Canadian Science Publishing | **Canadian Journal of Physics**

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Journal:	*Canadian Journal of Physics*
Manuscript ID	cjp-2019-0054.R1
Manuscript Type:	Article
Date Submitted by the Author:	21-Mar-2019
Complete List of Authors:	Fedi, Marco; Ministero dell'Istruzione dell'Universita e della Ricerca,
Keyword:	dilatant vacuum, relativistic kinetic energy, quantum relativity, Stokes-Einstein radius in a vacuum, relativistic mass
Is the invited manuscript for consideration in a Special Issue? :	Not applicable (regular submission)

Relativistic mass due to a dilatant vacuum leads to a quantum reformulation of the relativistic kinetic energy

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Relativistic mass change with speed is considered here as the effect of a viscous, dilatant vacuum, whose apparent viscosity is related to the Lorentz factor. Transient solidlike vacuum due to shear stress is presented as the reason why vacuum prevents the speed of massive objects from being indefinitely increased. Such a vacuum – that in a previous study allowed to exactly calculate the Pioneer anomaly, Mercury's perihelion precession and was shown to be compatible with stable planetary orbits – leads here to a quantum formula for the relativistic kinetic energy. A formula which distinguishes between the case of accelerated charges in a vacuum, for which a Stokes-Einstein radius comes into play, and the case of accelerated macroscopic bodies, for which the quantum potential term vanishes. In this way, incidentally, one obtains again correct results for the Pioneer 10, confirming the role of vacuum's viscous force. This description of a quantum mechanism underlying the relativistic kinetic energy may be also helpful in constructing a theory of quantum relativity and might tell us more even about the interactions of matter with the Higgs field and the dark sector: two issues which can be themselves linked to a dilatant vacuum.

PACS numbers: 95.30.Lz; 03.30.+p; 83.60.Rs; 41.75.-i
Keywords: dilatant vacuum, relativistic mass, relativistic kinetic energy, Stokes-Einstein radius in a vacuum, quantum relativity.

I. Introduction

In particle acceleration, there is an enormous discrepancy between the energy supplied to a particle and its resulting speed, which is much lower than expected, according to classical physics. Accelerated particles seem to meet an impenetrable wall, located at the speed of light: the greatest part of the supplied energy seems to get lost, besides a fraction of it, converted into synchrotron light. According to special relativity theory (SRT), this gap is due to energy which converts into relativistic mass. This concept is, however, still rather controversial [1, 2]. It is actually preferred to refer to relativistic kinetic energy, instead of relativistic mass, according to the well-known formula $E_k = mc^2((1/\sqrt{1-(v/c)^2})-1)$, but in any case, mass is the only parameter of this equation that can take charge of the supplied energy which is not converted into classical kinetic energy. At least if physical vacuum, the Higgs field or the dark sector are not taken into account and the equation is not modified into a quantum equation. In the light of modern physics, we could for instance hypothesize that massive bodies which are accelerated through the Higgs field are subject to its apparent viscosity and acquire further mass, which increases with the Lorentz factor. In this case the Higgs field could be interpreted as a dilatant fluid (a viscous, shear-thickening vacuum), capable of interacting also with macroscopic bodies. Indeed, a modified Stokes' law for a dilatant vacuum, in which the Lorentz factor has the function of a dimensionless viscosity term (in place of η in the classical Stokes' law), yielded direct and exact solutions [3] to the Pioneer anomaly and to Mercury's perihelion precession (which is the first classical test of general relativity). Section II recalls then the modified Stokes' equation for a dilatant vacuum, while Sect. III treats proton acceleration in a dilatant vacuum and allows to achieve a quantum formula for relativistic kinetic energy. Once pointed out that relativistic mass is actually the detected greater inertia (greater difficulty to accelerate a body) due to a dilatant vacuum which obeys the Lorentz factor ($\gamma-1$, precisely), the role of rest mass in the interaction with the vacuum is differently presented in Sect. IV via 3D plots of the Lorentz factor, which show that the factor progressively tends to a right angle, the larger the accelerated mass. In Sect. (V) calculations are presented as regards the acceleration of macroscopic bodies, highlighting the fact that they obey a different law, compared to that of charged particles, in which the quantum potential vanishes.

II. Reinterpreting relativistic mass via vacuum dilatancy

A modified Stokes' equation (MSE), obtained by replacing in Stokes' law the viscosity coefficient multiplied by speed, ηv, with $(\gamma-1)\kappa$, where γ is the Lorentz factor, here reinterpreted as a nonlinear, shear-depending term of vacuum dilatancy, has been able to exactly solve two famous anomalies [3] and should be therefore considered as an evidence that physical vacuum is a shear-thickening fluid. The MSE, expressing vacuum's viscous force, reads

$$F_\emptyset = -6\pi r\left(\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}-1\right)\kappa = -6\pi r(\gamma-1)\kappa = -6\pi r D\kappa \tag{1}$$

where γ is the Lorentz factor, κ is a unitary constant expressed in $\mathrm{kg/s^2}$, the subscript $\emptyset$ refers to the vacuum and

$$D = \gamma - 1 \tag{2}$$

is the term of vacuum dilatancy, whose asymptote refers to a transient solidlike state of the vacuum, due to sufficiently large shear stress, which prevents speed from being indefinitely

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increased. The relativistic kinetic energy can be therefore interpreted as the necessary additional energy an accelerated body needs to oppose the viscous force (1) of the dilatant vacuum, detected as increased inertia of the body. Indeed, Einstein equation for relativistic kinetic energy is actually nothing but the rest energy of a body multiplied by the term of vacuum dilatancy (2)

$$E_k = mc^2 D. \tag{3}$$

Section III describes this aspect and a quantum formula is therein obtained. Here, dilatant vacuum is therefore indicated as an essential step toward a quantum theory of relativity. Indeed, a previous positive test on the equation of general relativity expressing the perihelion precession of Mercury (first classical test of general relativity) was done in [3]. The opposite force (a drag force) exerted by physical vacuum is detected in synchrotrons as increased inertia, and consequently thought of as increased (relativistic) mass, but a quantum-oriented explanation is suggested below, in terms of the reaction of a shear-thickening vacuum, and it is confirmed by the calculations in Sect. V. Also the following reflection by Laughlin [4], 1998 Nobel laureate in physics, seems to support the fact that the vacuum behaves as a shear-thickening fluid: "Studies with large particle accelerators have now led us to understand that space is more like a piece of window glass than ideal Newtonian emptiness. It is filled with stuff that is normally transparent but can be made visible by hitting it sufficiently hard to knock out a part". Although Einstein's relativity yields correct *quantitative* results, a different *qualitative* picture, different from pure differential geometry and capable to be merged into a quantum formalism, is now necessary.

III. Particle acceleration in dilatant vacuum

Let us consider the case of a proton accelerated up to $E_k = 6.5\,\text{TeV} = 1.04141 \times 10^{-6}\,\text{J}$. According to classical physics it would have the following speed (v_c)

$$v_c = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \cdot (1.04141 \times 10^{-6}\,\text{J})}{1.67262 \times 10^{-27}\,\text{kg}}} = \\ = 3.5288 \times 10^{10}\,\text{m} \cdot \text{s}^{-1}, \tag{4}$$

where m refers, here and below, to *rest* mass. In particle accelerators a much lower speed (observed speed, v_o) is however detected. In this case

$$v_o = c\sqrt{1 - \left(\frac{E_k}{mc^2} + 1\right)^{-2}} = (299792458\,\text{m} \cdot \text{s}^{-1}) \cdot \\ \cdot \sqrt{1 - \left(\frac{(1.04141 \times 10^{-6}\,\text{J})}{(1.67262 \times 10^{-27}\,\text{kg})(299792458\,\text{m} \cdot \text{s}^{-1})^2} + 1\right)^{-2}} \\ = 299792454.8775\,\text{m} \cdot \text{s}^{-1} \tag{5}$$

and this fact is explained as relativistic kinetic energy. That could be also explained via the Higgs field. Indeed that field is like a viscous fluid which impedes particles and gives them mass. As for classical viscous fluids, we could hypothesize that acceleration through the Higgs field causes increasing apparent viscosity: that would result in the detection of increased inertia for accelerated particles. In this case, the Higgs field would not only give each particle its specific rest mass but it would also be the quantum cause of the phenomenon called relativistic mass. In this study, however, the more general concept of shear-thickening vacuum impeding an indefinite increase in speed is used, and the possibility that this kind of vacuum coincides with the Higgs field is only suggested and left to further investigations. Also considering the vacuum as a *doped* superfluid (e.g. dark matter particles scattered in a sea of superfluid dark energy) might account for vacuum dilatancy. It is furthermore interesting to notice that the T^{00} component of the stress-energy tensor in Einstein field equation may refer to both vacuum's mass density and relativistic mass per unit volume. Furthermore, the T^{21}, T^{31}, T^{32} components refer to momentum flux, which is related to viscosity (as momentum diffusion due to shear stress) via Newton's law for viscosity, while T^{12}, T^{13}, T^{23} directly refer to shear stress. In short, Higgs field, dark sector and Einstein field equations are closely related and it seems they point in the same direction [5]: a viscous, shear-thickening vacuum, whose apparent viscosity obeys the Lorentz factor.

In the example of a proton at 6.5 TeV, vacuum friction should cause the following speed decrease

$$\Delta v = v_o - v_c = -3.49882 \times 10^{10}\,\text{m} \cdot \text{s}^{-1}. \tag{6}$$

This can also be written $\Delta v = a_\emptyset \Delta t$, where $\Delta t = 1\,\text{s}$ and

$$a_\emptyset = \frac{F_\emptyset}{m} = \frac{-6\pi r_{h(\emptyset)} D\kappa}{m} = \frac{-6\pi(4.48164 \times 10^{-22}\,\text{m})\left(\frac{1}{\sqrt{1-\left(\frac{299792454.8775\,\text{m}\cdot\text{s}^{-1}}{299792458\,\text{m}\cdot\text{s}^{-1}}\right)^2}} - 1\right) \cdot 1\,\text{kg}\cdot\text{s}^{-2}}{1.67262 \times 10^{-27}\,\text{kg}} = -3.49882 \times 10^{10}\,\text{m}\cdot\text{s}^{-2}, \tag{7}$$

is the negative acceleration caused by dilatant vacuum. where $r_{h(\emptyset)}$ is the hydrodynamic Stokes-Einstein radius (being the vacuum the fluid at issue). The use of proton charge radius in this hydrodynamic context would be indeed nonsensical. The interaction proton-vacuum can be due to electric interactions with the virtual dipoles in the vacuum (particle-antiparticle pairs). Here is the derivation of the hydrodynamic radius for particles moving in a fluid, dilatant vacuum: from

Stokes-Einstein formula $r_h = k_B T/6\pi\eta d$, where k_B is the Boltzmann constant, T is temperature, η the viscous coefficient of the classical Stokes' law and d the diffusion coefficient, since $k_B T = pV/N$, where p, V, N are pressure, volume and the number of vacuum's quanta (respectively), the hydrodynamic radius reads $r_h = pV/N6\pi\eta d$. Moreover, $d = \mu k_B T$, where μ is the mobility coefficient (in this case of vacuum's quanta), so we can write the following rightmost equivalence $\mu = v_d pV/FN = v_o pV/F_\emptyset N$, where $v_d = v_o$ (with v_d being the terminal drift velocity) and the applied force (F) is vacuum's viscous force (1), according to Newton's third law (vacuum's reaction). The hydrodynamic radius becomes then $r_h = F_\emptyset/6\pi\eta v_o$. Finally, since to derive the MSE the substitution $\eta v = D\kappa$ was done (to describe a dilatant vacuum), one arrives to the equation $r_h = ma_\emptyset/(-6\pi D\kappa)$, where $a_\emptyset$ is negative, so the radius has a positive value. After some algebra, one can proceed with the following equivalences

$$r_{H(\emptyset)} = -\frac{ma_\emptyset}{6\pi D\kappa} = -\frac{m\Delta v}{6\pi D\kappa\Delta t} = -\frac{m^2c^2\Delta v}{6\pi E_k \kappa\Delta t} \tag{8}$$

where the term of vacuum dilatancy (2) contains v_o and $\Delta v = v_o - v_c$, as above.

Eq. (6) can also be expressed in terms of kinetic energy variation, that is

$$\Delta E_k = \frac{1}{2}m\left(v_o^2 - v_c^2\right) \tag{9}$$

and since

$$v_o^2 - v_c^2 = (\Delta v)^2 + 2v_o v_c - 2v_c^2 \tag{10}$$

we obtain the work done by the dilatant vacuum to brake the proton, also observable (with opposite sign) as the energy spent by the synchrotron to oppose vacuum dilatancy

$$\begin{aligned} W_\emptyset &\equiv \Delta E_k = \\ &= \frac{1}{2}m\left[(\Delta v)^2 + 2v_o v_c - 2v_c^2\right] = \\ &= \frac{1}{2}m\left[(\Delta v)^2 + 2v_c\Delta v\right] = \frac{1}{2}m\left[(a_\emptyset\Delta t)^2 + 2v_c a_\emptyset\Delta t\right] = \\ &= a_\emptyset\Delta t\left(\frac{1}{2}ma_\emptyset\Delta t + mv_c\right) = \frac{F_\emptyset\Delta t}{m}\left(\frac{F_\emptyset\Delta t}{2} + mv_c\right) = \\ &= \frac{(F_\emptyset\Delta t)^2}{2m} + F_\emptyset\Delta t v_c = \frac{(\int F_\emptyset\, dt)^2}{2m} + v_c\int F_\emptyset\, dt = \frac{J_\emptyset^2}{2m} + J_\emptyset v_c, \end{aligned} \tag{11}$$

where $J_\emptyset$ is the impulse due to vacuum's viscous force. The negative work of the vacuum on the accelerated proton in the example above is

$$\begin{aligned} W_\emptyset = \Delta E_k = \frac{J_\emptyset^2}{2m} + J_\emptyset v_c = &\left(-6\pi(4.48164\times10^{-22}\,\mathrm{m})\left(\frac{1}{\sqrt{1-\left(\frac{299792454.8775\,\mathrm{m{\cdot}s^{-1}}}{299792458\,\mathrm{m{\cdot}s^{-1}}}\right)^2}}-1\right)\cdot 1\,\mathrm{kg{\cdot}s^{-2}}\cdot 1\,\mathrm{s}\right)^2\cdot \\ &\cdot\frac{1}{2\cdot(1.67262\times10^{-27}\,\mathrm{kg})} + \left(-6\pi(4.48164\times10^{-22}\,\mathrm{m})\left(\frac{1}{\sqrt{1-\left(\frac{299792454.8775\,\mathrm{m{\cdot}s^{-1}}}{299792458\,\mathrm{m{\cdot}s^{-1}}}\right)^2}}-1\right)\cdot 1\,\mathrm{kg{\cdot}s^{-2}}\cdot 1\,\mathrm{s}\right)\cdot \\ &\cdot\left(3.5288\times10^{10}\,\mathrm{m{\cdot}s^{-2}}\right) = \\ &= -1.04133\times10^{-6}\,\mathrm{J}, \end{aligned} \tag{12}$$

part of which converted into synchrotron radiation: for the case of protons just a tiny amount. The corresponding equation for relativistic kinetic energy (total supplied energy) reads then

$$E_k = -W_\emptyset + E_k^{(class)} \tag{13}$$

where $E_k^{(class)} = mv_o^2/2$ is the classical kinetic energy. Thus

$$\begin{aligned} E_k &= mc^2\left(\frac{1}{\sqrt{1-\left(\frac{v_o}{c}\right)^2}} - 1\right) = \\ &= -\left(\frac{J_\emptyset^2}{2m} + J_\emptyset v_c\right) + \frac{1}{2}mv_o^2. \end{aligned} \tag{14}$$

where the minus sign allows us to observe the total energy amount supplied to the proton. In fact (14) yields the correct energy corresponding to 6.5 TeV. This energy amount can be interpreted as the classical kinetic energy the proton has at v_o, considering its rest mass, plus the work done by the accelerator on the particle to accelerate it through the dilatant vacuum and equal, for the third principle of motion, to the negative work of the vacuum, split into two components, one of which is revealed below as a quantum potential. Let us express (11) – with minus sign to refer to the supplied energy, which is specular with respect to the negative work of the vacuum – by resorting to momentum (p),

$$-\left(\frac{J_\emptyset^2}{2m} + J_\emptyset v_c\right) = \frac{(\Delta p)^2}{2m} + \Delta p v_c. \tag{15}$$

Let us define the momentum values $p_1 = J_\emptyset$ and $p_2 = mv_c$. The right member of (15) can be written

$$\frac{p_1^2}{2m}\left(1+\frac{2p_2}{p_1}\right) \tag{16}$$

In the momentum basis, switching to the momentum operator, $p = \hat{p}$, we obtain from (16)

$$\begin{aligned} &\frac{p_1^2}{2m}\left(1+\frac{2p_2}{p_1}\right) \Rightarrow \left[\frac{\hat{p}_1^2}{2m}\left(1+\frac{2\hat{p}_2}{\hat{p}_1}\right)\right]\Psi(\mathbf{r},t) = \\ &= \left[-\frac{\hbar^2}{2m}\nabla_1^2\left(1+\frac{2\nabla_2}{\nabla_1}\right)\right]\Psi(\mathbf{r},t) = \\ &= \hat{T}\Psi(\mathbf{r},t) \overset{\hat{V}\Psi(\mathbf{r},t)=0}{\Longrightarrow} = \hat{H}\Psi(\mathbf{r},t) = i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} \end{aligned} \tag{17}$$

where ψ is the wave function for the proton and the potential is zero, $\hat{V}\Psi(\mathbf{r},t) = 0$ (free particle). Eq. (17) therefore represents the quantum equation expressing the work of the vacuum on the accelerated particle and also the corresponding energy spent by the accelerator to oppose vacuum dilatancy. In Eq. (11), kinetic energy is expressed as the sum of two components and this fact will be important in Sect. V, analyzing the acceleration of massive, macroscopic bodies, a case in which the second addend (related to the quantum potential) vanishes and (14) produces correct results unlike SRT. The equations above, which resort to momentum for treating relativistic kinetic energy, also agree with Okun [2], who points out that referring to momentum to describe relativistic mass is more desirable. From (16) the expectation value for the kinetic energy is

$$\begin{aligned} \langle E\rangle &= \frac{\langle p_1^2\rangle}{2m}\left(1+\frac{2\langle p_2\rangle}{\langle p_1\rangle}\right) = \\ &= \int_{-\infty}^{\infty}\Psi^*(\mathbf{r},t)\left[-\frac{\hbar^2}{2m}\nabla_1^2\left(1+\frac{2\nabla_2}{\nabla_1}\right)\right]\Psi(\mathbf{r},t)\,d^3\mathbf{r}. \end{aligned} \tag{18}$$

As we know, the imaginary part of the Schrödinger equation in polar form, putting the amplitude squared equal to the probability density $\Psi^*\Psi = R^2 = \rho$ yields the continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\left(\frac{\rho\nabla S}{m}\right) = 0 \tag{19}$$

which can represent the sum of the observed kinetic energy variation and the energy which is transferred to the dilatant vacuum, due to viscosity as momentum transfer (that implies a ΔE_k), until the particle at issue collides ($\nabla\cdot\left(\frac{\rho\nabla S}{m}\right) = \nabla\cdot\mathbf{j}$, where $\mathbf{j}$ is the probability current, here interpreted as energy flux) and this yields the total, conserved, energy supplied to the proton by the accelerator. Energy conservation occurs via the exchange with the vacuum. The detected increased inertia of the accelerated proton is currently interpreted as relativistic mass or, in the Higgs picture, one can say that the apparent viscosity of the Higgs field, due to acceleration through the field, implies a further mass acquisition process, interpretable as relativistic mass, but actually still in form of increased

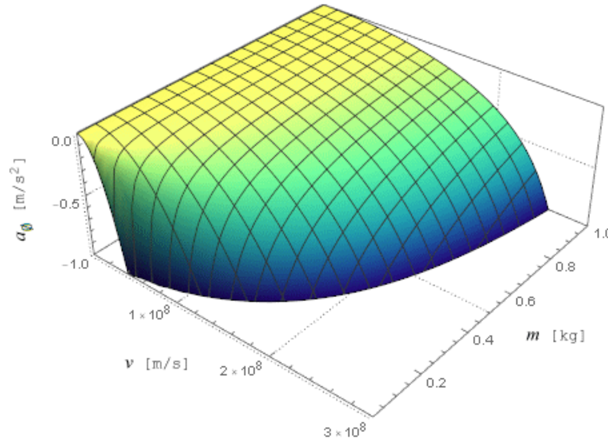

FIG. 1. Plot of $a_\emptyset = F_\emptyset/m$, in which $F_\emptyset$ contains the Lorentz factor in the form $\gamma - 1$ as a term of vacuum dilatancy. The greater the mass the lower the negative acceleration caused by dilatant vacuum.

inertia, due to field dilatancy. In this case, the Higgs field should be treated as a viscous, dilatant fluid and the work of the shear-thickening vacuum on the accelerated proton would be that of the Higgs field (ϕ), i.e. $W_\emptyset = W_\phi$.
The real part of the Schrödinger equation in polar form yields the quantum Hamilton-Jacobi equation

$$\frac{\partial S}{\partial t} = -\frac{|\nabla S|^2}{2m} + Q \tag{20}$$

where, in this case, potential energy is zero. Here S is the action (units are J · s) of dilatant vacuum on the accelerated proton, i.e. the work done on the proton, which settles on v_o. However, also the quantum potential

$$Q = -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} \tag{21}$$

is expressed as kinetic energy and has an analogous meaning. From (15), we deduce it refers to that part of $W_\emptyset$ corresponding to $pv_c = p_1 p_2/m$. In Sect. V, we will see that this quantum potential vanishes for macroscopic bodies, for which the Stokes-Einstein radius is not used.

IV. Lorentz factor 3D: role of mass

In the present framework relativistic mass has vanished, replaced by the viscous force of a dilatant vacuum, and a different role of mass emerges. Indeed, from Newton's second law, we see that the greater the mass, the weaker the acceleration: $a = F/m$. This is valid also for the negative acceleration caused by a shear-thickening vacuum. This fact means that bodies with a greater mass require of course more energy to be accelerated but they are less slowed down by the dilatant vacuum (Fig. 1). The greater the mass the lower the kinetic energy dissipation caused by dilatant vacuum. The fact that a larger mass implies lower kinetic energy dissipation (in this picture let us say *lower deceleration* caused by dilatant vacuum) is after all known in particle acceleration, when comparing electrons with protons: being the latter about 1836

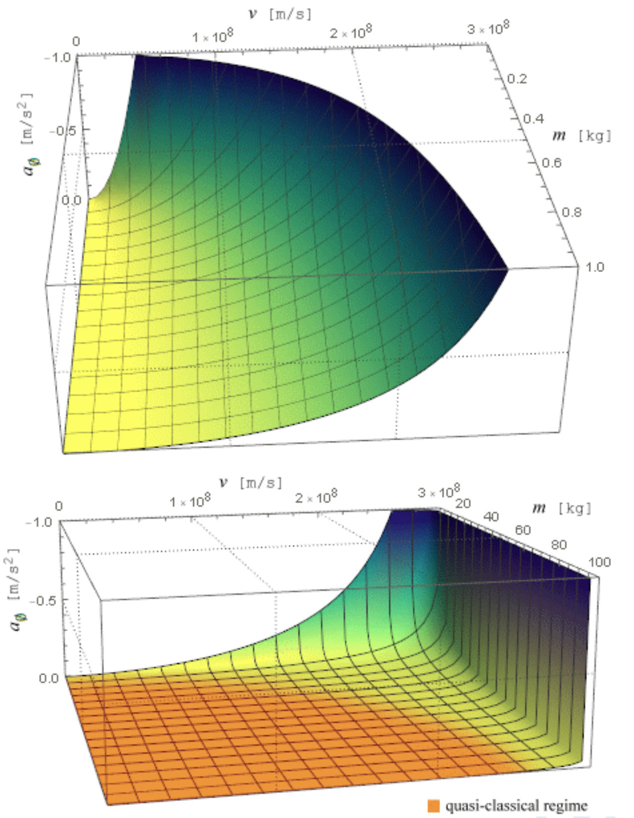

FIG. 2. Two graphs showing the negative acceleration provoked by dilatant vacuum as speed and mass vary. Above, a tiny mass accelerated toward the speed of light undergoes soon a great deceleration, while one kilogram mass is subject to a much lower vacuum-induced negative acceleration. Below, as mass increases, a body can reach quasi-luminal speed without suffering the relativistic effects of special relativity, i.e. without being appreciably decelerated by the physical vacuum. We see that the Lorentz factor-driven curve tends, indeed, to a right angle by accelerating larger masses. Quasi-classical regime extends then depending on mass.

times heavier than electrons, the dissipation of kinetic energy is much smaller. Indeed, the radiated power expressed by the relativistic Larmor formula

$$P = \frac{1}{6\pi\varepsilon_0 c}\frac{e^4}{m^2}\gamma^2\beta^2B^2\sin^2\alpha, \tag{22}$$

with acceleration perpendicular to velocity (where e is the elementary charge, ε_0 vacuum permittivity, β is speed and α the pitch angle), depends on the reciprocal of the squared mass. So, the smaller mass of electrons causes greater kinetic energy dissipation (as synchrotron radiation). As shown in Fig. 2, as a larger rest mass is considered, the Lorentz factor progressively bends at a right angle. Bodies with a larger mass can be therefore accelerated toward the speed of light remaining longer in a quasi-classical regime. That indicates an enormous difference, currently ignored, between the world of subatomic particles and that of macroscopic bodies, as far as the relativistic kinetic energy is concerned. By applying $a_\emptyset = F_\emptyset/m$, we have seen (7) that a proton close to the speed of light is decelerated in the order of magnitude of $-10^{10}\,\mathrm{m\cdot s^{-2}}$, while if the International Space Station, whose mass is 419 tons, were accelerated up to 0.99 c, it would suffer a deceleration of only $0.001\,\mathrm{m\cdot s^{-2}}$. Thus, missions based on ultra-light devices, such as the Breakthrough Starshot [9] are doomed to fail due to the enormous kinetic energy dissipation provoked by dilatant vacuum. If the vacuum in a transient solidlike state were not a rigid body, the collapse of its lattice under a sufficiently large accelerated mass would remove every obstacle to indefinitely exceeding the speed of light. The vertical asymptote at the speed of light is in fact explained here as a transient, shear-driven, quasi-lattice formation in the vacuum, which acts as an impenetrable wall.

V. Acceleration of macroscopic bodies: dilatant vacuum vs. special relativity

Below is shown how the new equation for kinetic energy (14) behaves as regards the acceleration of macroscopic bodies, for which the Stokes-Einstein radius is not taken into account. Let us consider the famous Pioneer anomaly, which was declared solved in 2012 [6]. The suggested thermal simulations which claim to have explained that anomaly are based on uncertain data, as declared by Turyshev himself [7], and produce an imprecise result affected by a large margin of error, i.e. $7.4 \pm 2.5\times 10^{-10}\,m\cdot s^{-2}$ compared to the acceleration detected by NASA, $8.74 \pm 1.33\times 10^{-10}\,m\cdot s^{-2}$ and to the direct and exact solution yielded by (26) and in [3], i.e. $8.74\times 10^{-10}\,m\cdot s^{-2}$. Unlike the thermal simulations, (26) can also explain why the anomaly decreased over time and why it was much weaker for the Pioneer 11, when the probe passed by Saturn: the answer is that the anomaly weakens if speed decreases (1) and the Pioneer 11 in 1979 was much slower than the Pioneer 10 in 1973. It is important to distinguish between the case of accelerated ions in quantum vacuum (23), as the case of the proton above, for which the Stokes-Einstein radius must be used, and the case of macroscopic objects (24), for which the normal geometrical radius is used and the term referring to quantum potential (20) vanishes

$$W_\emptyset^{(q)} = \frac{J_\emptyset^2}{2m} + J_\emptyset v_c \tag{23}$$

$$W_\emptyset^{(M)} = -\frac{J_\emptyset^2}{2m}, \tag{24}$$

where the superscripts (q) and (M) refer to charged particles and macroscopic bodies (respectively). Whereas Einstein formula for the relativistic kinetic energy makes no distinction between charged particles and macroscopic bodies. In the case of the Pioneer 10 spacecraft, the quantum-potential-related term vanishes and the work of the dilatant vacuum (11) on the probe, as subtracted kinetic energy, reduces to (24)

$$W_\emptyset^{(M)} = -\frac{J_\emptyset^2}{2m} = = -\left(-6\pi(1.371\,\mathrm{m})\left(\frac{1}{\sqrt{1-\left(\frac{36737\,\mathrm{m\cdot s^{-1}}}{299792458\,\mathrm{m\cdot s^{-1}}}\right)^2}}-1\right)\cdot 1\,\mathrm{s}\right)^2 \cdot \cdot\frac{1}{2\cdot 222\,\mathrm{kg}} = -8.479\times 10^{-17}\,\mathrm{J} \tag{25}$$

where 36737 m/s is the maximum speed of the probe after the Jupiter flyby [8] and 222 kg is the probe mass (i.e. 258 kg minus 36 kg burned hydrazine). In fact, by applying the classical equation $a = \Delta v/\Delta t$ to the energy value obtained in (25), we see that the acceleration exactly corresponds to that detected by NASA

$$a_P = -\frac{1}{\Delta t}\sqrt{\frac{2\,|W_\emptyset|}{m}} = -\frac{1}{1\,\mathrm{s}}\sqrt{\frac{2\cdot(8.479\times10^{-17}\,\mathrm{J})}{222\,\mathrm{kg}}} = \quad (26)$$
$$= -8.74\times10^{-10}\,\mathrm{m\cdot s^{-2}},$$

(see also [3]) where the minus sign compensates for the necessary modulus under root, to obtain negative acceleration. On the contrary, the formula of special relativity for relativistic kinetic energy, $E_k^{(rel)}$, once the classical kinetic energy ($E_k^{(class)} = mv_o^2/2$, with m rest mass) is subtracted, yields the following energy value

$$E_k^{(rel)} - E_k^{(class)} = mc^2\left(\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1\right) - \frac{1}{2}mv_o^2 =$$
$$= (222\,\mathrm{kg})(299792458\,\mathrm{m\cdot s^{-1}})^2\cdot$$
$$\cdot\left(\frac{1}{\sqrt{1-\left(\frac{36737\,\mathrm{m\cdot s^{-1}}}{299792458\,\mathrm{m\cdot s^{-1}}}\right)^2}} - 1\right) - \quad (27)$$
$$-\frac{1}{2}(222\,\mathrm{kg})(36737\,\mathrm{m\cdot s^{-1}})^2 =$$
$$= 1687\ \mathrm{J}$$

which does not account for the anomalous variation of kinetic energy of the Pioneer 10 and this value would imply an anomalous Δv of -3.89 m/s. NASA should at this point absolutely Doppler track other probes, maybe a simple probe expressly dedicated to verify the modified Stokes equation. Unfortunately, Doppler tracking was not considered for the New Horizons spacecraft, which recently reached Ultima Thule.

VI. Conclusion

This analysis suggests that the relativistic kinetic energy can be explained as the necessary additional energy to oppose the apparent viscosity of a shear-thickening (dilatant) vacuum. The present investigation also suggests the applicability of the Stokes-Einstein radius to charges which are accelerated through quantum vacuum, due to electric interaction with the virtual dipoles of the vacuum: this fact too may represent a further step in the understanding of the quantum aspects of the relativistic kinetic energy. This investigation has indeed led to a quantum formula for the relativistic kinetic energy containing a term which is related to a quantum potential and vanishes when the formula is applied to accelerated macroscopic bodies: for example, it has been shown that the exact solution to the Pioneer anomaly can be obtained. Considering all evidences so far collected about vacuum dilatancy, there are reasons to believe – let us repeat – that relativistic kinetic energy does actually correspond to the greater energy necessary to oppose vacuum dilatancy. Relativistic mass should be then interpreted as the increase in inertia of a body due to the progressive solidification of the vacuum as shear stress increases. Mass is actually unaffected, what varies is only the difficulty to accelerate it (inertia). The excess energy, with respect to classical kinetic energy, is transferred to the dilatant vacuum until particles collide: in that moment the supplied energy is expressed in the collision, according to what observed in particle accelerators. Such a viscosity of the vacuum may be due to the Higgs field itself or to scattered dark matter particles in a sea of (super)fluid dark energy. The new scenario illustrated in Fig. 2 can have consequences also for aerospace engineering: for instance, the acceleration of 1 kg mass to about the speed of light (e.g. 299792457m/s) would actually require 10^{16} J instead of 10^{21} J, predicted by special relativity. The greater the mass, the closer the kinetic energy to that of classical physics.

Acknowledgements

The author thanks Lorenzo Brandi and the anonymous reviewers for the useful suggestions.

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