	5
06-10 - Rechteckiger Grabstein	1
11 - Oben abgerundeter Grabstein	2
11 - Rechteckiger Grabstein	3
12 - Oben abgerundeter Grabstein	26
12 - Rechteckiger Grabstein	21
12-13 - Oben abgerundeter Grabstein	3
12-13 - Rechteckiger Grabstein	2
13-17 - Oben abgerundeter Grabstein	1
13 - Oben abgerundeter Grabstein	13
13 - Rechteckiger Grabstein	5
13-16 - Oben abgerundeter Grabstein	1
13-17 - Oben abgerundeter Grabstein	6
17 - Oben abgerundeter Grabstein	3
18 - Oben abgerundeter Grabstein	16
18-19 - Oben abgerundeter Grabstein	1
Total Oben abgerundeter Grabstein	72
Total Rechteckiger Grabstein	39
Total	111



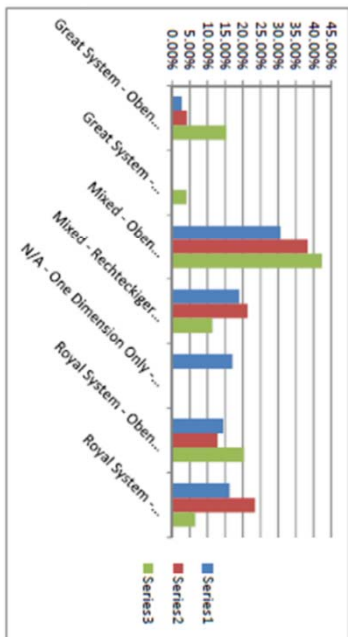
Percentage	Total Oben abgerundeter Grabstein 12th Dynasty	Percentage	Total Rechteckiger Grabstein 12th Dynasty
64.86%	26	55.32%	21
35.14%	21	44.68%	16
100.00%	47	100.00%	16

Addendum 1. Abydos Collection Item Distribution



Addendum 1. Overall Results

	Series 1	Series 2	Series 3
Overall System (Width_and_Height) - Classification Percentage	BM	BM 12th Dynasty	Abydos 12th Dynasty
Great System - Oben abgerundeter Grabstein	2.70%	4.26%	15.34%
Great System - Rechteckiger Grabstein	0.00%	0.00%	4.09%
Mixed - Oben abgerundeter Grabstein	30.63%	38.30%	42.33%
Mixed - Rechteckiger Grabstein	18.92%	21.28%	11.45%
N/A - One Dimension Only - Oben abgerundeter Grabstein	17.12%	0.00%	0.00%
Royal System - Oben abgerundeter Grabstein	14.41%	12.77%	20.25%
Royal System - Rechteckiger Grabstein	16.22%	23.40%	6.54%
	100.00%	100.00%	100.00%



Addendum 2

Purpose

The "False door or part of" category in the thesis database can be considered "statistical." It includes complete measurements of false door widths without heights, offering tables, and inscriptions not clearly demarcated by register lines. The purpose of this addendum is to remove the statistical entries from the previous "False door or part of " collection and perform an analysis of the clearly defined item heights (51 items or 21.25%) of the original collection. The items qualified as "rough" have been eliminated. The retained items (dating mostly from the Old Kingdom) in the collection have been categorized as *Scheintür*. All item references in the Cairo Museum *Scheintür* Collection spreadsheet's *Complete Scheintür Heights* worksheet include their respective bibliographic sources and hyperlinks to their respective descriptions (Cairo Museum). The heights are metric.

Methodology

In order to determine the percentage of royal and great measurements by system and by dynasty in the collection, the following methodology, already used in the first addendum to this thesis, has been applied.

- (1) The first addendum's British Museum *Grabstein* collection item heights have been used as a general reference benchmark against the *Scheintür* items contained in the *Cairo_Museum_Scheintür_Collection* on the CD.
- (2) The collection spreadsheet contains a worksheet (*Ordered Multiple Cubit Lengths*) including referenced and sorted divisions of the royal cubit and the great cubit over the length of six metres, corresponding to the length of 10 royal and great cubits. These 520 divisions are incremented by the length of a royal finger division (0.01875 metre) in the royal system in order to match the length of a great finger (0.025 metre) division of the great system. Division lengths shared by both systems are marked "Shared".
- (3) Each item height in the collection is compared to the cubit divisions of the *Ordered Multiple Cubit Lengths* worksheet. The computer assigns for each item height a lower and upper threshold

length and a system reference (royal or great) based on whether the item height is greater or equal to a cubit division (lower threshold) and smaller than the next cubit division(s) (upper threshold).

(4) The systems assigned to the item height thresholds are then adjusted. The adjusted system corresponds to the lower or upper threshold system the length difference of which, when compared with the item height, is the smaller. The adjusted height system (royal or great) of the item is assigned on the following basis:

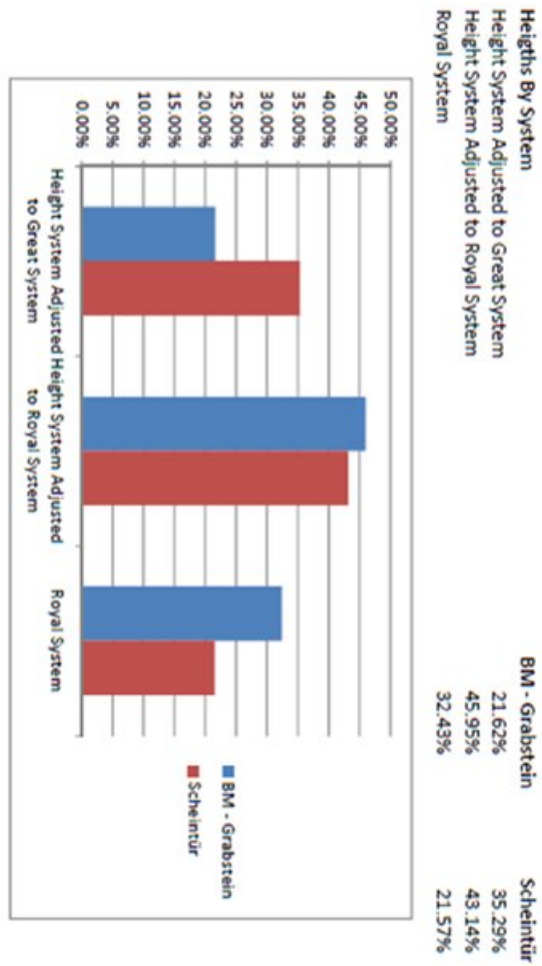
- i. Shared divisions by both the royal and great systems are treated as royal divisions.
- ii. When both thresholds are "royal" the item is considered to be a genuine royal system height and is marked "Royal System."

Interpretation

All items in both collections (Table 1 and Table 2) are "elite" items. As such, one would expect the heights to closely follow the royal system. Between 21 to 35% of the measurements are adjusted to the great system while the same percentages are genuine (not adjusted) royal measurements.

Within the limitation of the small number of items available, in Table 2, the great system Scheintür adjusted great system measurements show a substantial decrease from 75% to 21% - 32%, closer to the great system adjusted benchmark height percentage of the British Museum *Grabstein* collection (21%) of *Addendum 1*, dated to Dynasties 4 to 19. This decrease can be interpreted as a shift from the great system to the royal system in elite objects.

Addendum 2. Heights by System



Addendum 2. Heights by Dynasty

Heights by Dynasty
 Height System Adjusted to Great System
 Height System Adjusted to Royal System
 Royal System

