

Technological Pedagogical Content Knowledge: Secondary School Mathematics
Teachers' Use of Technology

By

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ABSTRACT

Although the Technological Pedagogical Content Knowledge (TPACK) framework has shown a lot of promise as a theoretical perspective, researchers find it difficult to use it in particular environments because the requirements of the framework change in specific contexts. The purpose of this study was to explore and produce more flexible ways of using the TPACK for inservice mathematics secondary teachers. Three such teachers at an urban public school were observed in their classrooms and interviewed about their experiences of teaching mathematics and integrating computer technology in their day-to-day activities. Each participant had over 10 years experience in teaching mathematics in secondary schools in Ontario, and expertise in using computers in mathematics curriculum. The research questions were: 1) How do secondary school mathematics teachers describe their ways of integrating technology? and 2) What difficulties do teachers have when they try to integrate technology into mathematics classrooms?

The findings from the first research question show that teachers displayed a high degree of integration of technology. Their activities were very clearly designed,

conferring clear roles to the use of integrating computer technology in mathematics classes. Teachers had specific approaches to integrate computer technology: a) to allow students opportunities to learn and experiment with their mathematical knowledge; b) to help them pass the content to the students in the process of teaching mathematics; and c) to assess and evaluate students' work, and give them feedback. The findings from the second research question reveal that teachers had difficulties in purchasing and maintaining the computer equipment. They had some difficulties in trying to integrate new technologies as these required time, preparation, and dedication. In addition, teachers had some difficulties in making students use computers in a significant way.

The implication for teacher education is that inservice teachers should have opportunities to update their computer and pedagogical skills, a long term perspective in integrating technology in mathematics education, and professional and technical support from teaching colleagues and administrators. Finally, the integration of computer technology in mathematics requires more intensive teamwork and collaboration between teachers, technical support staff, and administrators.

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CHAPTER ONE: INTRODUCTION

1.1 Overview

The purpose of this research is to better understand mathematics teachers' integration of computer technology in the secondary school mathematics curriculum. I am a secondary school teacher and a researcher in mathematics and educational technology. I try to understand, explore, and propose ways of improving mathematics education through the use of computer technology. In this research, I investigate teachers' use of computer technology based on a theoretical framework called Technological Pedagogical Content Knowledge, or TPACK (Koehler & Mishra, 2009). In this thesis, the notion of technology is understood as the use of graphic calculators, personal computers and associated devices, and software. This chapter delineates the premises for my research inquiry, the rationale of this study, the research questions, my research background, contributions to the field, and the outline of the thesis.

1.2 Research Context

Helping students understand mathematical concepts and being able to apply mathematical models in day-to-day problems is essential to mathematics education. Mathematics teachers are aware that knowing mathematics concepts is not enough to teach mathematics well. Mathematics education requires a specific way of knowing how to teach mathematics and not only general knowledge of 'pure' mathematics. Therefore, an important issue for teachers is acquiring this specific knowledge to teach mathematics. In this sense, Shulman (1986, 1987) introduced the Pedagogical Content Knowledge (PCK) framework as a modality to study knowledge of mastering the teaching of a

specific subject [see the definition in 1.5 and more explanations in the literature review].

More exactly, PCK is defined as the relationship between the teaching subject and

associated pedagogy. For Shulman (1987):

Pedagogical content knowledge identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue. (p. 4)

In mathematics education, Pedagogical Content Knowledge has been seen as an important aid for teachers' professional development (PD) (McNamara, Jaworski, Rowland, Hodgen & Prestage, 2002). In order to acquire and update their skills, teachers must keep pace with increasing educational requirements which necessitates a flexible strategy and a long time commitment. An important factor that can help mathematics teachers keep their promises is the use of technology in classrooms.

Many practical and scientific problems require an interplay among different domains of mathematics and technology. The integration of technology with the study of mathematics has been advocated for decades. For instance, the National Council of Teachers of Mathematics (NCTM) recommended that "mathematics programs must take full advantage of the power of calculators and computers at all levels" (NCTM, 1980, p. 8). More specifically, the International Society for Technology in Education (ISTE) recommends that teachers should start working in their profession by having consistent knowledge and exemplary models of pedagogy that integrate the use of technology with the content of the curriculum (ISTE, 2000). The Ontario Ministry of Education acknowledges the importance of integrating technology in classrooms as being able to offer "a range of opportunities and tools that can significantly extend and enrich teachers'

instructional strategies and support students' learning in mathematics. Teachers can use computer tools and resources both for whole-class instruction and to design programs that meet diverse student needs" (Ontario Ministry of Education, 2007, p. 37).

Research in the United States and Canada shows evidence of technology being implemented widely in classrooms (Cuban, 2001; Guzman & Nussbaum, 2009; Hew & Brush, 2007; Kincaid & Feldner, 2002; Lawless & Pellegrino, 2007; McCormick & Scrimshaw, 2001; Pea, 1997). However, studies have also shown that a great number of teachers remain unprepared to use computers in teaching (Cuban, 2001; Hokanson & Hooper, 2004; Robertson, 2003; Russell, Bebell, O'Dwyer & O'Connor, 2003). For instance, Hokanson and Hooper cautioned about failures that occur when educators mindlessly use computer technology as a panacea for improving education. After almost three decades of efforts to integrate technology in education, learners and teachers continue to struggle with issues of using educational technology in teaching and learning. As Abrami (2001) puts it, "there is much promise but less substance, especially long-term evidence, regarding the effective use of technology for learning" (p. 114). These issues are further complicated by attempts to enhance the pedagogical effectiveness of various disciplines with new technologies.

Mathematics education is also struggling with these issues. Researchers have criticized mathematics communities for deficiencies in implementing software technology in classrooms (Drier, 2001; Garofalo, Drier, Harper, Timmerman & Shockey, 2000; Kaput, Hegedus & Lesh, 2007; Manoucherhri, 1999; Milou, 1999). For instance, Kaput, Hegedus, and Lesh (2007) note that mathematics educators have a passive attitude toward technology and are uncritically accepting different products without being able to

facilitate suitable educational outcomes. As such, an important contribution of integrating technology in classrooms is educating mathematics teachers on how to effectively use technological tools in classrooms. For this reason, it is important to promote research and practices that are able to provide teachers opportunities to adequately reflect on and integrate technology into mathematics classrooms.

1.3 Rationale and Purpose of this Study

Researchers have acknowledged the lack of adequate theoretical and professional frameworks that provide help, guidance, and efficiency to teachers to integrate technology in classrooms (Koehler & Mishra, 2005; Mishra & Koehler, 2006; Niess, 2008; Niess et al. 2009; Valanides & Angeli, 2008). Many different approaches have been attempted in order to help teachers overcome difficulties of integrating technology in mathematics classrooms (Hew & Brush, 2007).

One of the most important ways of providing technological support is to use a framework for integrating complex problems of knowledge from pedagogy, content, technology, and different forms of interactions among these elements in classrooms (Mishra & Koehler, 2008). Adapted from the Pedagogical Content Knowledge model (Shulman, 1986, 1987), the Technological Pedagogical Content Knowledge (TPCK) model [see the definition in 1.5 and more explanations in the literature review] is a framework that treats technological integration in education “as a way of thinking about the knowledge [that] teachers need to understand how to integrate technology effectively in their classrooms” (Mishra & Koehler, 2008, p. 2). TPCK, later renamed as TPACK (Thompson & Mishra, 2007), is comprised from knowledge of content, pedagogy, and

technology, as well as skills to use the interactions among these components (Koehler & Mishra, 2008).

The TPACK theoretical framework has been adopted by different researchers in a multitude of educational areas, and is considered to have shown promising results in integrating technology in teachers' practices. In mathematics education, there have been several studies in using the TPACK framework. Most of the studies were developed in the US and focused on preservice mathematics teachers in secondary schools (e. g. Cavin, 2007; Harrington, 2008; Landry, 2010; Niess, 2006; Suharwoto, 2006). Few studies have been completed for inservice teachers in mathematics education, and at the time of writing this research, no study has been conducted in Canada on this topic.

This study uses the TPACK framework for inservice secondary mathematics education in Canada. More exactly, I will investigate inservice teachers integrating technology in secondary mathematics classrooms in Toronto, Ontario. For this purpose, I designed a qualitative research study for secondary school teachers to understand the process of technological integration in mathematics classrooms. This research aims to study the integration of technology in secondary school mathematics classrooms and explore the use of TPACK in order to improve the teaching of mathematics through educational technology. As such, the goals of this research are:

1. To understand existent pedagogical development ideas and pedagogical models of knowledge in the context of integration of technology into mathematics education by experienced teachers,
2. To document and analyze secondary school mathematics teachers' choices in integrating technology use,

3. To propose improvements to pedagogical practices based on the advancements provided by this model.

1.4 Research Questions

The aim of this study is to explore how integrating computer technology into mathematics teaching affects teachers' development, how TPACK is perceived, and how it can be improved. The main question for this study is “How do mathematics teachers develop their TPACK as they integrate technology throughout mathematics classrooms?”

There are two specific questions emerging from the main question. They are:

1. How do secondary school mathematics teachers describe their ways of integrating technology?
2. What difficulties do teachers have when they try to integrate technology into mathematics classrooms?

1.5 Definitions of Terms

The terms used in this study are defined here:

Educational Technology – “The study and ethical practice of facilitating learning and improving performance by creating, using and managing appropriate technological processes and resources” (Richey, 2008, p. 24).

Integrating Technology –Integration of technology means:

curriculum integration with the use of technology [that] involves the infusion of technology as a tool to enhance the learning in a content area or multidisciplinary settings...Effective integration of technology is achieved when students are able to select technology tools to help them obtain information in a timely manner, analyze and synthesize the information, and present it professionally. The technology should become an integral part of how the classroom functions—as accessible as all other classroom tools. (ISTE, 2002, p. 6)

Pedagogical Content Knowledge (PCK) - Blends the content and the pedagogy into understanding of how these are associated for successful teaching (Shulman, 1986):

The category of pedagogical content knowledge includes the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations, in a word, ways of representing and formulating the subject that make it comprehensible to others...Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult; the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (p. 9)

Technological Pedagogical Content Knowledge (TPCK) - The use of technological tools that help in delivering PCK (Mishra & Koehler, 2006).

TPACK – The new acronym for TPCK (Thompson & Mishra, 2007).

Technology – Technology is defined as “the practical application of knowledge especially in a particular area” (Merriam-Webster Dictionary, 2010). In Dictionary.com, technology is “the branch of knowledge that deals with the creation and use of technical means and their interrelation with life, society, and the environment, drawing upon such subjects as industrial arts, engineering, applied science, and pure science” (para. 1).

In this study, technology is understood as computer based technology and manipulatives. More exactly, the physical devices used in this research are personal computers, video projectors, graphic calculators (Texas Instruments, 1992, 2007), SMART Boards (SMART Technologies, 2009), SMART Clickers (SMART Technologies, 2009) and manipulatives. The mathematics software programs used for this study are Fathom (Key Curriculum Press Technologies, 2009), Geometer's Sketchpad (Key Curriculum Press Technologies, 2009), Graphmatica (kSoft, 2008), GraphCalc (Arrison & Fields, 2003), and Gizmos (Explore Learning, 2010). This research also

considers the use of Microsoft Office software (Microsoft, 2003), such as Excel, Power Point and Word, for teaching and learning in classrooms.

1.6 Background of the Researcher

I was born in Romania and spent the first 35 years of my life there. I have completed an undergraduate program and teaching certification in mathematics and computer technology. I arrived in Canada in 2002 and lived here with my family, taking graduate programs in education, being an instructor in computer technologies, and tutoring my children. It is clear to me that teaching and learning mathematics and educational technology are different in Romania and Canada, and they were taught very differently as well.

I have enjoyed learning mathematics since my early years of school. Both of my parents and several relatives were educators. My father had been a math and science teacher in middle and high schools. He was a successful and passionate professional, teaching for almost 40 years, mostly as a middle school mathematics teacher. Therefore, I started to love mathematics as I was a high achiever, and regularly trained and participated in mathematics Olympics. I started to be aware at an early age of the ways mathematics is taught and how students learn it. After finishing high school, I attended a well-known Romanian university, the University of Bucharest, and pursued a double major in mathematics and computer science.

As a learner, one of the most important aspects I noticed was the lack of integration of mathematics with all of the other disciplines. Mathematics education was not integrated with technology. The educational style was teacher-centered, oriented toward the transmission of information. Another important aspect was that mathematics

was viewed as the most important discipline in the national curriculum. Students who were doing well in mathematics were considered classroom leaders and those from the humanities, athletics and so on did not receive any recognition.

Computer technology and science education were second in importance, but they were usually treated only as applied disciplines of mathematics. Science education was appreciated if the student could use correct formulas, calculate accurately, and obtain correctly the final numerical result. There was not much place left for scientific inquiry and scientific interpretation, since very limited time and importance were allocated for them. Computer technology was taught in a similar way. Mainly, the high school curricula consisted of learning some mathematical theories such as Boolean logics, mathematical equations, and some very basic algorithms. Nothing was taught about integrating technology in other disciplines such as mathematics and science. The computers were outdated and scarce. In high school, I saw only pictures of computers, and I was never able to see how a PC works as an undergraduate student in the computer science department. Doing science and computer technologies as ‘applied mathematics’ meant there were few benefits for mathematics education. As a result, failing to adequately integrate mathematics, technology, and science was an unfortunate situation with major consequences for the national system of education in Romania.

On the positive side, this intensive style of thinking and doing mathematics made many Romanian immigrants able to adapt easily to mathematics teaching and to study in graduate programs when they relocated to the US, Canada or Western European countries. I found myself privileged, with a high level of confidence in mathematics, and

my knowledge in mathematics was well appreciated by Canadian colleagues, peers, and teachers.

My native country had a communist regime at the time I was student. After the fall of the regime, Romania started to be more open to social, political and educational exchanges. Reforms from Western democracy started to be implemented courageously. Romanian education started to take a child-centered perspective, eliminating communist dogma from its curricula. As a result, educational technology started to be implemented in the early grades. In classrooms, mathematics, technology and science started to get integrated.

In 1991, after finishing my undergraduate study right in the midst of these social changes, I taught for over seven years in computer technologies and mathematics in high schools. I had my students in national Olympics, I wrote three textbooks on computer technologies, and I had the honour to be selected for the Romanian National Committee of Teachers to elaborate the national curriculum. I participated as a juror in different educational contests and scholarly discussions on national committees. I also taught part-time as an instructor for universities, a teacher for elementary and middle schools, an expert teacher for inservice teachers, an associate teacher for preservice teachers, and I worked for several companies as a trainer in computer technologies.

While I was a Romanian teacher, I believed it was crucial to use computer technology as an interdisciplinary facilitator. I saw computer technologies as an important agent in teaching and learning, as a way to address learners and communities' needs, and not as 'math applied'. For me, computer technology could help mathematics education and vice versa. As a teacher, I was interested to use interdisciplinary

approaches, trying to integrate technology, mathematics and science in different ways. My textbooks on computer technologies were very well received for promoting these points of view.

Another personal experience in my professional development in educational technology was when I worked for several years in software design and programming. An important part of my work specifically involved designing educational software. I tried to think as a software designer, to see how technology can be used for cognitive and social purposes, as a tool to promote communication, collaboration and learning. Being a former teacher, I had many important insights in designing educational software. I realized how difficult it is to adequately design and produce software for students' use, when they are at a specific age, and how challenging it is to empower teachers and students from different contexts and communities.

An important stage in my life was when I decided to immigrate to Canada with my family. In my graduate programs in education, I took courses in curriculum, educational technology, research methods, science and mathematics education, constructivism, and social theories of education. I wrote some papers for journals and conferences in educational technology, mathematics education and science education, and I enjoyed being a researcher and scholar in education. In Canada, I became more aware of the child-oriented approach in Ontario, constructivism, and integrating technologies in education. I was very enthusiastic when I noticed many mathematics and science teachers trying to integrate computer technologies in classrooms.

I discovered that many mathematics educators, computer programmers, and graduate students struggle in developing rigorous inquiries in mathematics classrooms.

Integrating technology in mathematics classrooms is a difficult process in which even expert teachers in mathematics encounter serious challenges. Therefore, teaching ‘the right thing in the right way’ for students and teachers still remains an open and passionate pursuit for me. Selecting an adequate teaching framework such as TPACK for the purpose of integrating technology in mathematics classrooms is an important topic not only for me, but for many teachers and researchers in mathematics education. This is why I selected the TPACK for inservice teachers in mathematics as the topic for my doctoral dissertation. In many ways, I feel that this doctoral thesis is a logical continuation of my previous work as a teacher.

1.7 Significance of the Research

This research provides an important way to describe relationships between mathematics education and educational technology. The TPACK model describes experiences, challenges and solutions that might be adapted by any teacher involved in integrating technology in mathematics education. This study can prove advantageous in several ways. First, it provides participants and readers with multiple professional development benefits and will add to the body of research on mathematics teachers. In addition, a pedagogical model designed for supporting technology and integrating it with teaching and learning practices and mathematics knowledge is provided. This can lead to valuable insights into improving mathematical education by using educational technology.

1.8 Limitations of the Study

This study has several limitations. Developing research for the TPACK framework based on only three case studies of inservice teachers in mathematics limits

the generality of the findings. Also, this pedagogical study was designed for mathematics teachers already involved in integrating technology in teaching mathematics. Therefore, these findings and recommendations might not be valid for preservice teachers or inservice teachers who are inexperienced in using technology. Also, TPACK findings are limited to the specific topics of curriculum and technological products used in this study. However, some findings can be extended for other teachers involved in teaching mathematics through integrating technology.

1.9 Structure of the Thesis

The first chapter is an overview of the research. It provides the context of the study and presents the research topic by introducing the importance of implementing educational technology in mathematics classrooms. It also provides the research questions, the roadmap of the dissertation, the researcher background, the contributions to the research, and some limitations that the current research might have.

The second chapter provides a literature review. This section presents the main steps in integrating technology in mathematics education and some issues that this integration poses, problems in professional development in teaching mathematics, and the presentation of the pedagogical models that integrate curriculum, mathematics content, and technology.

The third chapter presents the methodology used in this research. This chapter contains a rationale for using qualitative research, the process of selecting participants, the process of collecting data, the analysis of data, procedures ensuring the trustworthiness and the validity of the study, as well as ethical considerations.

The fourth chapter presents the main findings of the current research. First, the environment of the secondary school is presented, together with some academic, social, and cultural aspects. Next, I will detail each of the three case studies of secondary school mathematics teachers in integrating technology.

The fifth chapter summarizes the findings of this research. This chapter presents the cross case analysis, summarises the research questions, discusses the major findings of the current study, and suggests future paths of development for researchers, administrators, mathematics teachers, and technological personnel who intend to integrate technology in secondary mathematics classrooms.

CHAPTER TWO: LITERATURE REVIEW

2.1 Introduction

In this chapter, I will review previous research about integrating educational technology in mathematics classrooms. In the second section, I will describe some pedagogical difficulties in teaching mathematics. In the next section, I will present general aspects of using computer technology in classrooms. Specific contributions in integrating technology into mathematics education are reviewed in the fourth section. In the fifth section, I will discuss some challenges and results in professional development for mathematics teachers. In the next section, I will review some main important pedagogical models related to TPACK in mathematics and will present the rationale for selecting the TPACK model. In the last section, I will draw an overall view about implementing educational technology in mathematics by using TPACK and will articulate some directions and contributions based on current research.

2.2 Mathematics Education in Secondary School

In this section, I will describe some general challenges in teaching secondary mathematics curriculum and some particular difficulties in teaching specific areas of secondary mathematics curriculum such as statistics, geometry, and functions. The importance of mathematics education in formal education has always been emphasized by offering it a major role in the overall curriculum. From kindergarten to the college level, mathematical instruction retains an important role. Yet, researchers in mathematics education have reported numerous cases where mathematics is taught using inadequate pedagogical skills as teachers are often relying on intensive drill activities and rote

memorisation of formulae (Garofalo, Drier, Harper, Timmerman & Shockey, 2000; Lockart, 2009). As Lockhard (2009) puts it, it is important for teachers to gain more pedagogical insights in order to give students opportunities “to pose their own problems, to make their own conjectures and discoveries, to be wrong, to be creatively frustrated, to have an inspiration, and to cobble together their own explanations and proofs” (p. 29).

According to the National Research Council (NRC) (2001), teachers are recommended to develop an “elaborated, integrated knowledge of mathematics, a knowledge of how students’ mathematical understanding develops, and a repertoire of pedagogical practices that take into account the mathematics being taught and how students learn it” (p. 381). In other words, NRC expects teachers to be not only proficient in knowing the curriculum, but also be capable to foster students’ mathematical learning and be able to manage classrooms where students can experiment with their mathematical ideas.

Taking into account the benefits of peer discourse and collaboration that these pedagogical approaches are capable of bringing to mathematics teaching practices, Powell, Lai and O’Hara (2009) recommend giving up on tendencies to explain theoretical subtleties of a problem, or giving students direct instructions to follow as a narrow path in order to complete a predetermined solution. Instead, they suggest that teachers should help students a) communicate mathematical ideas by explaining to each other their understanding of the problem; b) discuss their interpretation of the problem; c) exchange their personal ways of visualizing the problem; and d) describe patterns that they found. In other words, for authors “mathematics is the art of explanation” (p. V) and not a mechanical approach.

Another important approach in teaching mathematics is to consider connecting multiple dimensions of representing a mathematical concept. This is emphasized by the research of Sfard (2008) and Healy and Hoyles (1999). For instance, in order to introduce the concept of function in the secondary curriculum, mathematics teachers are recommended to familiarize their students by representing the same phenomenon in multiple ways. For instance, students can work with the same function developing different ways of representation such as formulae, graphics, narratives, or tabular forms. Therefore, for teachers, it is important to introduce students to a mathematical concept and understand it through various forms, to demonstrate how these representations are interrelated, and how they are being used efficiently.

Fosnot and Dolk (2001) see the way students learn mathematics as a complicated labyrinth. Therefore, it is important that teachers be able to give students advice in this complicated *landscape of learning*. Teachers should be aware of the mathematical concepts required and be prepared to understand the *landmark strategies* that students may use in order to reach an understanding of a mathematical concept. Once teachers know where students are in their mathematical thinking, they must acknowledge these strategies and offer students opportunities to explore the connections between their strategy and the required problems. In this way, a skilful teacher understands not only the mathematics but also the ways in which students learn mathematics by helping them explore these complicated labyrinths.

Schoenfeld, Smith and Arcavi (1993) consider that the creation of most graphs requires a significant amount of time as they require many points to be calculated, plotted, and sketched. Another issue noticed in teaching advanced functions in secondary

mathematics curriculum is that students and teachers did not focus on critical aspects of understanding. For instance, Potari, Zachariades, Christou, Kyriazis, and Pitta-Pantazi (2006) found that teachers participating in their study did not explain the major steps. In addition, they showed that during their teaching, the teachers did not stress the significant aspects of functions such as tangent and critical points of the graph and, therefore, students tended to overlook these important stages of understanding.

The most advanced course in statistics in secondary schools in Ontario, the Data Management course designed for Grade 12 Academic, has as its main goals solving “problems involving one-variable data by collecting, organizing, analysing, and evaluating data and [to] determine and represent probability, and identify and interpret its applications” (Ontario Ministry of Education, 2006, p. 76). Cobb and Moore (1997) recommended some basic guidelines for statistics educators such as: a) emphasizing statistical thinking; b) more importance given to data and concepts and less to theoretical aspects; and c) a priority in fostering active learning.

In the US, some general advice for teaching statistics was developed in Guidelines for Assessment and Instruction in Statistics Education (GAISE) (Franklin & Garfield, 2006). These recommendations are:

1. Emphasize statistical literacy and develop statistical thinking;
2. Use real data;
3. Stress conceptual understanding rather than mere knowledge of procedures;
4. Foster active learning in the classroom;
5. Use technology for developing conceptual understanding and analyzing data;
6. Use assessments to improve and evaluate student learning.

Difficulties in teaching statistics were recently noticed by Moore (2005) and Roseth, Garfield and Ben-Zvi (2008). These difficulties pose serious challenges as teaching statistics typically requires students to learn concepts and mathematical ideas, to use computer software, and to make them collaborate. Garfield and Everson (2009) note that “for statistics teachers, this is knowledge about ways to effectively teach important statistical ideas and skills, ways to help students use statistical software and technology tools, and ways to help prevent or overcome typical misunderstandings and misconceptions about statistical concepts” (para. 5).

Another difficult areas to teach in secondary mathematics education is geometry (Knuth, 2002). Geometry was found by students and teachers from secondary mathematics education as the most challenging curricular area (Newman & McCoy, 2009). Explanations are multiple, such as students dislike making an adequate effort and regularly find it without practical value, and extremely challenging.

Healy and Hoyles (1999) showed that students tend to develop two different types of tendencies of developing geometrical proofs. The first type is to adopt the arguments for themselves and accept them only if they are convincing and discover that logically these make sense. The second tendency is to develop technical arguments (even when they are not sustainable) and quickly accomplish the mathematical problem in order to get good grades and other social benefits. Therefore, for teachers, balancing these two aspects for their students is a difficult pedagogical task.

Because of different knowledge levels, students cannot be easily grouped to work and do not acquire informal deduction at the same pace, which is very problematic.

Therefore, it is important for teachers to negotiate a pace of instruction and a way of collaborating with their students appropriate to each particular context.

2.3. Integrating Technology in Classrooms

In this section, I review the requirements and effects of integrating technology in classrooms. First, I discuss the technological expertise required in classrooms. I then show curricular aspects of adapting the curriculum to the new technologies. Next, social issues relating to the introduction of the curriculum are presented. Finally, some aspects related to integrating technology in order to facilitate interactions and innovations in classrooms are exposed.

2.3.1 Technical Expertise Aspects in Integrating Technology in the Curriculum

Educational technology has been praised for offering support for performing differentiated instruction, collaboration, multiple intelligences, and constructivist teaching and learning (Kelly & Tangney, 2006; Stoilescu, 2005). For instance, Bereiter and Scardamalia (1996) showed evidence that using two different media concurrently (text and visual diagrams) is an effective aid for learning. Overall, educational technology might have positive benefits in classrooms if educators do not view it as a panacea and are knowledgeably adapting the technology to concrete settings (Kimmel & Deek, 1995; Ringstaff & Kelley, 2002).

Lewis (1999) states that the importance of educational technology is making connections between different components of the school curriculum. Therefore the relationship between technology and curriculum is one that should produce deep integration:

If schooling is to have desired meaning for children, then the various elements of the curriculum must cohere. Lessons learned in one subject must be amplified in others. To take its place squarely in school curricula, technology education must establish itself not just in its own right, but crucially in relation to other subjects. Thus, the relationship of technology to other subjects in the curriculum is a fruitful area of inquiry. The field has to understand integration better. (p. 49)

Technology is changing at a very fast pace. Although researchers pointed out that computers might be used in education to improve learning (Roschelle, Pea, Hoadley, Gordin & Means, 2000), this is not easy to accomplish in practice. One of the reasons is that keeping up with the latest technology is very difficult for educators. For instance, some have cautioned about the risk of computer technology becoming obsolete in short time (Anderson, 1992; Kaput, 1992; McCrory, 2006). This fact means that the computers and the already installed software in classrooms become very rapidly outdated. In addition, because of the high cost of periodically purchasing software, there is constant pressure for educators to produce better student learning outcomes.

Another difficulty that has been frequently reported is teachers' lack of computer skills. According to Feistritzer (1999), teachers have missed opportunities to integrate technology while they were learning in K-12 education or in higher education. Even after finishing their preservice teaching program or taking other inservice teaching coursework, teachers do not acquire enough computer skills to make them confident to use computer technology in classrooms. Nicassio and Overstreet (1993) mentioned some of the difficulties that teachers encounter: a) challenges in access and utilization of existing technologies to their entire potential, b) lack of technical personnel, c) lack of opportunities for professional preparation, and d) inability to counteract unanticipated negative consequences of the current technology.

Another concern is that the software is not designed taking into account discipline-specific requirements. Research from the 1990s tried to find general rules about technology, ignoring specific disciplines and contexts. Kenny (2002) argued that technology still does not focus enough on a specific subject. Therefore, many technologies and programs remain ineffective and the technology is not used efficiently with pedagogical practices.

A possible cause is that teachers do not encourage a useful transfer of information among students. Researchers focusing on constructivism, situated learning, and place-based education have shown that the school system in place is not interested in developing students' skills for communication, practical life, and community problems (Brown & Duguid, 1995; Gruenewald, 2008). For instance, Brown and Duguid (1995) believe that technology does not necessarily guarantee better student learning outcomes. They criticized schools' tendency to hold isolationist views and cautioned that any reform to promote technology is futile:

The more educational technology is constrained to “essentials” and “individuals” the more it resembles a nugatory ‘delivery system,’ ... A preferable goal, it seems to us, is to design technology that provides an underconstrained ‘window’ onto practice, allowing students to look through it onto as much actual practice as it can reveal, to see to increasingly greater depths, and to collaborate in exploration. (p. 15)

2.3.2 Pedagogical Aspects of Using Educational Technology

For many researchers, the attitudes and beliefs of the teachers influence their approaches to integrating technology in classrooms. For instance, Ertmer, Addison, Lane, Ross, and Woods (1999) state that technological beliefs of teachers shape their reasons for using technology in classrooms. More specifically, when teachers view technology with low interest, the integration of technology became less important than the content

knowledge and, as a consequence, is viewed as peripheral. In this sense, Ertmer et al. found that teachers who use technology just to keep their students busy do not see the relevance of technology.

Teachers' use of technology is mediated by their beliefs about learners, about what counts as good teaching, and about the role of technology in learning. Negropontes, Resnick and Cassel (1997) reported that poor pedagogical techniques are often employed when using educational technology. They noticed that digital technologies are used to present educational perspectives rather simplistically. Instead, the authors recommended that educational environments using computers should offer: a) direct explorations, b) direct expressions, c) direct experience, d) multimodal exploring, e) multicultural settings, and f) multilingual technologies. Each of these factors is important in addressing multiple learning perspectives and social equity, which are integral parts to assuring an equitable classroom environment.

In order to efficiently integrate technology in classrooms, Earle (2002) recommends adjusting instructional practices to the new content delivered by computer technologies. He argues that the practice of integrating technology is not about focusing on mastering specific software products. Rather, it is about reviewing the content of the curriculum and pedagogical practices involved in the process of curricular design:

Integrating technology is not about technology – it is primarily about content and effective instructional practices. Technology involves the tools with which we deliver content and implement practices in better ways. Its focus must be on curriculum and learning. Integration is defined not by the amount or type of technology used, but by how and why it is used. (p. 8)

Watson (1993) asserted that teachers' beliefs changed very little in their teaching and computers were simply used as additional tools supporting pre-existing pedagogical

content and teaching styles. Overall, the belief in the universality of the benefits of technology in classrooms has not gone without major criticism. The tendency to exaggerate these benefits of technology in American cultural life was harshly criticized by Postman (1995), who called it technological adoration. Among other things, he contested the present capability of technological products to be widely accepted by children of different ages and backgrounds. For instance, most of the educational software products used in classrooms was designed for adults or high achiever students. Cases were reported when top officials did not have realistic expectations, being not aware of children's specific ways of acting and reasoning. Therefore, Postman contended, software designers, classroom teachers, and educational administrators failed to cooperate in order to produce adequate software programs.

Integrating educational technology with mathematics has a proven track record of enhancing teachers' ability to cover multiple subjects with increasing precision. In particular, this is demonstrated in new pedagogical approaches in laboratory work and inquiry (Hofstein & Lunetta, 2004) which include more accessible classroom environments and the possibility to use mathematics and technology in science classrooms. No longer is the lab geared toward specialized students or those destined for a scientific career in academia. Integrating technology in science and mathematics classrooms has made it possible to conduct classroom experiments in ways more relevant to the daily experiences of students of different backgrounds.

Pea (1997) notes that introducing computer technology requires teachers to change their teaching methods in classrooms. Yet, Cuban (2001) noticed that only 60 percent of teachers acknowledged this need for changing their pedagogical strategies,

with less than 20 percent actually trying to change them. Tyack and Cuban (1995) cautioned the adopters of technology as a fast fix about the difficulties required for this reform to be successful. Ultimately, they cautioned that this is not a top-down reform. Instead, teachers should individually decide if they need the technology or not and, if so, to start implementing it according to their individual settings.

In order to describe difficulties that teachers encounter in implementing technologies in classrooms, Ertmer (1999) introduced the notion of first-order barrier and second-order barrier as a way to locate if difficulties are generated from teachers or from external factors. According to this distinction, first-order barriers are difficulties external to teachers (e. g. lack of computers and software) while second-order barriers are difficulties internal to teachers (e. g. teacher is not familiar with a specific software). These levels of difficulty are useful for a systematic analysis of the challenges encountered by teachers and learners in classrooms.

2.3.3 Social Aspects of the Integrating Technology in Classrooms

There are several social issues for teachers promoting technology. In the process of integrating technology in classrooms, a teacher should consistently consider equity issues. Some researchers have shown that technology might be used in classrooms in order to provide social equity (Evoh, 2007; Pea, 2004; Tapscott, 2008). Although new technologies were noticed being implemented at a fast pace (e. g. virtual reality, video, mobile technology, CAD/CAM) and with possible positive potential of encouraging social equity, the way these technologies are taught creates concerns. New technologies have not created the sharing of power as some utopians predicted (Toffler, 1990). On the

contrary, these technologies have potentially facilitated power relationships that are more fortified and autocratic (Kincheloe, 1999).

Technology has been accused of blurring the line between consumer and citizen as parts of consumption. In this way, while technologies might offer many new possibilities, these opportunities might distort facts or amplify inequalities. For instance, using educational technology in teaching science, Layton (1988) noticed that technology has often been viewed as minimizing the accomplishments of scientific thinking by superficially and mechanically modelling scientific reasoning. Also, sometimes the prices for many software and hardware products are prohibitive for many schools and universities (Moyle, 2003).

Socio-economic status (SES) was found to be an important factor in shaping the ways computer technology is used. Becker (2000) and Warschauer, Knobel, and Stone (2004) assert that high-SES students are more likely to use computers because they tend to have a better technical and human network of support. In contrast, ESL students, usually placed in schools with low-SES, are less likely to use computers at home than their native English speaking classmates. Warschauer et al. found that high-SES students benefit more academically from having home computer technologies than do low-SES students. These findings were supported by studies developed by Attewell and Battle (1999) and Cuban (2001). As for students with low-SES, they are more likely to be directed into skill drills training than to using technology for fostering complex thinking (Warschauer, 2002).

Another issue is that using technology might have a gender bias. Technology has often been seen as more prominently used by males. In the past, females have

traditionally had a smaller amount of time and opportunity to access computers in classrooms. For instance, when technology was insufficient, male students tried to occupy the computational resources to the detriment of female students (Jenson, de Castell & Bryson, 2003). Also many software products were found to be biased against women and minorities (Chuck, 2002; Friedman & Nissenbaum, 1996). Even when these resources were accessible, male students had opportunities to receive better ideas and opportunities from their peer networks in order to perform in classrooms or in informal settings (Stoilescu & McDougall, 2010).

Another factor that has an important social role is the constant pressure that educational technologist educators receive. For instance, Artigue (2000) asserts that, because technology involves a great amount of money, educators are pressured to be accountable to help students achieve a high level of performance in a short time after purchasing a new technology. This creates a huge and often unfair pressure, and can result in failures, if not a vicious circle.

2.3.4 Integrating Technology to Support Interactions and Innovations

Using technology in classrooms has had an important educational impact and therefore has been studied in different theories of interactions and innovations. Some researchers are skeptical in the possibilities that the technology offers to improve qualitative processes of learning. For instance, Becker and Ravitz (1999) reported that the use of computers in mathematics education often takes place more as supplementary tools and does not offer opportunities for creative approaches for students and teachers. They recommended transforming the use of technology in order to foster more creative

approaches to curriculum, mentioning that the role of technology should shift from tool to “mindtool” (p. 358).

One of the most important aspects of education is to facilitate individually tailored learning to students. For instance, researchers have proposed using technology by designing delivered content for the specific personal use of students (Glenn, 1997; Pea, 1997). In this vein, Hodgson (1996) and Wilson (2005) demonstrate that educational technology has produced a shift in the delivery of information and the exchange of communication, and, therefore, the cognitive role and the importance of teaching have been drastically challenged. More exactly, if teachers are not delivering the information because the computers are, teachers might have more time to provide feedback to students. Teachers can now focus more easily on high order skills such as simulation, proof, real applications, and critical thinking aspects. They might leave the uncreative parts to be performed by computers.

Dede (2002) considers that educators are similar to engineers in that technology now offers multiple pathways for solving different classes of problems. In this way, it is the responsibility of teachers to transform the process of solving problems into efficient ways of exploring a software product and reflecting on the specific implications. Emerging educational technologies create an opportunity to change from the transfer of information to creation and mastery of knowledge (ibid.).

Another important aspect of educational technology is the role played by interactions (Dede, 1999). Technology reshapes the role played by social interactions so that technology is essential for the cognitive development of learners and for the design and the development of the curriculum’s content. Through an adequate use of technology,

students might be able to receive support from a teacher and other knowledgeable learners. Therefore, adequate interactions between technology, teachers, and students might foster more opportunities to achieve an adequate culture of schooling and learning opportunities (Dede, 1999).

2.4 Integrating Technology in Mathematics Education

In this section, I will present aspects of integrating technology in mathematics classrooms. I will present some general benefits that educational technology might bring to mathematics classrooms and I will describe ways in which curricular and social aspects in mathematics education are affected by educational technology. In the last section, cognitive aspects of using technology in mathematics are considered.

2.4.1 General Challenges in Integrating Technology in Mathematics Classrooms

Integrating technology in mathematics education has been praised by many researchers and practitioners for providing a great number of advantages. For instance, integrating technology in mathematics classrooms can help learners to master concepts such as probability, risk, and certainty and allows them to make complex decisions in a modern democracy with various social or personal implications (McFarlane & Sakellariou, 2002). Kaput, Noss and Hoyles (2008) state that technology attracts mathematicians and mathematics educators because it offers an empowering representational infrastructure for simulation, visualisation, and modelling for both adults and children. Kaput et al. mention that technology might be used to improve mathematics education by “enlarging the limited processing power of human minds affording new

domains of knowledge with new representations to populations who previously did not have access” (p. 713).

Beyond the immediate advantages of using the software in manipulating quantitative data and software for the office (word processors, PowerPoint presentations, spreadsheets), researchers are looking at new ways to integrate these benefits in order to generate scalable inventions in society and improve the quality of educational achievements (Dede, 1999). More specifically, the National Council of Teachers of Mathematics (NCTM) has been promoting since the beginning of the 1980s the process of integrating technology in mathematics classrooms. McDougall (1997) recommends developing a ground for careful understanding that encourages teachers to learn how to use new technologies and empower them in order to create a safe environment for experimenting. The NCTM (2000) recommends that teachers use appropriate computer technology in mathematics classrooms in order to improve the quality of learning. “Students can learn more mathematics, more deeply with the appropriate and responsible use of technology” (NCTM, 2000, p. 25).

Technology provides students with the opportunity to simulate different complex scenarios, processes and phenomenon, to generate visualisations and explorations, and to connect dynamic notations, linked representations, and operations with symbols (Kaput & Shaffer, 1999). Therefore, integrating technology in mathematics classrooms should not be simplistically perceived as using computers to avoid laborious calculations; rather it is a tool for mind, representation and modelling. With adequate technology, diSessa (2000) argues that mathematics teachers can help their students to develop their metarepresentational abilities.

As in any domain, the improper integration of technology in mathematics classrooms has a long history. Much research from the 1990s was focused on presenting cases of inadvertent use of technology in mathematics education. For instance, Wenglinsky (1998) reported on cases of inappropriate uses of technology in mathematics classrooms, emphasizing that only by using adequate pedagogies can teachers help students to perform better in mathematics through computer technology.

The mathematics curriculum, topics, and problems are changing with the integration of new technologies. Developed around mathematical topics, technological tools designed from pedagogical perspectives are capable of offering effectiveness in supporting students' efforts to understand mathematics (Chazan, 1999). Hodgson (1996) mentions the necessity that mathematics teachers discriminate skills of minor importance from the essential skills. The role of teachers is changing with the advent of introducing technology in classrooms (Goos, Galbraith, Renshaw & Geiger, 2000). In this respect, computers might help to focus attention on those skills that are considered important. Therefore it is vital that adequate educational software specific to mathematics teaching and learning be put into place. In this sense, researchers such as Dreyfus (1993) and Yerushalmy (1999) have offered specific principles for designing mathematics educational software for children.

McDougall (1997) emphasizes the importance of having a mentor or a coach available to provide technical and mathematical expertise. It is important that teachers are aware of their preferences such as classroom settings, ways of managing the classroom, collaboration styles, and levels of tolerance. Teachers should have enough time to familiarize themselves with the teaching process and become able to “focus on the shift

in control” (p. 18). In addition, mathematics teachers should receive support from colleagues, principals and district officials.

Educational technology is not reduced only to an individual act. This is especially the case of using it in learning mathematics. In order to be successful, the integration of technology in mathematics should be correlated with other disciplines such as science education or social sciences.

Investigating the process of development of ideas for students learning mathematics, Carraher and Schliemann (1998) noticed that, although pedagogical interventions are very important, some restrictions are necessary. For the purpose of learning, in order to offer better opportunities for tuning into inquiries about different mathematical concepts and problems, these mathematical domains should offer some restriction that open, programmable systems do not have (Hoyles & Noss, 2003).

2.4.2 Curricular Aspects in Implementing Technologies in Mathematics

Some researchers feel that, because of integrating technology in mathematics classrooms, teachers are required to redesign the curriculum. Kaput (1992) recommends that, in order to offer benefits, the use of computer technology requires teachers to rethink and redesign the curriculum and also the context in which learning mathematics takes place.

In implementing computer technology in mathematics curricula there are numerous positive examples. For instance, Noss and Hoyles (2006) state that learners using Logo might obtain major benefits by having good qualitative assistance for the purpose of enhancing their insights in mathematical conceptions and practices. Another

example is given by Olive (2000), who designed a computer software tool in order to provide children help in understanding and manipulating fractions.

Therefore, implementing technology in mathematics classrooms is becoming more important. In particular, the Ontario Ministry of Education has made some recommendations for integrating technology in mathematics classrooms (2005, 2007). Some of the ways in which mathematics teachers should use technology are:

- Communicate and exchange opinions in classrooms, from home and with other classrooms and schools,
- Locate, disseminate, and access different Internet resources,
- Use databases, spreadsheets, word processing, presentations, multimedia documents,
- Manipulate large quantitative data, reduce the time for routine mathematical tasks allowing students more time to think for conceptualisations and designing solutions,
- Use graphical software, computer algebra systems (CAS), statistical software, and
- Practice simulations and computer assisted learning modules for supporting mathematical inquiry.

Artigue (2000) notes that the process of integrating computer technologies in mathematics is marginal and gives three main reasons. First, introducing computers in mathematics increases the level of complexity. As such, the challenges to be proficient in an enriched technological mathematics classroom are greater. Second, there is an opposition between the technical and the conceptual activity in mathematics. Computers'

outcomes are considered easy as automatically solving some tasks and as not having properties of enhancing conceptual and theoretical problems. Third, students' needs are not adequately recognized as teachers shift from the traditional mathematics curriculum to the new synthesis of mathematics curriculum that relies intensely on technological integration.

In mathematics classrooms, researchers found that access to computers and software is difficult and problematic (Galbraith, Goos, Renshaw & Geiger, 2001). They consider expensive to design classroom-based research able to explore and develop technological integration. Because the technology is advancing fast, the associate research with the technology becomes easily outdated so the price to replace old technology became an important issue (Andersen, 1992; Kaput, 1992; McCrory, 2006).

Kaput and Thompson (1994) notice three important advantages of using technology in mathematics. First, dynamic media (video, audio, multimedia) offers better learning and knowledge opportunities than static media by giving learners opportunities for dynamic interactions and inquiries with different components of the product. Second, new representational infrastructure offers opportunities to reintegrate previously achieved knowledge. Third, new systems of knowledge might be designed by employing infrastructures based on technology. Unfortunately, introducing educational technology in mathematics education is a difficult process. Kaput and Thompson (1994) also mention inadequacies where the same software is used for different purposes and for audiences:

Calculators built for shopkeepers, shoppers, or engineers often entered an unchanged curriculum; uses of spreadsheets often led to a curriculum that fit the characteristics of a tool built initially for accountants. Only recently have calculator manufacturers sought the advice of educators in the design of their products. And only very recently have computer-based tools been specifically designed for learners. Often, however, such tools are designed by people steeped

in the technology but without deep insight into the problems of mathematics education. (p. 681)

Calvert, Zack and Mura (2001) notice that learners often take visual snapshots or simplified solutions presented by computers as proofs. In this way, they are not able to find an adequate demonstration of the mathematical problems. Hoyles and Noss (2003) confirm that students tend to use technology in order to avoid the cognitive load that learning mathematics requires. They raise an important distinction between mathematics learners and mathematics users. Generally, mathematics learners try to understand, structure the results and eventually look to generalize them, while mathematics users want 'just to get the job done'.

Sometimes, the creation of software creates challenges in establishing an adequate agency representing learners. For instance, some mathematics software might prevent students from establishing proper connections and proofs (Van Herwaarden & Gielen, 2002). Likewise, Hoyles and Noss (2003) report cases where students believed that, by having computers in classrooms, there was no need to think mathematically. Another challenge of software agency was reported where students' use of the software becomes rigid and students cannot have the support of the initial authors to modify the product (Magala, 2006). Not being flexible enough for deep inquiries and complex interactions with learners, in this case, the software becomes a barrier between learners and software authors.

Another problem with software agency is the great number of software products based on drill and practice principles to the detriment of software focused on higher-order thinking skills. Although the drill and practice products might be attractive and easy to

use, they do not enhance students' abilities to reflect and find adequate answers. The software products are based mainly on a mechanical way of generating standard answers. For instance, calculators do mathematics for students but they do not teach students to think mathematics (Ronau et al., 2008).

Hoyles (1989) suggests that technology requires a change in the learning objectives. This does not mean mechanically exchanging a set of problems with another set, but it suggests changing the philosophy of learning to teach mathematics. For instance, collaborative software is difficult to be used in teacher-centered classrooms, where students do not receive opportunities to make their decisions in order to explore and simulate themselves technologically. In this case, technology cannot be used creatively or properly.

Sometimes innovations are poorly implemented by being adopted uncritically and without paying attention to the specific local settings. For instance, Kaput and Thompson (1994) criticize the passivity of mathematics educators in implementing educational technology:

In some ways, the mathematics education community participates in the same conservative attitude as do schools. With few exceptions, the mathematics education community, and especially researchers, have had a passive attitude towards technology. The latest technological innovation, often a tool created for another audience and set of purposes, is too commonly accepted uncritically, leading to sometimes awkward marriages between learning environments and technological innovation or to retrofitting curriculum and instruction to accommodate the innovation. (p. 681)

Similarly, Cavanagh and Michelmore (2000) mention that learners tend to uncritically accept graphical images, without relating other sources and ways of information and communication. Hershkowitz and Kieran (2001) notice that secondary

school students obtain graphical representations from computer programs without considering the algebraic properties of the functions.

Technology might improve math equity. For instance, Kaput, Noss and Hoyles (2008) mentioned that “new computational media offer the opportunity to create democratizing infrastructures which will redefine school knowledge” (p. 713). For instance, for geometry education, Kaufmann and Schmalstieg (2003) developed the Construct3D software that allows students to experiment with geometrical constructions and improve their special skills. Another example where computers radically change mathematics education is in statistics education (Garfield & Everson, 2009; Gourgey, 2000; Rossman & Chance, 1999) where computers can help in doing calculations with a large amount of data and allow students to do calculations instantly and concentrate directly on inferential aspects rather than on the tedious aspects of calculating the numbers collected.

In Calculus and Advanced Functions courses, the importance of using computers in instruction has been shown by Arnold (2004). Ayub, Mokhtar, Luan, and Tarmizi (2010) found that some software packages such as MACCC (Mastering Calculus Computer Courseware) and SageMath (educational software designed by Sage Software for mathematics classrooms) help students to improve their performance in calculus. They ran ANOVA tests by comparing the results of the students who were using SageMath and MACCC software in their mathematics classrooms with students who had traditional instruction in calculus and did not use computers at all. The results indicated that the students from the groups who used computer software in tutorial class scored

better than students in the traditional group. This research shows that used correctly, computers can improve teaching and learning results in calculus classes.

2.4.3 Sociocultural Aspects in Integrating Computer Technology in Mathematics Education

Technology might change not only the content of the curriculum but also improve the social accessibility of mathematics education. Researchers and historians agree that the history of mathematics education was for thousands of years a social process targeting intellectual elites (Kaput, Noss & Hoyles, 2008). As Ernst (n.d.) put it: “Many people have come to feel that mathematics is cold, hard, uncaring, impersonal, rule-driven, fixed and stereotypically masculine. Evidently there is a strong parallel between the absolutist conception of mathematics, the negative popular view of mathematics, and separated values” (para. 38).

Kaput, Noss and Hoyles (2008) assert that “changes in representational infrastructure are intimately linked to learnability and to the democratization of the intellectual power” (p. 713). Similarly, Wenglinisky (1998) notes that the quality and the nature of instruction received by educational technology is affected by students’ socio-economic status. In mathematics instruction, classrooms in poor neighbourhoods received technology instruction more as drill and practice but not as a way to improve higher-order thinking skills.

Another major disadvantage in using computers in mathematics classrooms is the limited availability of the software. Becker and Ravitz (2001) surveyed 1,215 schools with 4,100 mathematics teachers of grades 4 to 12 to see patterns and frequency of using computers. The results show that, during 30 weeks, only 11 percent of participants

reported using computers more than 20 times. Even in these cases, the use of computers was for playing games, typing in word processors, or reading materials from CDs.

2.5 Aspects of Professional Development in Mathematics Teaching

In this section, I will present PD challenges by focusing on teachers' requirements for integrating technology in mathematics classrooms. I describe some general challenges in providing PD programs and general recommendations for mathematics teachers. I will present some particular issues of PD when teachers try to integrate educational technology in classrooms. Finally, I will present aspects and issues of PD when teachers attempt to integrate technology specifically in mathematics.

2.5.1 Identifying Challenges for Teachers in Professional Development

One important challenge that teachers face is isolation. If teachers become isolated, they will have more dull routines in their practices (Sachs, 2003). Moreover, they will tend to be hostile to any new challenges to their established practices. Therefore, new professionalism policies should create opportunities for teachers to avoid separation and take responsibility for practical actions (McLaughlin, 1997).

Another problem in Ontario teacher professional development is implementing centralized directives and policies without consulting local structures. Often teachers are required to implement content and strategies developed by ministries or universities that did not involve teachers in the design process. Models such as top-down implementations, centre-to-periphery, and research development diffusion adoption (RDDA) have been criticized as widening the gap between theory and practice. These reforms left teachers outside the innovational efforts (Pedretti, 2003). Shulman, (1987) argues that a simplistic theory-to-practice model is ineffective in teaching. Similarly,

McDougall (2000) considers that learners and teachers produce the success and not the curriculum manipulation.

One of the greatest problems that preservice and inservice teachers are confronted with is separating theoretical PD courses from practical fieldwork teaching. Practice has shown that taking courses in isolation is inadequate for teachers' preparation (Putnam & Borko, 2000; Szabo, Scott & Yellin, 2002). Therefore, looking for conceptual and structural alternatives that can help to improve teacher preparation, the researchers advised to link coursework with field experiences for the benefit of both. This is especially valid when teachers learn technology in their PD programs and technology is introduced in classrooms.

Another challenge for represented by professional development is imposing managerial and commercial agendas in teaching. Public school systems from Canada, Australia, the UK, and the US have been scrutinized for these managerial agendas (Sachs, 2003). These attitudes often regard public services and infrastructure as inherently inefficient and require them to be reformed by applying management laws such as effectiveness, efficiency and economically-based accountability. This is a very important aspect when teachers have to make decisions in educational technology, since a large part of the existent software is commercial software.

The importance of transmitting teaching achievements and making them valuable for the whole school was noticed by Fullan (2001). He recommended that staff development lead to improvement of school organization and not only to improvement of the individual skills of teachers. Therefore, efforts should not be focused on training specific or isolated skills. Professional development programs need to be grounded in

teachers' work, their feelings, experiences and interactions with communities. An example in this sense is the model proposed by Marx, Blumenfeld, Krajcik and Soloway (1997) called CEER (Collaboration-Enactment-Extend-Reflection). This model helps teachers adapt constructivist innovations to community settings under four directions: a) collaboration with others; b) enactment of new classroom practices; c) extended efforts to instantiate change; and d) reflection on practice.

2.5.2 Professional Development in Teaching Mathematics

One promising program that helps mathematics teachers to become familiar with technology is called California Mathematics Education Technology Site (CMETS) (Guerrero, Walker & Dugdale, 2004). CMETS is a three-year program designed to improve technical skills for mathematics teachers. This state-wide program has a main goal of establishing and sustaining a network for middle and secondary school mathematics teachers for the purpose of helping them develop technological integration in mathematics classrooms and fostering mathematics teacher leadership. CMETS is designed to include long-term planning for the development and implementation of policies for promoting teacher leadership in integrating technology in mathematics education.

Fullan (2006) mentions that teachers' professional development must lead to improvement of school organization, not only the skills of individual teachers. Efforts should be focused on nurturing the required abilities of learning and not only in training for specific or isolated skills. Professional development programs need to be grounded in teachers' work, feelings and experiences.

Researchers noticed a great gap between curriculum theorizing and curriculum development. Eisenhart, Cuthbert, Shrum, and Herding (1988) stated that the difference between theorizing and development in designing curriculum falls outside the comfort zone. They mentioned that, instead of paying attention to designing the content of curriculum, teachers simplify the act of teaching by adopting a technical approach to the teaching act by refusing to negotiate the content of the curriculum. This experience remind us of Dewey's (1916) warning of a "disposition to take considerations which are dear to [our] hearts... and set them up as ends irrespective of the capacities of those [being] educated" (p. 108).

Professional development for teachers has often been criticized for relying on general educational theories to the detriment of taking concrete steps for the existent problems. For instance, Morgan (2001) and Smulyan (2004) argue that teacher training programs should be more oriented toward schools and real problems and less for educational theories.

Another issue is related to teacher involvement in social aspects of the community and society. Many researchers consider that today an adequate professional development model implies also the teacher's involvement in the community and in its social aspects (Hargreaves, 2000; Hodson, 2003; Sachs, 2003). In a democratic country, teachers have to fight to eliminate or reduce inequities and oppressions such as poverty, racism, sexism, and ableism. The fact that the teaching profession is inevitably a social and political act was substantiated by Hargreaves (2000), Hodson (2003) and Sachs (2001). For instance, Sachs (2001) recommends that teachers incorporate the following principles in teaching: a) inclusiveness rather than exclusiveness; b) collective and collaborative; c) effective

communication of aims and expectations; d) recognition of the expertise of all parties involved; e) creating an environment of trust and mutual respect; f) ethical practice; g) being responsive and responsible; h) acting with passion, and i) experimenting with pleasure and having fun.

Shulman (1986, 1987) proposed a body of knowledge that teachers should have in order to be successful including knowledge of: a) content, b) general pedagogical principles and strategies, c) curricular programmes, d) pedagogy of specific content domains, e) learners, f) community, social, and cultural context in which the school/class exists and with which the teaching takes place, and g) general educational aims.

2.5.3 Professional Development in Integrating Technology in Classrooms

In the 1990s, reports showed that teachers have many problems in integrating technology. The Office of Technology Assessment, (OTA, 1995) mentioned that a great number of teachers are not prepared to incorporate computers in instruction: "...it is becoming increasingly clear that technology, in and of itself, does not directly change teaching or learning. Rather, the critical element is how technology is incorporated into instruction" (p. 57). Some provinces and states have introduced rigorous criteria for selecting teachers' proficiencies in using technology. For instance in Texas, the Texas Education Agency (TEA) developed a standard for computer skills that teachers should have in order to be considered proficient. For this purpose the TEA elaborated the Texas Essential Knowledge and Skills (TEKS) containing computer applications in K-12 curriculum. TEKS asserts that teachers should master several skills such as producing graphics multimedia, desktop software, and other software (e.g. communication, databases) in order to help express ideas to students and communicate ideas to them.

There are several professional development organizations for teachers designed to integrate technology into schools: the National Council for Accreditation of Teacher Education (NCATE), the International Society for Technology in Education (ISTE), the Association for the Advancement of Computing in Education (AACE), and Interstate New Teacher Assessment and Support Consortium (INTASC). These educational organizations have designed important standards of methodologies and pedagogies to prepare professional teachers who are attempting to integrate technology in their classrooms and also improving the skills of the teachers who are already doing so. For instance, ISTE (2002, 2007, 2008) has worked to improve teaching and learning by integrating technology in teacher education. One of ISTE's most accomplished contributions is the design of the National Educational Technology Standards for Teachers (NETS-T) in the following directions:

1. Technology operations and concepts,
2. Planning and designing learning environments and experiences,
3. Teaching, learning, and curriculum,
4. Assessment and evaluation,
5. Productivity and professional practice, and
6. Social, ethical, legal, and human issues (ISTE, 2002).

Introducing educational technology in teaching has been a difficult challenge. For instance, Magala (2006) was concerned about the spread of technology without adequate preparation for teachers. Thus, a critical aspect in teachers' professional development is the ability to scale up innovations, in our case integrating technology in classrooms.

Blumenfeld, Krajcik, Marx and Soloway (2000) state that implementing new

technologies should be critically evaluated. The technological invention is not the problem per se but the way technology is promoted in classrooms is often an issue. Designing rigorous criteria for evaluation is difficult, since that invention just started to be adopted. In other words, an ability of a new technological invention to succeed in a specific classroom and school should be considered in its entire environmental complexity.

Implementing educational technology has also been viewed as a radical reform that requires a lot of investment in time, energy, and money. Some researchers such as Cuban (2001) are famous for their very scepticism in accepting the success of implementing technology in education. Often the problem of having teachers prepared for using technology has been treated as a problem between generations. For instance, Yokochi et al. (1993), quoted in Hodgson (1996), see the problem as replacing an old generation with a new generation of teachers with an open-minded approach to technology. Hodgson (1996) sees the problem differently. For him, the time required to wait for a generation change is unacceptable; likewise the probability of a new generation automatically having the knowledge required to teach using computer technology is unlikely to occur by itself. Therefore, Hodgson asks for a gradual training in integrating technology for both inservice and preservice teachers in order to change their skills and attitudes.

As in any effort to implement a new invention, researchers admit that there will be teachers excited about using technology, while other teachers might be reluctant (Pierson, 2001). Also, a large percentage of school teachers are likely to become later adopters of technology. In studying personal roles relating to technological change,

George, Sleeth, and Pearce (1996) mentioned that novices and sceptics should be specifically contacted, in order to increase their chances of integrating technology.

Administrators also have a key role in sustaining technological reforms in classrooms. Gray (1997) notes that school administrators can affect the success of implementation by providing teachers opportunities for training and learning how to implement technology in classrooms. Lam (2000) states that experienced teachers using technology should shift the way they use computers in classrooms. As such, this change should be from drill and practice to an instructional tool.

Hoyles (1989) mentions that teachers should know how to use technology before starting teaching: “My proviso, though, is that these mathematics teachers must have adequate in-service support. Teachers need to experience the power of the software for themselves and appreciate some of the obstacles that pupils will face before they can use computers” (p. 2). Cornu (1996) also asserts that teachers should receive more opportunities for knowledge acquisition to develop computer competence and pedagogical skills. He emphasizes that these areas of PD training should interact with each other in order to have successful teacher professional preparation. In particular, in the case of integrating technology into education, the emphasis should not be reduced only to learning the technical skills required to manipulate a specific technology; rather the preparation should focus on preparing to use that technology in the classroom.

There are several models of adapting innovations and, in particular those that can help assess the maturity of implementing technical innovations. Among them, the models developed by Rogers (1996) and Hardy (1999) are the most well-known. Both models have a five-step process of implementation and will be presented separately.

Rogers' *Model Analysis* (1996) explains teachers' integration of technology in classrooms through a five-stage developmental process, in our case computer technology in teaching and learning mathematics:

1. Recognizing (knowledge), the persons only theoretically acknowledge the invention that might be introduced, but they do not have any previous experiences, and they are still reluctant to risk.
2. Accepting (persuasion), where persons form a favourable (or unfavourable) attitude toward invention. In our case, if the opinion is positive, teachers only acknowledge that the technology might be useful in mathematics classrooms and decide to start a procedure of integration in classrooms.
3. Adapting (decision), where persons actively engage with that invention to be implemented in a specific context and make the required adaptations. In our situation, this is an in-progress case when teachers start already to introduce the technology in mathematics classrooms, but no results are reported yet.
4. Exploring (implementation), where teachers actively integrate that invention in their settings. In our case, teachers appropriately master that technology in mathematics classrooms.
5. Advancing (confirmation), where the evaluation is done and the decision is positive. In our case, teachers evaluate that the technology in classrooms produces benefits for their students and themselves.

In order to assess the computer use and the integration of technology, Hardy (1999) also designed a model that contained, similar to the Rogers model, five distinct

stages: entry, adoption, adaptation, appropriation and invention. In more detail, these stages are:

1. Entry – At this stage there is only theoretical knowledge. In the case of teachers, they have just started transformations and they are beginning to be accustomed, though often critical of the goals and methods of instructional technology.
2. Adoption – The adoption of technology goes in the direction of drill practices.
3. Adaptation – At this stage technology is integrated in classrooms, teachers are well trained and alternative teaching styles start taking place.
4. Appropriation – The moment when personal mastery of technology is achieved.
5. Invention – Learning becomes a creative and social process.

Each stage of these two models are very similar each other. Overall, these two models are very useful when researchers are trying to evaluate the degree of successful integration in mathematics classrooms. Without attaining the last level (“invention” in the Hardy model and “confirmation” in the Rogers model), it can be inferred that there is not full confirmation of success in integrating technology in mathematics classrooms.

2.5.4 Professional Development in Integrating Technology in Mathematics Education

Mathematics education was traditionally oriented toward intellectual elites. The studied subject was considered to be static and so was the support or media in which the subject was written. Because of the increasing number of years of schooling, and high school becoming mandatory, mathematics education no longer targets only intellectual elites. In addition, the quantity and complexity of mathematical problems have increased in life and in practical settings. Furthermore, computers offer an environment that allows

a dynamic context and lets instruction become more engaging and interactive (Kaput & Shaffer, 2002).

Steen (1991) found that teachers and administrators do not commit themselves to a broad vision of instructing mathematics. It was found that the study of mathematics is disconnected from the study of the other disciplines. In addition, mathematics is taught mechanically, without letting learners understand proofs and general patterns of solving problems. Also, parents, coaches and teachers themselves have low motivation in engaging pupils in mathematics activities. This is why technology has additional challenges when introduced in mathematics classrooms.

In the US, administrators and officials considered that all programmes of professional development for mathematics teachers should offer opportunities to build “three kinds of knowledge [that] are crucial for teaching school mathematics: knowledge of mathematics, knowledge of students, and knowledge of instructional practices” (National Research Council, 2001, p. 370). Educational reforms require a profound understanding of premises on which change takes place (Edwards, 1994a). For Hart (1996) and Lord and Miller (2002), mathematics reforms are a difficult choice when intended for large scale change. Mathematics teaching has been considered to be based extensively on teacher-directed processes and a transmissionist model of passing knowledge from teacher to students (Chapman, 2002). For Chapman, changing teachers’ orientation from teacher-centered to student-centered is not an easy or direct process. Even though teachers might be willing to change their teaching style, there are a variety of barriers that might interfere with their initial approaches.

The danger of imposing a preestablished curriculum or being guided by excessively centralized mathematics curricula was evidenced by Smart and Hoyles (2009). They emphasize that all ready-made solutions are not enough for mathematics teachers, since they need to tailor their mathematics discourse to the specificities of their classrooms and communities. Therefore, it is important that teachers be able to learn how to particularize these changes according to their own requirements. They designed a program at London Mathematics Centre consisting of ten days of professional development for secondary mathematics teachers in London schools. This program helps teachers develop their own professional agency in order to obtain mathematics confidence by designing materials for the purpose of responding to the needs of their students. This PD program has four stages:

1. Teachers personally reflect on the purpose of mathematics. This stage is before any classroom activity.
2. Starting their mathematical practice based on research designed around mathematical themes such as generalising or proof. Reviewing this research in mathematics teaching is important to give teachers new ideas and insights.
3. Considering the previous research results, teachers will design some initial teaching points, and reflect on how to adapt these to their particular settings.
4. Supporting and sustaining the practical development in classrooms of their new curricular ideas.

Blume (1991) says that mathematics teachers are prone to failure when they are attempting to learn new materials and instructional approaches and teach them in short time in their classrooms. Instead, he recommends that the focus of teacher training should

be on learning mathematics when computers and software are already available in classrooms and the teachers know how to use these devices. In order to be efficient in inservice or preservice programs, teachers should have opportunities to share their knowledge and experiences. Therefore, Blume recommends a professional model that has three phases:

1. Teachers learning the use of technology,
2. Teachers reflecting on their learning and practices, and
3. Teachers translating their own plans that were facilitated by their instructor in a technology rich environment.

In Ontario, the Learning Consortium, a partnership between the University of Toronto and several school boards from the Greater Toronto Area (TDSB, TCDSB, Peel DSB, York Region DSB) designed a program of integrating technology for secondary school classrooms (McDougall & Jao, 2009). Some of the goals of the current study are to investigate Grade 9 Applied mathematics classrooms for the purpose of:

1. Investigating school and teacher improvement, and
2. Understanding collaborative inquiry as a way of professional development.

This study gives a chance to teachers not only to learn new technologies (such as wikis or Smart Boards) but also the opportunity to practice them in schools and share their experiences in integrating technology.

2.6 Frameworks of Teaching Knowledge

This section familiarizes the reader with the main teaching frameworks required to approach this research and the improvements that these models bring in teaching mathematics. I detail the following two important models of teacher knowledge: Pedago-

gical Content Knowledge (PCK) and Technological Pedagogical Content Knowledge (TPACK). Based on the advantages and limitations of these models, I build a rationale for using the TPACK model for facilitating the integration of computer technology in teaching mathematics.

2.6.1 Pedagogical Content Knowledge Model

One of the most often used frameworks in teacher education is Pedagogical Content Knowledge (PCK) (Shulman, 1986, 1987). Shulman (1987) asserts that the knowledge of teaching has to have a specific focus and, in consequence, designs a framework called Pedagogical Content Knowledge (PCK):

Pedagogical Content Knowledge identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue. (p. 4)

This model was initially defined as an intersection between pedagogy and content knowledge, which a teacher should have in order to produce an efficient outcome. “By definition, Pedagogical Content Knowledge is both an internal and external construct, as it is constituted by what the teacher knows, what the teacher does, and the reason for the teacher's actions” (Baxter & Lederman, 1999, p. 158).

Overall, the PCK model proved to be useful not only in designing subject-based topics, but also in investigating teachers' attitudes, crafts, experiences and activities in various school settings. It has had a great impact in the educational world. Among them, Grossman made a major contribution by systematically defining contextual settings in analyzing the PCK framework (Grossman, 1990; Grossman & Stodolsky, 1995). She

recommended that teachers should be less concerned about distinct forms of knowledge and beliefs, and instead undertake a more holistic approach (Grossman, McDonald, Hamerness & Ronfeldt, 2008). Furthermore, teachers should focus more on classroom practices, emphasizing the importance of integrating theoretical principles with the enactment of practice.

The following diagram illustrates how the model works:

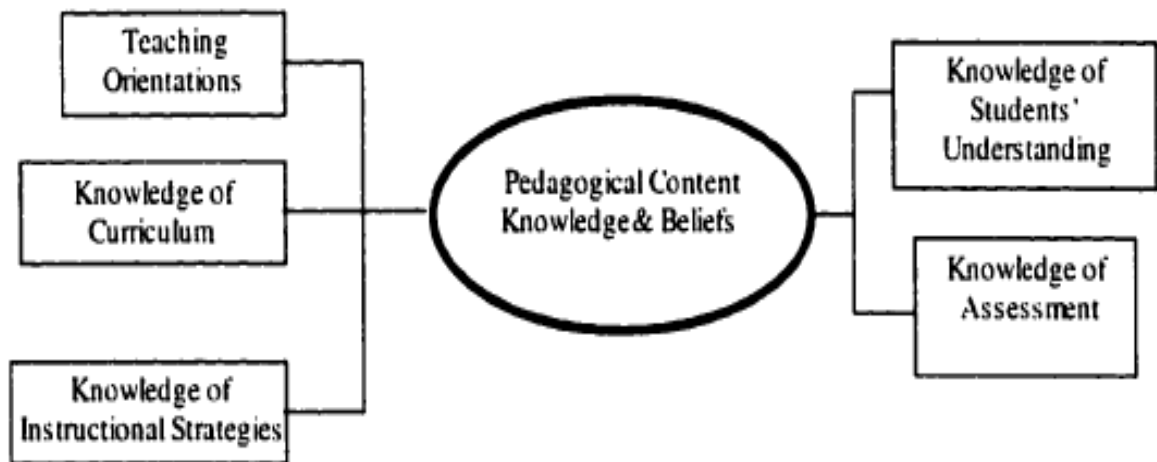


Figure 1: The PCK Model (Abel, 1996)

This framework should not be considered as an attempt to find universal laws of pedagogy (Verloop, Van Driel & Meijer, 2001). Rather, this concept helps educational researchers study groups of teachers with common characteristics. PCK differs from place to place (Carlsen, 1999); for instance, a PCK model from a rural school might be totally different from a PCK model from an urban school.

Other researchers choose to extend the PCK framework. For instance, Grossman (1990) extended the PCK model by defining a pedagogical model based on the following:

- General Pedagogical Knowledge,

- Subject Matter Knowledge,
- Pedagogical Content Knowledge, and
- Knowledge of Context.

The representation of Grossman’s model of teacher knowledge is presented below:

Subject matter knowledge			
Syntactic Structures	Content	Substantive Structures	
General Pedagogical Knowledge			
Learners and Learning	Classroom Management	Curriculum and Instruction	Other

Pedagogical Content Knowledge		
Conceptions of purposes for Teaching Subject Matter		
Knowledge of Students Understanding	Curricular Knowledge	Knowledge of Instructional Strategies

Knowledge of Context		
Students		
Community	District	School

Figure 2: Model of Teacher Knowledge (Grossman, 1990)

There is debate over whether and in what ways teachers’ beliefs help them to construct a PCK framework. With respect to teachers’ background, Hashweh (1996, 2005) found that teachers holding a constructivist approach in teaching developed a richer PCK. Therefore, Hashweh considers that PCK does have a belief component that should be considered.

With respect to the PCK framework, there are several challenges. One of them refers to the context factor. Although the PCK considers the context, there is not a

systematic approach in this line. In different contexts, researchers use the framework differently and change the importance of the components according to the specificities of current research and practices. This creates difficulties when we try to integrate different pieces of PCK obtained from different contexts.

Another critique refers to the PCK complexity. Due to the fact that relationships between cognition and action are multiple and complex, the PCK framework is difficult to examine thoroughly. McEwan and Bull (1991) argue that a PCK is hard to accomplish, if not impractical, because content knowledge and pedagogic content knowledge are hard to separate in certain domains. Marks (1990) highlights the ambiguities in pedagogical content knowledge: in particular the fact that it is derived from other forms of knowledge, so that interpretation of teaching knowledge becomes a matter of focus.

Many researchers proposed additional components in order to extend PCK. Therefore, this framework has become difficult to manage since many different structures and identities have been produced, with different interpretations from one to the other. In this respect, Hashweh (2005) mentions that, over time, too many aspects and components became embedded in the PCK framework:

It had been transformed from that special amalgam of subject matter and pedagogy that Shulman (1986, 1987) described to a category of teacher knowledge that curiously seemed able to encompass all other categories of teacher knowledge and beliefs—such as knowledge of subject matter, orientations, student characteristics, aims and purposes, resources and pedagogy. Additionally, PCK seemed to have lost one of its most important characteristics, its topic specificity, and was being thought of as a broad and general form of knowledge ... If PCK, according to some researchers, has become that generic all-encompassing form of teacher knowledge and beliefs then we do not need the term PCK—it has become synonymous with teacher knowledge and beliefs, and even practices for some. On the other hand, many researchers have empirically examined teacher PCK and concluded it includes some or all of these components. It seems we have reached an impasse regarding the nature of PCK and its utility as a theoretical construct. (p. 274)

Carlsen (1999) affirms that the PCK is linked with structuralist paradigms and does not leave much space to track students, teachers and community influences in their efforts to negotiate school activities. Therefore, argues Carlsen, this framework does not offer adequate possibilities to study student identities, and even teachers themselves do not benefit from this perspective. Even Shulman (1987) recognizes that the framework has been improvised and might miss some important aspects. Yet, with all of these critical aspects, PCK still remains the most popular teaching framework in educational research and practical teaching settings.

2.6.2 Other Frameworks of Teaching Knowledge Derived from the PCK

Teachers are often unaware of the knowledge they possess, as it is often contextualized and associated with particular students, events, and classrooms (Kagan, 1990). Therefore, there were numerous attempts to produce extensions of the PCK model in order to adapt it to different practical settings. Leading scholars such as Cochran, Deruiter, and King (1993), Fernandez-Balboa and Stiehl (1995), Grossman (1990), and Marks (1990) included knowledge of representations as well as strategies and knowledge of student learning and conceptions under the PCK banner. Grossman (1990) included knowledge of curriculum and educational media while Cochran, Deruiter and King (1993) excluded curriculum knowledge while adding knowledge of educational contexts and general pedagogical knowledge. Connelly and Clandinin defined Personal Practical Knowledge (Clandinin, 1985; Connelly & Clandinin, 1988) a theoretical framework very different from PCK.

There are also other pedagogical models such as Pedagogical Content Knowing (PCKg) (Cochran, DeRuiter & King, 1993), Pedagogical Practical Knowledge (PPK)

(Clandinin, 1985; Connelly & Clandinin, 1988), Pedagogical Context Knowledge (PCxK) (Barnett & Hodson, 2001), and Teacher Pedagogical Constructions (TPCs) (Hashweh, 2005). These models were created for various purposes. For instance, Pedagogical Content Knowing was created to support constructivist approaches in teaching by integrating “understanding of the four components of pedagogy, subject matter content, student characteristics, and the environmental context of learning” (Cochran, DeRuiter & King, 1993, p. 266).

Pedagogical Context Knowledge was designed to explore teachers’ development by using various public and private settings. Hashweh (2005) proposed to view PCK as a collection of Teaching Pedagogical Constructs (TPCs), defined “as a form of knowledge that preserves the planning and wisdom of practice that the teacher acquires when repeatedly teaching a certain topic” (p. 290).

These discussions were very useful for refining the PCK framework by adding new components and perspectives. In particular, the use of technology in teaching stimulated educational researchers to look for opportunities to discuss extending the traditional PCK framework that would provide better feedback for integrating technology in classrooms. Technological Pedagogical Content Knowledge (TPACK) was considered to be one suitable answer.

2.6.3 Technological Pedagogical Content Knowledge

Researchers argue that computers do not make the difference in teaching, rather, the difference is made by the pedagogical methods that teachers use when instructing with computer technologies (Clark, 2001). Learning is most effective when pedagogical strategies are carefully developed, with computer technologies used as mediators of

learning and incorporated into the pedagogical strategy. Consequently, adequate pedagogical models are necessary in order to explicitly describe and support the use of technology.

When introducing educational technology in classrooms, researchers noticed that the PCK framework did not explicitly support technology. There were some attempts to adapt the old PCK framework. Some of them, such as TPACK, offer adequate support for technology and offer more opportunities to see how integration of the technology takes place. Since the end of the 1990s, there were several attempts to adapt Pedagogical Content Knowledge to the use of educational technology. For instance, Margerum-Rays and Marx (2003) developed the PCK of educational technology.

Angeli and Valanides (2005, 2009) used the concept of ICT-related PCK, Slough and Connell (2006) defined Technological Content Knowledge (TCK), Mishra and Koehler (2006) developed Technological Pedagogical Content Knowledge (TPCK and later called it TPACK), and Stoilescu (2010) defined the term Technological Pedagogical Context Knowledge (TPCxK). From all, the Technological Pedagogical Content Knowledge (TPACK) framework developed by Mishra and Koehler (2006) is the most well-known.

TPACK is an extension of the PCK, defined as a systematic approach to joining technical expertise in teaching with pedagogical content knowledge (see Figure 3). TPACK is an emergent model resulting from the intersection of technology, pedagogy and content. This model considers the context as an important aspect. Teaching practices are very important as a source of learning and not just as a consequence of applying a set of learning theories.

The TPACK framework contains components of the PCK (i.e. PK – Pedagogical Knowledge, CK – Content Knowledge, PCK – Pedagogical Content Knowledge). In addition, technological aspects are systematically considered in the following ways:

- Technology knowledge (TK): understanding information technology broadly;
- Technological content knowledge (TCK): understanding technology in a specific subject or discipline; and
- Technological pedagogical knowledge (TPK): understanding how technology can shape the ways of teaching.
- Technological Pedagogical Content Knowledge (TPACK). As Schmidt et al. (2009) mentioned, TPACK “refers to the knowledge required by teacher for integrating technology into their teaching in any content area. Teachers have an intuitive understanding of the complex interplay between the three basic components of knowledge (CK, PK, TK) by teaching content using appropriate pedagogical methods and technologies.” (p.125).

The TPACK framework offers many insights into how technology should relate to other components of education in order to be successful. This framework offers clear explanations of why technology should not be treated in isolation but related with required pedagogy and content. For instance, in explaining numerous failures to implement technology in classrooms, Mishra and Koehler (2006) provide a useful explanation of why implementing technology in classrooms fails so often:

In terms of the TPACK framework that we have proposed, context neutral approaches are likely to fail because they overemphasize technology skills (the “T” in the model) without developing pedagogical technology knowledge, technological content knowledge, or technological pedagogical content knowledge. (p. 1033)

A schematic of the model is presented below:

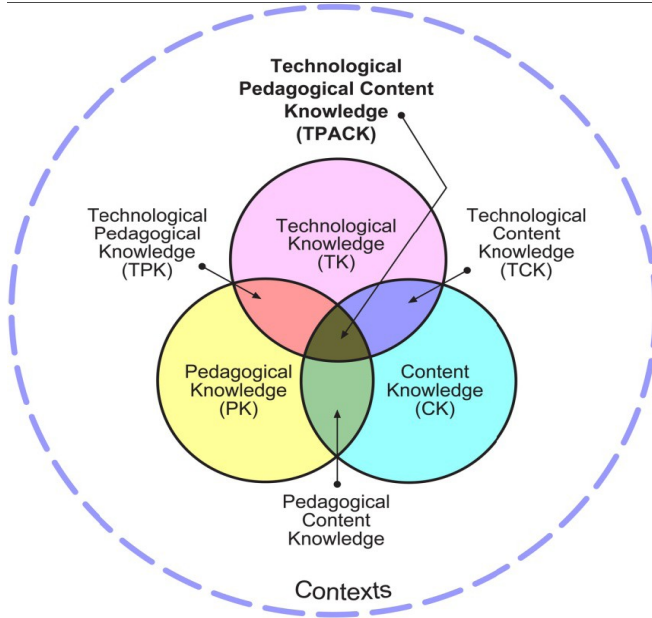


Figure 3: The TPACK Model (Koehler & Mishra, 2009)

Some limitations that this model might have are that it was originally designed by studying graduate students working with professors. It was based originally on only one practical study. However, this model was redesigned and tested in different contexts and domains: arts, science, mathematics, literacy, social studies, preservice and inservice teachers, and physical education.

Angeli and Valanides (2009) believe that, while TPACK can be considered an analytical framework for guiding and explaining teachers' thoughts and practices, the further value of TPACK remains unclear. For instance, they consider that it is unclear if TPACK is a distinct form of knowledge or only an achievement of one of its components. Also, the researchers consider that TPACK does not offer enough guidance to explore adequate integration of technology. Therefore TPACK should be disseminated more carefully and more components and facts are required to be introduced in order to obtain

greater clarity. In addition, more discussions and views from different paradigms and specific research studies are required in order to clarify the opportunities that the framework offers.

2.6.4 Technological Pedagogical Content Knowledge in Mathematics Education

Important contributions in research using TPACK framework in mathematics education have been produced by Niess (2005, 2006, 2008) and her students Suharwoto (Suhawoto & Lee, 2005; Suharwoto, 2006) and Harrington (2008). Another contribution has been the doctoral research of Cavin (2007). All these studies were based in the US (in Oregon State University and Florida State University) and were designed for preservice teacher candidates in these universities who were enrolled in secondary mathematics education classes.

In mathematics education, one of the earliest contributors to TPACK for inservice mathematics teachers was Grandgenett (2008). He asserts that mathematics teachers with strong TPACK have six main characteristics:

1. Opening to experimentation with technological tools and willingness to experiment with new lessons using technology;
2. Staying on task and not being sidetracked when teaching mathematics topics with technology;
3. Offering clear pedagogical strategies by knowing where students are academically, what students need to know, and how the lesson should be taught;
4. Helping students understand why technology is important;

5. Using technology for teaching, assessment, and classroom management;
and
6. Being comfortable and optimistic about changes in technology.

In studying preservice teachers in mathematics, Niess (2005) recommends that the TPACK framework should take into account four important aspects:

1. An overarching conception of what it means to teach a particular subject integrating technology in the learning process;
2. Knowledge of instructional strategies and representations for teaching particular topics with technology;
3. Knowledge of students' understanding, thinking, and learning with technology; and
4. Knowledge of curriculum and curriculum materials that integrate technology with learning.

2.7 Chapter Summary

In this chapter, literature regarding integrating technology in teaching and learning in mathematics education was discussed. Since integrating technology can occur in many ways in various environments, successful curriculum development might provide teachers with ideas on how to teach mathematic concepts in a mode that includes collaboration, exploration, inquiry, discovery, and construction. An adequate development strategy for mathematics curricula would provide a better understanding of the classroom settings and the use of technology in pedagogical activities that encourage teachers to learn and create new teaching and learning environments and instructional materials. The instructional objectives are designed to empower teachers to create ideal

settings for learning mathematics in the classroom. After obtaining these views from the literature review, I will compare them with data obtained in my study and discuss if these aspects will apply to this study. In the final chapter I will discuss and compare the results obtained in my study with the main ideas drawn from the literature review.

CHAPTER THREE: METHODOLOGY

3.1 Overview

The purpose of this study was to explore experiences and challenges that secondary school mathematics teachers are encountering and, based on these explorations, to interpret, describe, and explain their TPACK. The nature of the research questions influenced the methodology selected for this research. The following chapter elaborates and explains the research approach, why I selected a qualitative study, and, in particular, a multiple case study approach. The first part of this section outlines the rationale for the qualitative research. This is followed by a description of the multiple case study methodologies. I present details about my researcher role and the process of selecting and interacting with the research participants. Data collection and data analysis are described. In the end, I discuss ethical considerations and issues of validity and credibility.

3.2 Rationale for Research Design

Qualitative and quantitative research draws on different philosophical orientations (Bogdan & Biklen, 2003; Carr & Kemmis, 1986). Quantitative research explores facts that are transformed into numbers and analyzed using statistical methods (Mertler & Charles, 2005). Qualitative research differs from quantitative research in that it does not involve the accumulation of facts based on data for the sake of elaboration and verification of theory. It is informed by a different view of the nature of reality. Instead of purely factual data, qualitative research takes into account factors such as the purpose of doing research, ideological and political practice, and the type of knowledge to be

produced. Thus, it embraces a large set of paradigms, methodologies, and methods, and does so from different perspectives. According to Bogdan and Biklen (2003), qualitative research design is not simply putting together methods of data collection, or the analysis, or the report writing, but contains the whole research process, from figuring out the research problem to building the findings and writing the narratives.

There are several reasons for considering qualitative research appropriate in this study. First, the purpose of this study is to gain an in-depth understanding of how secondary school mathematics teachers incorporate technology into the curriculum. In particular, this study seeks to evaluate the implementation of integration of technology in mathematics curriculum and the evolution of the processes of this implementation.

Second, the holistic approach afforded by qualitative methods allows for a flexibility and openness to details and implications that may otherwise be ignored by quantitative research, which typically follows a narrow, predetermined and detailed plan of execution (Bogdan & Biklen, 2003; Creswell, 1998). This means looking beyond simple causal relationships to interrogate processes and data using broader questions. The immediate benefit is an avoidance of assuming the result and then proving it. A greater benefit is the opportunity to work with the few cases yet remaining open to many variables. There is no need to restrict analysis to the fine line between understanding and explanation, which is assumed in quantitative research. The design of quantitative research is predetermined and has a detailed plan of operation, while qualitative research is flexible, evolving and the researcher has only a hunch about what she or he is going to happen (Bogdan & Biklen, 2003).

Third, in order to understand the change process, this study is aimed at including perspectives of persons who were experiencing or had experienced the change process rather than having these persons receive treatments in a contrived circumstance that takes place at a specific time and a designated location. In short, qualitative researchers “study things in the natural setting, attempting to make sense of or interpret phenomena in terms of the meanings people bring to them” (Denzin & Lincoln, 1994, p. 2).

Fourth, a qualitative strategy of inquiry uses “skills, assumptions, and practices that the researcher moves from the paradigm to the empirical world” (Denzin & Lincoln, 2005, p. 25). Specifically, qualitative research seeks to understand something that cannot be done experimentally. In my case, I will proceed by exploring how the TPACK framework functions with inservice secondary school mathematics teachers.

In order to study the TPACK model of technology integration into secondary math courses, this research takes place in a public secondary school. In an effort to understand the complex changes that technology involves in math classrooms, this approach requires consideration of contextual factors, such as the culture of the school, teachers’ actions and skills in technology, the development of the projects in mathematics, types of hardware and software tools used, the relationships between the use of technology and mathematical problems, and the degree of integration between educational technology and mathematics curriculum.

Fifth, Denzin and Lincoln (2005) describe the process of collecting data and analysis as follows: “The [qualitative] researcher does not just leave the field with mountains of data of empirical materials and then easily write up his or her findings” (p.

26). Although researchers might use computers for coding and analysis, the researchers themselves remain the main instrument and qualitative software.

By visiting classrooms, I collected and reviewed observations and relevant documents. I then refined the protocol's interviews. After doing the interviews and writing and analyzing the transcripts, I revised the interview questions, generated new interview questions, and raised further questions with the interviewees. By analyzing the data, I was able to check what additional documents I should obtain. As the study evolved, I clarified and summarized the findings. I presented the story about the change process from my interpretation and analysis of these cases. Finally, if necessary, I discussed and readjusted the theoretical model resulting from these case studies.

3.3 Research Context

The research took place in a high school in the Greater Toronto Area. The mode of research involved the placement of the researcher in each of the classrooms where the educators were teaching. Participants were recruited from among secondary school mathematics teachers currently using technology in classrooms. Three teachers from a public secondary school participated in this study. Each has over fifteen years of experience in teaching mathematics, as well as a background in educational technology that includes experience in integrating technology in their teaching practices. Teachers were expected to use different computer technologies. No technology was imposed by me, so I let them use whatever technology they were willing and able to use. In this research, I took the role of observer. I only went into classrooms, made notes, recorded observations, and analyzed different documents without having interactions with students or being involved in classrooms activities.

3.4 Methodological Considerations

The main research process was qualitative consisting of multiple case studies. The process was deductive-inductive (Merriam, 1998). In the first step, the initial TPACK construction and the data from the three case studies are evaluated separately. After, the data are examined in terms of fit and possible discrepancies. The results of these evaluations then highlight strengths and areas in need of improvement in terms of teaching practices. Finally, the usefulness of the TPACK framework itself is discussed. This framework eventually assists in making recommendations for improving educational technology skills in integrating computer technology into mathematics curriculum.

3.4.1 The Role of Multiple Case Studies

“Case studies concentrate attention on the way particular groups of people confront a specific problem, taking a holistic view of the situation” (Shaw, 1978, p. 2). In each case study, mathematics teachers were considered experts not only in teaching mathematics, but also in integrating computing technology in mathematics. Merriam (1998) describes three types of case studies: descriptive, interpretative, and evaluative. Descriptive cases use existing theories for the purpose of studying samples to detect differences relative to the theory. Interpretative cases analyze facts in order to create a new theory. Finally, evaluative cases combine the previous two, updating a theory and incorporating judgments at a final stage. This type of inquiry allows for openness and enables researchers to use a variety of models and techniques according to the particular settings.

This research is an evaluative case study starting with the theoretical TPACK model obtained from the literature review. The theory was updated after comparing the three

case studies. In developing multiple case studies, Merriam recommends considering two stages of data analysis. The first stage consists of within-case research whereby each case is treated extensively and in isolation. After the temporary findings are released, the second stage begins. This stage is called cross-case analysis. Data, contexts and findings are then compared across cases.

3.4.2 Selecting Participants

Participants selected for this study were three secondary mathematics teachers from an urban public school. Two of them were selected from among inservice teachers' participants registered at Learning Consortium workshops, a professional development program jointly organized by the Ontario Institute for Studies in Education of the University of Toronto (OISE/UoT) and public schools from the Greater Toronto Area (GTA). They recently participated in inservice workshops, were trained to use different pedagogical strategies to integrate technology in classrooms such as interactive whiteboards and Web 2.0 technologies, and were committed to implement these new technologies in their daily teaching practices. The third teacher was selected from the same school, as she was already using SMART Board technology and regularly used computer technology in the classroom. The written consent for the Ethical Review Process between OISE and for the District School Board was obtained in September 2009.

The process of selection was done as follows. I described my research project of integrating computer technology in mathematics, and stated my research interests, and the role of participants in this project. I searched for secondary mathematics teachers who presented themselves as interested and committed to integrate technology in mathematics

in secondary classrooms. I selected the teachers and the school that already used computer technology in the classroom. In this research, I used pseudonyms for the names of the teachers and the name of the school.

3.4.3 The Role of Researcher

I was a passive observer, choosing not to change or interfere with activities happening in mathematics classrooms. My main role was to discuss with the mathematics teachers different modalities entailed by the integration of computer technology in mathematics curriculum. Teachers' practices varied according to settings. Therefore, I spent a considerable amount of time reading different curricular and educational research literature for the purpose of understanding various solutions designed for the classroom. Also, I was required to be a reflective teacher and a critic able to evaluate interactions between teacher and students and group activities in order to understand the role of technology and the mathematics curriculum. In these stages, building, openness and the sense of sharing were essential.

3.5 Data Collection

In this research, data collection and analysis are not separate stages, but interrelated in on-going processes. Data was collected from observations in classrooms, document analysis, and interviews. This covers two units of mathematics curriculum or two projects. I asked teachers to assist me in the process of collecting the data by asking them for clarity of observations, perceptions, and interpretation.

3.5.1 Interviews

Each teacher had two individual interviews designed to explore their insights about the use of technology for implementing the technological integration. These interviews were semistructured. That is, some of the questions were selected before the interviews and additional questions were asked based on answers given by each participant. The interviews lasted around 50 minutes each, were recorded with a digital voice recorder, and were fully transcribed. The protocol for each interview was designed to explore previous experiences and insights, the experiences evolving during the mathematics units, issues, and closing comments. Follow-up discussion and debriefing were carried out with each teacher participant.

The first interview was designated to determine the teachers' background and previous experiences. Teachers were asked about reasons for studying mathematics, reasons for adopting technology in class, personal motivation that made them select teaching mathematics, and their perspective on the integration of technology in mathematics classrooms. The questions addressed in the initial interview are detailed in Appendix D.

The second interview took place in the week after teachers finished teaching the mathematics units. It contained questions about current experiences and how these helped to foster integration of technology in the curriculum. Teachers were asked to reflect on what they expected to be different when they were using computer technology. They were asked to observe difficulties encountered in classrooms when they tried to integrate technology in the mathematics curriculum. They were asked to reflect on ways their

teaching practices could be modified in order to improve computer use in mathematics. The questions addressed in the second interview are detailed in Appendix E.

3.5.2 Observations

In studying educational settings, Bogdan and Biklen (2003) suggested that some preliminary aspects be described about the school environment (i.e. community setting, economic, social and cultural milieu), human environment (i.e. teachers, students, administration, staff, and parents) and learning environment (i.e. learning situations; teacher-students relationships, discipline, and control). I observed each teacher's classroom for at least 20 hours to see how they were using computer technology in their classes. Field notes from the mathematics classes and meetings were recorded.

3.5.3 Document Analysis

In document analysis, I started first with considering curricular documents about secondary school mathematics and educational technology from the province of Ontario. I considered handouts on teachers' ways of designing mathematics curriculum and ways of using technology in teaching mathematics. These were followed by analyzing some students' assignments and projects. After I analyzed these documents, I discussed individually with teachers for debriefing.

3.6 Data Analysis

I analyzed the accuracy of the data as a preliminary step. As with any qualitative research, the process of analysis was on-going (Bogdan & Biklen, 2003). I took the transcripts from initial interviews and observations and started the process of coding. I identified major themes and categories that emerged in literature review. From the

temporary data collected, I listed them, and looked for correspondences with the literature.

I created a set of codes based on adapting the findings from the literature review with the preliminary data obtained in this study. The pieces of data were grouped according to the following categories: TPACK background, curriculum planning and assessments, knowledge of students, and strategies of integrating technologies. As mentioned before, data came from interviews, classroom observations, and documents from various sources (e.g. school website, computer technologies, teacher handouts, and student artefacts).

3.7 Ethical Review

A formal letter seeking participation of secondary mathematics teachers was given to each teacher. They were informed of their right to withdraw at any time without any consequence and also about their options to refuse to answer any question that they did not feel comfortable with. I asked participants to sign the consent form, emphasizing that the study was not evaluative and that the findings and conclusions would not be used against them.

Except the supervisor of the thesis committee and me, the identities of the subjects involved in the study were not revealed to anyone. Therefore, I replaced their real names with fictional ones from the start. All of the work involved in this study was conducted after achieving their written consent. After collecting the data, I reviewed the findings with the participants to achieve better trustworthiness, and to be sure that the findings from my research did not undervalue their teaching and human experiences. The consent form is presented in the Appendix A. The letter for the principal of the school is

presented in Appendix B. The letter requested by the school board to inform parents about my purpose in visiting their children in their classrooms is presented in Appendix C.

I followed the recommendations of Shank (2002) who mentions the following premises required for ethical contact: a) to not harm, b) to be open, c) to be honest, and d) to be careful. In addition, Mills (2003) recommends the identification of broader social principles that define the teacher as a contributing member of the school and of the community. In addition, Mills recommends accuracy as a central concern of research, discuss personal biases and leave no space for deception. In my case, as I am tempted to overemphasize the influence of computer technology in providing constructivist instruction and promoting integration of computers in mathematics classrooms, I was prepared to accept cases where integrating computer technology in mathematics classrooms fails to accomplish the stated goals, and to explain why.

I also followed some ethical prescriptions of Flinders (1992) who mentions four important concerns in the process of developing ethical qualitative research: a) utilitarian ethics, b) deontological ethics, c) relational ethics, and d) ecological ethics. More specifically, these are:

- Utilitarian perspective recommends that research should be “for the greatest good for the greatest number” (p. 94).
- Deontological perspective requires that people participating in research should respect duties and obligations in-place. In addition, this case asks researchers to clarify their ethical views and those of the research participants, so that cases of feeling exploited or deceived can be avoided.

- Relational ethics requires the research benefit individuals, community and society at large.
- Ecological ethics recommends carefulness when the research tries to balance different perspectives such as “being democratic, equitable, liberating and life enhancing roles” (p. 95).

3.8 Validity and Credibility Issues

For credibility and validity, Lincoln and Guba (1985) recommend pursuing the following main aspects: prolonged engagement, persistent observations, structural corroboration, referential adequacy, member check, and triangulations. Prolonged engagement is attempting to overcome possible distortions presented by the researchers’ presence and to test their perceptions, biases as well as those of their participants. By extending their presence in the researched classrooms, respondents became adjusted to my presence and confident that this study was not criticizing their teaching approaches and classrooms’ interactions. Persistent observations are used to identify pervasive characteristics from atypical ones. As a result, I eliminated from analysis aspects that were irrelevant, while I continued to observe critical aspects.

Structural corroboration means ensuring that there are no internal conflicts and contradictions. Referential adequacy requests testing of interpretations against different sources of data. Member check means discussing the findings with participants to see if there are contradictions or discrepancies. In particular, I was interested to check if my observations and interpretations were not overlooking participants’ intents.

In this research, the triangulation theory was very important. Considering types of triangulation used in qualitative research, I used three distinct types of triangulation

recommended by Patton (1990): data triangulation, triangulation through multiple analysts, and theory triangulation. For data triangulation, I checked to see if information from different sources of data (interviews, meetings, field observations, paper and electronic documents) confirmed each other. Also, my supervisor had access to the entire data set. I verified if the findings from different sources corresponded. In addition, I discussed the findings with participants to see if there were major discrepancies.

CHAPTER FOUR: FINDINGS

This chapter describes the findings of the three secondary mathematics teachers' expertise of integrating technology in mathematics education. First, I will present the settings of the school in which the three mathematics educators were teaching. I will detail each case by presenting the teacher's biography, activities performed and expertise during this time. For each case, I will describe motivations, reasons and solutions to integrating technology into mathematics classrooms.

4.1 George Goodwin School

The school is a secondary public school in a large urban district and has a population of about 2,000 students, the majority of them being foreign-born. On the school's website, the administration defines the school as able to offer high academic quality for their students and community.

We have established a tradition of excellence and a reputation for our academically challenging and personally supportive learning environment. The success of each student is the focus of our staff and programs. Our curriculum focuses on performance and prepares students for university, college and the work environment. You will have the opportunity to explore your interests, develop your talents and build new leadership and service skills. Our school is unique. In spite of being a large facility, our school has a community feeling fostered by personal and social responsibility and student leadership. The school is home to a large English as a second language (ESL) population. (School website)

The school offers classes from grade nine to grade twelve and all three types of educational levels offered in Ontario high schools: academic, applied, and essential. Courses at the academic level are dominant in this school and the students from these classrooms obtained excellent results in mathematics and science education. Their alumni have graduated famous universities such as Harvard, MIT, Yale, and the University of

Toronto. The students from this school exceeded the average level in many academic activities. For instance, in mathematics, the school offers many courses in personal finance, information management, probability, and statistics. There are tutoring services provided by senior students to junior students and technology and mathematics clubs. In the Mathematics Department office, there were over 70 trophies displayed. These trophies were obtained from regional and national contests over the last twenty years: Canadian International Mathematics Challenge, Canadian Open Mathematics Challenge, Canadian Mathematical Olympiad (CMO), Fermat, Descartes, and many other events.

The neighbourhood consists of immigrants, the majority of them from East and South Asia. At the time of the study, teachers unanimously agreed that the number of academic students had slightly decreased, as an increasing number of applied students are registering at this school. The ethnic structure of the school has changed and the socioeconomic status of students coming to this school has also changed. Mark, one of the mathematics teacher participants in this research, describes the current state:

A few years ago, our school was dominated by East Asians and there were only a few students from other races. And you can see right now that the South Asians, I am not sure that they are dominating, but they are now very visible in our school now. So, the community is changing continuously and even the students' social status is changing. When I started working here in 1997, I was told then by the teachers, that it was already changed. There was a lot of immigration from Hong Kong. The families that would come to Canada from Hong Kong were financially well off, the parents were usually well educated. The immigration was very selective then, so to speak. They chose only those people who could afford emigration. Now, I would have to make an assumption and guess that an average family emigrates. The parents are not necessarily well off. Now, the parents are busy trying to survive. They might not have as much time for the kids. Then, again, I am just guessing. Again, yes, the feeling is that we all seem to agree to this in our department, that the students who are coming right now are not as strong and not as motivated, as they were few years ago. If you look now at the number of applied classes that we are offering now compared with a few years ago, the number is increasing. The number of students that select applied classes

is greater than it used to be. So is it a trend? I do not know. (interview, Dec 18, 2009)

The existing schools around the community changed their profile or even disappeared. Struggling schools are closing, and their students, together with some problems, are taken on by this school:

Some schools are closing. For instance, [a local school] was closed, and that was the school where most, if not all of the math courses, were at the applied level or essential level. This school which was not situated that far from where we are. So, I guess that the students are coming from that community, the community is still around. The students have to go somewhere so they will be absorbed by other schools that would include our school. I guess, we are going to see more of the shift that I was talking about before. We are going to see more of an increase in applied sections in grade 9 and 10. (Mark, interview, Dec 18, 2009)

The school offers various disciplines where technology is integrated in different areas. The Science and Mathematics Departments offer such examples where technology is integrated in their classrooms. There are several technological programs where students have opportunities to complement their education at the post secondary level or find employment in the following fields: communication technology, technological design, transportation technology, and construction technology. These courses offer the opportunity to learn different skills such as computer multimedia, robotics, architectural design, engineering design, automobile technology, and residential and commercial electrical wiring. In addition, computer science courses are available as credit courses.

There are 15 full time secondary teachers in this school that teach mathematics and most of them teach only mathematics. Part of the high quality of achievements in mathematics is due to the dedication of the teachers. Also, they have a special room for all the mathematics teachers. "Mathematics is something weird for many so we are

considered weird also. Welcome, among the weirds” was said to me on my first visit to the department office.

The use of computers in mathematics is still an issue in Ontario public schools. Although the official curriculum unconditionally promotes the use of computer technology in classrooms, the effective use of integrating technology in classrooms remains difficult. This school has several classrooms specially designed for teaching mathematics and a computer lab with over 30 personal computers equipped with mathematical software designed for teaching mathematics.

While some math teachers are currently struggling in their efforts to integrate computer technology in their classrooms, I selected three mathematics teachers who stand out as their expertise shows not only a high quality of teaching mathematics but also a high degree of integrating technology in mathematics. They each have over 15 years in teaching mathematics and also a high degree of expertise in technology. Their names (pseudonyms) are Cecilia, Lawrence, and Mark. In the following sections, I will present each case study of integrating technology in teaching in secondary mathematics classrooms.

4.2 Lawrence

4.2.1 Background

Lawrence was born in Canada, although his parents have an Eastern European background. He has 32 years of teaching experience, mostly in secondary schools, and has been teaching at George Goodwin for twelve years. Before that, he taught secondary school students and adults. Although he is close to the age of retirement, he regularly participates in workshops and seminars. He is very informal, laughing often, and makes

astute comments in classrooms and in the Mathematics Department room. He likes to comment about sports when teachers are gathered in the Mathematics Department.

I observed him in classrooms while he was teaching Academic Mathematics in Advanced Functions (MHF4U) for Grade 12 and Foundations of Mathematics for Grade 9 Applied (MFM1P). Earlier he spoke about his beginnings in teaching:

I studied math as my major and science as a minor. Why did I become a teacher? I don't know (laugh). I sort of fell into it; I was in the third year in the university and I did not know what I was going to do. My father was a teacher, my sister was a teacher and, yeah, maybe I will be a teacher. I really did not know until third year university. (interview, Nov. 6, 2009)

I asked Lawrence how he accumulated his expertise in integrating technology in classrooms. He mentioned that having taught computer science before might have helped him to handle the computer technologies required in mathematics classrooms. But the present software used in the math classrooms was learned alone, without formal workshops and training seminars:

The graphic calculator, I just self-taught. I learn everything by myself, except for Fathom. For Fathom, I learned for two weeks in a summer program. Computers? Well I was a computer science teacher. I taught computer science for Grade 11 and Grade 12. (interview, Nov. 6, 2009)

When I asked about support they get from the local community, Lawrence noted that the administration helped them purchase graphic calculators and personal computers. Asking if the administration is providing enough money for computers, he said that there will never be enough computers, but the administration provided a reasonable amount of money for equipment. In addition, he said that most math teachers in his school are willing to use technology in mathematics.

When I asked about the administration, he mentioned that they encourage the use of technology in the mathematics instruction and give them the right amount of money:

The administration supports us because they give us money to buy computers... We always want more. The more labs you have, the more computers you want. It is never enough. What they do, they give you something, as much money they can afford to get you right. We stand on technology. Most teachers seem to be quite open to technology, to use technology. I do not know other schools. Because no schools are the same. (interview, Nov. 6, 2009)

Lawrence believes that the main reason the administration is interested in providing technology for the school is that the technology is seen as improving the Education Quality and Accountability Office (EQAO) tests:

The administration does help. You know the numeracy committee? They try to put money in technology equipment. You know ... calculators, computers. Basically, the focus is EQAO, to get better results and do whatever it takes to improve EQAO. (interview, Nov. 6, 2009)

As for collegial support, he considered teaching as a team an important part of working with his colleagues in order to collaborate and discuss different ideas about assignments, technology or mathematics content.

All the teachers that teach the same course, work in a group, and collaborate. They share, they discuss technology, math, assignments... any situation that comes up. You should discuss any situation with fellow teachers who teach the same course. (interview, Nov. 6, 2009)

4.2.2 Overall Strategies of Integrating Technology in Classrooms

I asked Lawrence why he is using the technology. In particular, I discussed with him the differences between teaching mathematics with and without technology. He said that teaching without technology might be successful for a limited number of people, while teaching with technology might attract an additional number of students who were

not traditionally able to understand explanations just by listening to the teacher and looking at blackboard:

If you do without technology, all you do is in an old fashioned way. You write on the blackboard, you do it all algebraically, it is all academic exercise. This is the old way. Nowadays you can do that but you can add technology to bring a visual, hands-on like demonstrations of what is going on. This is a problem because a lot of people cannot understand these equations. [The students] do not get it. But if you do a graph, then they come together and this is a different way of showing a problem and the solution of the problem... like solving absolute values, solving an equation. Then you graph with the calculator and then you say ‘Where is the function now’? And they can see it. (interview, Dec. 15, 2009)

Most of the time, Lawrence uses graphic calculators. He explains that graphic calculators are affordable, easy to use, and resilient even when students are not using them properly. In addition, graphic calculators offer important visual clues for users who experiment with mathematical ideas: “Graphics calculators are visual. Some people comprehend better when they see them, because some people learn differently” (Lawrence, interview, Dec. 15, 2009).

I noticed that some teachers were using interactive whiteboards, called SMART Boards, and video projectors. Lawrence did not use a SMART Board in his classroom, but he used a video projector that was connected to a graphic calculator. In this way, students could see the steps required to solve the problem, observe the solution from the graphic calculator, and display it through the video projector to the rest of the class.

For teaching, Lawrence sometimes uses PowerPoint presentations and a video projector. “We do use PowerPoint, that is another one. We do a lot of PowerPoint” (Lawrence, interview, Nov. 6, 2009). He mentioned that offering the essential things on PowerPoint helps him to present these points to the students, especially for Grade 9

Applied, and to visualise some details. In the Advanced Functions course, Lawrence does not use a personal computer at all with his students.

For Lawrence, technology is a tool for teaching and learning. Therefore we should not substitute technology for teaching, but rather use technology to help students better understand various mathematical problems:

Technology is just a tool. It is a way of showing something. Technology does not teach anything. It is a different approach. When I teach, I try to do it visually, I try to use everything. I am touching everybody's style. Depends on the situation, sometimes the technology is good, sometimes not. Depends what you want to do, I guess and to pick your moment. What is good, when, and where? (Lawrence, interview, Dec. 15, 2009)

When I asked him what technology does not cover in mathematics instruction, he asserted that not all technologies are suitable for all teachers. Some technologies are suitable for some students, while the same technologies might be ineffective for other students. Likewise, some technologies are well designed for some types of teachers but not suitable for others. Therefore, there are not universal solutions because the technologies in place are for particular teachers and students. For instance, the SMART Board was not suitable for his teaching style, especially when he taught the Grade 12 students.

Technology is not covering everything. For instance, the SMART Board is limiting. It does not really apply to my course very well. Maybe to Grade 9 applied. The problem with SMART Board is that they move easily. When they move, you have to reset it. Setting up the SMART Board every day takes time. And it has too many glitches that can make them wrong. I find graphic calculators have fewer cases to go wrong whereas with SMART Board... But I think SMART Board will be used in the future, when we do them more properly, when we do projectors, access the Internet and then to SMART Boards. We will see more use for it. (interview, Dec. 15, 2009)

I asked Lawrence if he was using the same strategies for the Grade 9 applied classrooms as for the Grade 12 academic level classrooms. Overall, he felt that the teaching style should be different as the classrooms have different experiences and learning styles.

Some approaches are better than others. But I think that graphically, if they see something, if they see that thing, it makes a lot of sense for them. This is why the graphics are nice. Because they see when they use them... That one is lower than this. This is better because it is more visual. I think that nines are learning better visual, whereas academic classes tend to learn better analytically doing it algebraically. (interview, Dec. 15, 2009)

When I asked Lawrence what he experienced in teaching these units with technology, he noticed that the technology was reshaping the teaching, offering new perspectives and a radical departure from the old way of teaching at the blackboard. For him, technology has the potential to make students more attracted to mathematics. Eventually, technology might make mathematics more understandable for students who do not benefit when teachers chose to teach mathematics only by talking and writing on the blackboard:

What did I experience? I think technology is quite successful. It helps [students] to get the concepts. Other than old traditional teaching at the blackboard, you know, with all the examples on the board. I do this anyways. But when I do it algebraically, I like to do it visually, and when I do this, I use technology. So it is now from all angles. They [students] do not ask any question. They just do it. (interview, Dec. 15, 2009)

In interpreting the curriculum, Lawrence is masterful, as he is using his whole experience of over 30 years in education. Therefore, Lawrence is able to adapt curriculum for different students, contexts, and provincial requirements. In particular, although he might not use the most recent technologies, technology was well-integrated

in his pedagogical approaches, so that his pace was smooth, with a lot of dedication to answering individual questions and concerns.

4.2.3 Using Technology in Teaching, Interactions, and Communications

During my classroom observations, Lawrence used video projectors often while SMART Boards were used rarely. This might be due to various reasons. One is that the mathematics department received only three SMART Boards, and one of them was permanently in the computer lab. Another reason might be that Lawrence was only recently trained to use SMART Board. Lawrence has only experimented with SMART Board, bringing his Grade 9 Applied students to the computer lab to use Gizmos.

Lawrence explained that he is still reluctant to use SMART Board in teaching, since, if it is not mastered well, it wastes teaching time and effort. Asked about SMART Boards, he noted that the technology might be tricky and efforts to tame it might be futile. His regular classroom did not have a SMART Board, so he did not have the opportunity to use this tool much and accommodate his teaching practices:

You see SMART Boards and it looks like fun. Maybe I will try it. But you learn it when you start using it. The only way you really learn is by using. You have to learn to use it. But that takes a lot of time and effort. You got to learn how to use it. You got to find what works and what ruins it. You know because of the SMART Boards. If it moves -- and it does move -- nothing works anymore. You have to start working again. (interview, Nov. 6, 2009)

When I was in his Grade 9 Applied classroom in the computer lab, Lawrence occasionally used a SMART Board to show them some commands. Still, he prefers to discuss with each student individually, instead of talking in front of the class.

4.2.4 Technological Knowledge in Curriculum Planning and Assessment

Lawrence considers himself a person who knows how to approach his students. For him, teaching cannot be rigid, but should be considered a flexible strategy designed for the students' benefit. Hence, the crux remains the art of being able to perform meaningful ways of teaching.

I just go in and just do it. You know, with the calculus I can do everything. I can set up, show this, show that. I've been doing it for so long and I am good at that... I have to know how to manage people. Take the lead and keep people doing what they are doing. There are good ways of management and there are bad ways. I use whatever works under the situation, but I assess all the time. I always read my students. When I teach a class, if I know that they do not know anything about what I am talking about, I stop teaching that. I go back teaching what would get them to do where I want to go. Because there is no sense talking about where they do not have a clue... Because you got to be flexible, you got to be adaptable. You got to read the situation, you got to read the people. It's like playing football; you see what the defence is doing. They change something, and then you got to change something. (Lawrence, interview, Nov. 6, 2009)

Knowing the content represents only the starting point, Lawrence recommend that teachers should be able to adapt their curricular material to the students' needs:

You got to adapt your strategies, you have to know your stuff, and you have to know your material. Knowing the material is not teaching; knowing the material is just the basics. While you know your stuff, how to teach this in a meaningful way that they actually get it, this is for me the art of teaching. Being smart doesn't mean to be a good teacher. You can be a genius and not be able to teach. Or you can be not too smart but an excellent teacher. (interview, Nov. 6, 2009)

Lawrence considers that, in this school, graphic calculators and software products such as Fathom and Geometer's Sketchpad should be the main technological tools for mathematics instruction. Indeed, in his personal case, the use of graphic calculators in teaching was dominant. In addition, he found some new products such as Gizmos and Clips, tested them and used them in the classroom. He mentioned his interest in using TI-

Nspire, the new type of graphic calculator that is more complex than the TI-85, the graphic calculator that his Mathematics Department currently uses:

We use technology. We all use graphic calculators a lot. Also, we have the computer lab which means quite a bit. And basically that's it: the Geometer's Sketchpad, Fathom, and graphic calculators. Those are the programs that we use most. And now Gizmos and Clips in the computer lab but 80 percent of the time it is graphic calculators. And now with the new TI-Nspire calculator, which is a new higher end. ... We got the TI-Nspire calculators, but I haven't used them yet, and probably I will not use them because I will leave. There is not much software. And Fathom, I taught it in data management (interview, Dec. 15, 2009)

Lawrence noticed that some mathematics teachers might be fearful of technology. Especially when the technical support is limited, the teachers should be able to work with technology on their own. Therefore, in order to overcome these difficulties, rigorous preparation is required for any teacher willing to implement computer technologies in classrooms. He said that teachers helped each other:

If people are comfortable with it, they might use it... Secure enough to go to the classroom! Otherwise, they might look stupid and they might not know how to use it. So, a lot of people have the fear factor with technology because they are not familiar with it... You are on your own. We help each other; we try to help other people but teachers are on their own. They have to learn by themselves, practice by themselves, and learn how to do by themselves. This takes time and effort. You have to learn the technology, how to set it up and how to work it out. (interview, Nov. 6, 2009)

Having these two classrooms with different academic backgrounds and requirements, Lawrence taught very differently. In the following sections, I will detail each of these teaching approaches in a separate section.

4.2.4.1 Grade 9 Applied Classroom

Lawrence felt that the technology envisaged for this course helped the Grade 9 applied to explore and try new things: “[it] let them try more, maybe whereas the idea of

group-work is to try to get them to do by trying different things and collaborating” (interview, Nov. 6, 2009). When he compared this course with previous versions of the curriculum, he noticed that the present Grade 9 applied course had some parts removed to make it more accessible. Yet, the curriculum remains pretty ambitious since the students are not coming prepared properly with the knowledge from previous years of study. I asked him what he thinks that the Ontario curriculum emphasizes. He mentioned that,

The Ontario curriculum in Grade 9 applied is basically linear algebra, linear equation, direct variation, partial variation, that kind of stuff. Straight lines, basic algebra, basic geometry like angles, and things like that. Triangles, the use of linear models, solving linear equations: one step - two step solving, proportions, graphics. And that’s about it. I don’t think there’s anything else about the curriculum. (interview, Nov. 6, 2009)

Lawrence noticed that the mathematics curriculum encourages technology for the essential reason that technology dramatically improves the way students can do simulations, investigations, and models. “Kids like simulations and make their thoughts come to life easier. They actually see them happen” (interview, Nov. 6, 2009). Later, Lawrence explained the structure of applied curricula that, in his opinion, is based on linear relations.

This whole unit is basically on linear equations, direct versus partial variations. Graphics. I guess looking at data in terms of numerical values, then graphical. Lines. Algebraic in terms of equations. Using graphic calculators, graphs, doing algebra on paper or numerically on the chart... Because they look to find the answers, they match when they look to the chart. They compare the numbers, they see whether they match, so it is a different way of looking at the problem. (interview, Nov. 6, 2009)

In the office of the Mathematics Department, I noticed old textbooks from the former course, Fundamentals of Mathematics for Grade 9 Applied. Looking at these books, comparing with the current curriculum documents, and discussing with Lawrence

and other teachers, I noticed that these old versions were overloaded with information. In addition, these old versions did not include details about using computer technologies, and they did not offer much information about working in groups. Lawrence argued that the new resources called Targeted Implementation and Planning Supports for Revised Mathematics (TIPS4RM) (MoE, 2005) are more flexible, allowing for more collaborative strategies and group work. Unfortunately, there is no textbook designed for the new curriculum and therefore good teaching resources have not been designed yet. However, being in a digital format, these files can be easily used in the classroom either by printing papers for each student or by displaying them with a video projector.

Lawrence had some positive comments about cooperative learning activities that TIPS4RM offers. He mentioned that, at this time, he is only using this material for his Grade 9 applied class. In his opinion, these pedagogical plans are more realistically designed, taking into account some deficiencies that the students from applied might have:

The only thing we have is TIPS4RM, which is supposedly hands-on and experimental. It is like group work based on collaboration and discovery... It is more manageable! Kids are working together. More freedom to work... They can move around so they can talk. So it makes them better behaved. As long as they focus on the topic, then something is done. If you want to teach nine applied the traditional ways, it doesn't work because they become bored so quickly. So try to teach them at the blackboard! Give them something to do! They cannot focus. They can actually work better together, they can talk, they can discuss, and they can manipulate. They like playing with it. It is visual so that they can see what's happening. You can do the trace features, yeah. I think it helps. But if you just give them the calculators they cannot do it because they do not know what to do. You have to really direct them. They learn the routine of working with it. They get better and better. (interview, Nov. 6, 2009)

The teacher asks them to stay in groups and work on the list of problems on linear relations. In a problem, two workers have different rates of salary. One starts with zero

dollars and has a constant amount of pay for each hour worked. The other has a starting amount per each day and a lower wage rate per hour. The teacher asks students to calculate how much time each of the workers needs in order to gain \$72 and to represent and describe their graphics.

Lawrence avoids lecturing and tries to discuss and answer individual questions. He sometimes gives brief presentations by using PowerPoint connected to a video projector. He noticed that the texts for some of the problems are too long for the students who have limited reading ability. The main problem remains motivation, which is essential for the class's success. "They do not want to take it apart. They want to go out of class" (interview, Nov. 6, 2009), said Lawrence.

Overall, Lawrence believes that the actual curriculum is too ambitious for Grade 9 Applied Mathematics. He believes that students can solve these problems from the official curricula only up to a certain point. Once, Lawrence had a problem requesting students to determine a triangle made by three intersecting linear processes. These three processes have three distinct points of intersection, which finally can determine a triangle. The problem seemed to be too complex for his class. Lawrence was unsure so he talked with other teachers from the Department of Mathematics.

Lawrence: Look at these pages! How can they solve them? They are supposed to calculate these three intersections and have obtained a triangle.

Mark: I think that it might be too difficult, even for some students from academic.

I recorded this discussion in my field notes:

Another teacher asked about the amount of English required for the Grade 9 Applied students. There were two pages to read in order to understand the whole problem. I asked him if the level of English required for this problem might be too demanding for many of his applied students. He confirmed that this might be too difficult for some. Cecilia thought that he might split the problem into several easy tasks in order to make it understandable. Lawrence said he would consider

her suggestions further, but he still seemed to be reluctant to ask students to solve the entire problem. Lawrence said: ‘I think I will do it only for one intersection first and I will see’ (Field notes, Nov 19, 2009)

Lawrence preferred to start only with the easier problem. He took only the problem with two linear events and only one intersection and made photocopies for students. He would see how this easier problem went. If the problem went well, Lawrence would give students the problem with all three intersections in the next hour.

4.2.4.2 Grade 12 Academic classrooms

I talked several times with Lawrence about the curriculum in place for the academic level courses. Lawrence asserted that the academic curriculum has been seriously damaged by removing Grade 13, the fifth year of secondary school in Ontario. Since then, he mentioned that only perfunctory changes have been made, so that deficiencies in preparing students for higher education inevitably remain:

To me, the curriculum seems to have a lot of holes into it. When they took out Grade 13, it changed a lot. Dramatically! Especially in academic classes, it was more challenging. I think now there are holes in the curriculum. There are gaps. When they try to compress it in four years, there are gaps inevitably. Kids that are coming in Grade 11 or 12 do not have the skills that they should have. They should get them in Grade 9 and 10. There are things missing and I don’t know. They [curricula for academic courses] are poorly written. I don’t know. I’m not the guy who wrote them. (Lawrence, interview, Dec. 15, 2009)

For academic level courses, Lawrence’s concern for eliminating Grade 13 from the Ontario curriculum was confirmed by a column from the local paper, which I found in their Mathematics Department’s room. This column contained short interviews from major Ontario universities, where several professors deplored the lack of preparation in mathematical inquiry for new registered students.

The Advanced Functions course was designed to “extend students’ experience with functions. Students will investigate the properties of polynomial, rational, and trigonometric functions; develop techniques for combining functions; broaden their understanding of rates of change; and develop facility in applying these concepts and skills” (MOE, 2007, p. 85). One of the main directions of the course recommends the integration of computer technologies in classrooms.

The Grade 12 Academic class used an Advanced Functions textbook from McGraw Hill Ryerson, recommended by the school board. This textbook offers many pictures showing how to use personal computers and graphic calculators in order to solve specific problems. I looked at the textbook and noticed there were many problems from mathematical contests. The book also contains problems from various areas and contexts. For instance, there are many applications with subjects from finance, transportation, cinematography, construction, ecology, Earth climate, physics, and entertainment. There are many career connection discussions about ways of using mathematics further in different professions. The Geometer’s Sketchpad appeared many times in the book, but I did not see any problems solved with it in Lawrence’s classroom. For the period of time he was observed, he used only graphic calculators. It might be that the schedule of this hour was overlapping with Mark’s and Cecilia’s classes and they had to share the only math lab. The most probable explanation, however, is that Lawrence believed that graphic calculators were able to give enough support for the Advanced Functions course. For the Data Management course, he used to use Fathom and Excel, but for the Advanced Functions course, Lawrence relied entirely on graphic calculators.

4.2.5 Knowledge of Students

Lawrence believes that currently, the use of technology makes a great difference for society and schools. In particular, introducing technology in mathematics instruction offers a great advantage in helping students to understand, to visualise results in different ways, to make connections, and to proof. Lawrence regards technology as part of mathematics education. For him, technology gives people new ways to communicate and express their mathematical thoughts. In turn, the students and the community expect that computers be used in education, in particular in teaching mathematics. He also felt the technology is part of students' lives:

We are in the computer and information processing era. I think that these kids are so used to computers that they expect to use them in math also. I think if you don't use it, you are limiting the students. I think you have to incorporate computers in classes... They will be proficient in Fathom or graphic calculators. We don't use the Geometer's Sketchpad as much as we used to. There are better programs now. But, for me, the beauty of technology is to help them demonstrate, not only visually make sense. They can do it visually and so they can make connections. I think this is opening their minds... anything that will turn the bulb on. (interview, Nov. 6, 2009)

For Lawrence, when teachers are expected to give students adequate guidance, technology is part of the support that might help students build their knowledge:

Well, basically you lead them through. You have to give them instructions on how to do them. Until they get it. Once they get it, they will do more. With the technology, you will use what you learned and then let them investigate. Because the teacher is more of a facilitator. I like to open the door and let them investigate. See this, this and this. Help them to discover so it's now more open-ended. (interview, Dec. 15, 2009)

When I asked what Lawrence expects from his students when they use computer technologies, he asserted that the main goal remains the understanding of concepts and not mastering a specific technology per se.

I expect [students] to learn the concepts. I do not expect them to be experts in using technology. I use the technology to make them understand the concepts. This is the mandate. Not to be an excellent graphic calculator user. This means nothing to me. Do they see the math? Do they see the concept? This is why it is important. You do not have to be a computer programmer. (interview, Nov. 6, 2009)

When asked about students' technological expectations from him, Lawrence understood that students expect him to make more accessible pathways to understanding mathematics through technology:

[Students] want to learn the concepts. If the technology makes it easy for them to grasp the concepts, then this is good, technology might help them. The whole idea is how to make them learn in a more efficient way and in the most noncomplex way. To try to get them to understand the concepts - this is what it's all about. And how you present it might make a difference on how well they learn. (interview, Nov. 6, 2009)

4.2.5.1 Students from Grade 9 Applied classrooms

When I asked Lawrence about Grade 9 students, Lawrence mentioned that “they seem to understand concepts but they cannot do anything with them” (interview, Nov. 6, 2009). He was aware of the process of selecting students in mathematics applied courses and believed that the current strategies of integrating technology are getting more effective for them:

They are weak. They are always weak. Nine applied are never strong. But I think that [integrating technology], they get to learn more. I cannot prove that. But I think that their EQAO scores are a little better, which means that the program is actually working. The classes are going now a little smoother and the students are less nasty, mean, or belligerent. They are more focused if they are going better. (interview, Nov. 6, 2009)

He stated that these students had difficulties in elementary school the mathematics curriculum in important topics such as working with integers and fractions:

What skills have the students from the Grade 9 applied? Well, you saw me yesterday working with integers. They are really weak. I have the feeling they were not properly prepared in elementary school. I continually see poor integer work. But definitely, it is the fractions and integers they are horrible with. (interview, Dec. 15, 2009)

When students do not fully understand the elementary mathematics curriculum, they have additional challenges. The teacher felt that part of the problem was that many students have problems in reading and writing English. He recognized several issues that might be related with the lack of achievement. He said that some of them have a limited knowledge of English as they are studying English as a Second Language (ESL). Some of them have problems focusing and have Attention Deficit Hyperactivity Disorder (ADHD). He argues that they are at different levels and some of them are not even at the applied level. He thinks that often his students cannot stay focused for more than ten minutes of teaching.

When asked about students' expectations, Lawrence reiterated the difficulty of teaching something new when previous curricula was not understood. "The only thing they learn is going over and over again" (interview, Dec. 15, 2009), mentioned the teacher. Although it was not expected from him to teach the elementary curriculum again, he attempted to find opportunities to review the elementary curriculum.

If they do not know what I am teaching, I go back and I suppose they know nothing. I try to teach them how to do it. I basically add to the curriculum. It is not in the curriculum, but I just add to the curriculum. (interview, Dec. 15, 2009)

One of the problems some students in applied classrooms have is attendance. Many students, especially male students, were skipping mathematics classes. One of the students had only attended the mathematics course twice in over two months of school. I

asked Lawrence if the student had any chance of passing his course. He replied that, although he is capable, his attendance made it impossible for him to pass.

For Lawrence, one of the main reasons that his students were not performing well was the lack of motivation: “They just don’t want to think about anything” (interview, Dec. 15, 2009). Because their students are over 14 years old, the role of parents is now limited. “With parents, we don’t get involved” (interview, Dec. 15, 2009), mentioned Lawrence. Therefore, a discussion with parents about students’ academic deficiencies was not realistic. Usually, the parents only appeared in school over disciplinary issues.

Sometimes, children had enough skills to be in a classroom with an academic profile, yet parents did not support them enough. For instance, I noticed in the applied classroom a student who was capable of performing well. However, the student was reluctant to have any involvement in mathematical activities and was doing just the minimum activities required to pass. Lawrence was concerned about why his parents registered this student for the applied classrooms when he was able to work at the academic level. He told me, “I can’t force them to take academic classes” (interview, Nov. 15, 2009).

For applied students, there were also other multiple challenges. One issue was that students could stay motivated for only a very limited period of time:

Technology keeps them busy because the problem with Grade 9 is that they get bored. So that you have to make them interested. So, everything that gets them interested will work... They won’t pay attention so long. I can only teach them for ten minutes. Ten minutes maximum and then they lose their interest. They don’t want to do anymore. They want to do something hands-on... They want to do group work or hands-on; otherwise they get bored. (interview, Dec. 15, 2009)

Students cannot stay focused for the entire period in order to participate in math classrooms so technology, in this case, is supposed to motivate them. Asked if technology can help them to focus, Lawrence mentioned that some help from technology might be possible. Often students do things other than learning mathematics. They often talk about things that are unrelated to the classroom activities. Sometimes, they do not take care of graphic calculators and drop them on the floor. Sometimes, they prefer to listen to music or eat. Some students come late or do not show up at all. They often ask to go to the washroom but the teacher allows only one student at a time.

The discussions in classrooms are usually noisy. The teacher allows the students to make noise. He told me he cannot expect them to be quiet. However, he tries to catch their attention. While he lets them talk, he insists on discussing mathematics problems based on linear models, so that students understand these types of problems, model these from processes to equations, and eventually use graphic calculators to solve them. In the following, I describe one of the classes that might be considered typical:

Slowly, the students come to class. After 5 minutes, some girls come to class. After 10 minutes, another student shows up. There were still three male students and one female student missing. They take the graphic calculators from the container but they sit with their back to the teacher and talk. Some students listen to the teacher. A girl listens to an iPod. Some boys eat candies. They spill them on the floor. (Field notes, Classroom observation, Nov. 25, 2009)

4.2.5.2 Grade 12 Students

When I asked Lawrence about his Grade 12 students who registered for the Advanced Functions course, he stated that being at this stage means that they will pursue higher education degrees. They were described as gifted and able to accomplish complex mathematical problems:

This is a different matter. I treat them like university students. I try to give them interesting problems. I try to challenge them. I try to give them problems that are not obvious. Something that requires a bit of thought so they can play around with it. So I like to make them puzzles. ... The textbook is pretty straightforward. I try to make them more open-ended. (interview, Dec. 15, 2009)

Asked if the students understand their lessons, he praised their results and dedication:

Most of the time they do this right away; it's no problem for them learning and doing it. It's pretty straightforward... It's hard to judge but based on tests. Most of them do well. Most of the kids are well prepared. They know the stuff; they get at least 80, 85 percent. If they do not know, they come and ask me, whatever the misunderstanding is. Like I always go back, review, and repeat. I constantly repeat. Whenever one comes up you do. You know? You must be adaptable. Flexible. You teach what is required to them. Right? (interview, Nov. 6, 2009)

He thought that most of the students understood this course, and, because almost all of his students get over 80 to 85 percent, this shows that these students understood the concepts of the course. Also, Lawrence noted that students know how to use computer technologies in their mathematical projects:

Most of them get it. Like the weak students will not get it very well. But the good students will do well. And they will understand. Usually most of them are successful... They know how to graph... They know how to find maxs, mins, and intersections. They know almost everything. Especially for Grade 12, there isn't much on calculators they can't do. You know, most of the time it's TI-84 and TI-85. This is what we have. Now we will use more TI-Nspires, whenever it is available. You have to adapt it and use what's there. But then, you let them use and discover. (interview, Nov. 6, 2009)

4.2.6 Involving Technology in Math Classroom Activities

In this section, I will present ways of integrating technology in mathematics classrooms. First, I will detail a typical activity from Grade 9 Applied using graphic calculators in the classroom. After, I will describe a small project in the classroom in which students from Grade 9 Applied used graphic calculators to make posters. The third

activity presented is about Grade 9 Applied doing assignments in the mathematics computer lab. The last activity presented is about Grade 12 Academic using graphic calculators in regular classrooms.

4.2.6.1 Using Technology in Grade 9 Applied for Small Exercises

In regular classrooms, Lawrence gives students a paper with four or five problems. These problems are straightforward, as they require having knowledge about linear relations. He usually asks students what the equation is. After the students find the equation, they are required to draw the graph and explain what type of correlation and what type of variation it is. Sometimes, if the theory is new, the teacher uses PowerPoint presentations and manipulative materials such as algebraic tiles, building blocks, geometric shapes, protractors, or rulers. In the followings, I will reproduce some problems solved in a classroom in a regular day.

In the first problem, a company charges \$50 initially and then 20 cents per kilometre. The teacher asked them to find the mathematical model. First, Lawrence asked them to determine the equation for the cost. For this, he discussed using the blackboard and explained how they can select several particular values for the number of kilometres travelled and how to calculate the associated costs. I wrote about this interaction:

The teacher encourages the students to use graphic calculators and showed them how to manipulate them and how to interpret the results. He explains to them some commands by using the video projector connected with his graphic calculator. The teacher first introduces a table of values that shows how the values of the function change. After, he discusses with the class why this problem is a linear relation and how it should be solved. He shows them the commands again that they need for the graphic calculator and asks them to solve this problem using the graphic calculator. Some individual questions are asked and Lawrence, who is walking to see students' work, answers them. (field notes, Nov 12, 2009)

The second problem is about the money that Holly has in her account. Initially, she had \$250. Each month, Holly takes \$50 out of her account. How much will be left in her account? When will her money run out? I describe the class in my notes:

Again, the teacher discusses the problem. A student is at the blackboard and tries to solve the problem. With other students, the teacher helps them to use graphic calculators and interpret the results. He asks them to describe the process. A student replies that this case is called perfect negative linear correlation. The students are again showed how to make a table of values. The teacher explains to them how to properly define a window to scale the graphic so that it can be visible on their display. They are asked to perform first by themselves. After a period of time, the teacher goes to the projector and shows the solution. Students were already bored. Some of them were reluctant to continue. (Field notes, Nov 12, 2009)

After 15 minutes, the students already start to lose interest. The other two problems are similar to the first, but the students are reluctant to solve them. Therefore students start discussing noisily in small groups, talking about unrelated topics and repeatedly ask to go to the washroom.

4.2.6.2 Using Technology in Grade 9 Applied in a Small Project

Lawrence chose to do the project in two different days. First he asked students to find only one intersection of two linear processes. He gives each student the worksheet and asks students to work on the problem in teams. He invites them to take graphics calculators, liners, markers, and large sheets of paper for posters. He lets them split into teams with a maximum of four members. At the end of the hour, the group will receive a mark for their poster and presentation. There are only ten students in the classroom, forming three groups, two teams of four girls and one team of two boys. A girl from the right side comes up with the solution of the problem and shows it to the teacher. He praises the group of girls for working well.

The students explain the solution and the teacher agrees with them. The girls start to work on the paper and draw tables and graphics. Finally, the girls finish working on the sheets of paper. But they delay presenting the entire problem and avoid going to the table. The other group of female students from the middle finish the problem first. They go to the blackboard and do the presentation. First, they stick the sheets of paper on the blackboard. There is still enough space to write and draw on other blackboards. They had to make some corrections and write some additional comments.

A girl is furious with her peer because she missed something while she was trying to present the solution. Now that student, who usually is very vocal, is quiet and scared. One student fully contributed solving the problem, while the others were only a little involved and mainly were waiting for her. But overall, the presentation goes well, all the students understood the problem, and everybody gets the maximum number of points.

The group of girls from the left, who started well, come up and give a very clear presentation. The writing is done well; there are good explanations, and a lot of drawings with many colours. As in the first group, one student made an important contribution to solve the problem and the others were mainly helping her with minor contributions. I believe that only that student could solve the problem individually. But, as a group assignment, the members understood the solution of the problem. This presentation went well and once again everyone got the maximum number of points.

I noticed that the group of two boys are struggling. They were the only male students in the class. Being only two students in a group, they were advancing slower than the other groups. One of them understood the concepts, but it was difficult for him to elaborate and write the final solution on paper. When he does not understand, he

desperately asks for help from the teacher. However, whenever he starts to understand, he starts making fun of the teacher. This happens several times. Overall, the students understand the problem, they know the solution, but they are not able to write consistent explanations. They are late to class, and ultimately, they are not able to finish the assignment. The teacher, who was initially reluctant to leave the assignments for the next day, has to postpone this assignment. In the next class, two other boys joined the group. The group formed from the four male students were able to finish the problem.

Overall, the teacher was satisfied with their work and attempted to give the problem with the three intersections to his students. In the next hour, the teacher distributed the papers with the three intersections. There was a group of four girls, a group of five girls, and a group of four boys.

The groups of students are working well in this class. They discuss the problem with the teacher and he tries to help them using what they learned in the previous hour. The big sheets of paper are filled with drawings, tables, and explanations. They make better presentations to the class, although the problem is more challenging. The teacher is very enthusiastic about their work. He keeps the sheets to discuss with other teachers. Few were able to solve this problem alone, as this problem might be challenging even for some students from the academic profile, but working in groups, these students were able to solve it.

4.2.6.3 Grade 9 Applied Students in the Mathematics Lab

Lawrence brought his Grade 9 Applied students to the computer lab. He instructed them how to log in and assigned them passwords. Being new to this lab, they spent a considerable amount of time logging in, open the browser, and connect to the Gizmos

software. First they did some tutorials for the current unit. They read the first tutorial and after a period of time, they tried to solve the assignment. If the students did not score enough points, Lawrence reset the software, so that the students could do it again with a similar assignment. If students finished the current assignment, they could choose at their discretion other assignment from previous units, either from the grade 9 or from the elementary curriculum.

I found that Gizmos helps students to learn concepts and test themselves on their skills. This was very useful for the teacher; for instance, when he used Gizmos for Grade 9 applied classrooms, he gave students opportunities to update skills from elementary classrooms. Lawrence said that some of these products are useful for his courses:

Some are good, some are well designed, easy to understand, make sense. Some are not well designed and don't really make much sense and are kind of difficult to understand what to do with them. So, I think they must be written properly. You have to put the right student at the right level, so that makes sense. I think that this is great, because it's hands-on, because they like to do things. Especially Grade 9 applied students like to have catching and feeling, doing stuff. (interview, Dec. 15, 2009)

When I asked him about different technologies that he might consider in teaching for the next semester, Lawrence mentioned Gizmos as an important tool for assessment and instruction fit for use in his classrooms.

Not much in this moment. The only thing I see is using Gizmos, so they can set up to learn by using computer tutorials. Graphically, they visualize, they do problems. And there are a lot of problems, a lot of Gizmos. I used a few, but I think that this is going to be a thing in the future. Using these little programs called Gizmos. But that requires that the kids have individual computers. I can bring one computer only in a regular classroom, so I have to bring them to the lab. And to have everybody work on that it is a bit of a problem. There are not many labs. It's only one lab. You got to share. (interview, Dec. 15, 2009)

At the end of the semester, Lawrence succeeded in putting this idea into practice. He took his Grade 9 applied mathematics students into the computer lab. The teacher let them use Gizmos to review different curriculum units from elementary classrooms in parallel with using tutorials and tests for the current course's units. Asked if he will use TI-Nspire, he mentioned that he would be interested when he has more opportunities that will allow him to get familiar with it.

4.2.6.4 Grade 12 Academic

In his Grade 12 academic classroom, Lawrence gives them problems from the textbook. He selects these problems from the homework and discusses them with the students. He relies on graphic calculators to help with mathematical modelling for these functions. When asked about how he uses computer technology in Grade 12 Advanced Functions, Lawrence acknowledged that graphic calculators remain the main tool in this course. Also, the same tool, in this case graphic calculator, has different roles in different grades:

We use them [graphic calculators] a lot in Grade 12. I showed them how to do regressions, graphs and how to analyze. I showed them how to do it and we use the graphic calculators every day. They can use graphics calculators. They can see it from that context not just algebraically. There is a different way of solving... Grade 12 students use it as a tool to solve problems. It is not like Grade 9. In Grade 9, they use it to demonstrate, whereas the Grade 12 they are using it as a tool to find a solution. They are expected to know how to use the calculators and how to apply it. Because a lot of them use calculators, you would like that.
(interview, Nov. 6, 2009)

In connecting different topics with mathematics, Lawrence was adept at relating mathematics with models from various areas. For instance, one of the problems was about the ratio between deer and wolves. The teacher gave a very inspired interpretation. He noticed how some of the data about adult deer and adult wolves were inaccurate and

he tried to present it in a serious but, in the same time, silly way. “If there are more wolves than deer, this is terrible. Because the wolves eat deer”. He solicits interpretations of the data from the students to try to explain the results and understand the content. “What does the data say?” The students noticed that the number of deer decrease when the number of wolves increase. Indeed at one point, the ratio of wolves to deer becomes too high (5.40 to 3.40).

He asks students to develop an analysis of the pattern of data for the requested period of time. After analysis, students acknowledge and accept the teacher’s interpretation. They try to interpret the inaccuracies reported in the textbook. I asked if he likes to look for problems from real life and look to interpretations. Lawrence looks for real world problems as these types of problems motivate students to learn the current math curriculum. When I discussed this with Lawrence, he described why he likes these kinds of real-life problems (in this case ecology):

We use them. Well, in the curriculum, they try to bring them from different facets. I guess meaningful to the real world. They want to connect the math to the real world so that they can see the usefulness of math. Otherwise, they are asking why they have to use math. So they take problems from the real world, applied mathematics. They like this approach... Because they understand better... I think this is what we tried to do. (interview, Dec. 15, 2009)

At times, students become bored and do not like to work on graphic calculators. But overall, Lawrence succeeded in capturing their interest and even, in the last hour before holidays, some students showed up at the blackboard and discussed different problems. His pedagogical experience made the difference when he succeeded in getting them interested in this course, despite the fact that they worked only on older models of graphic calculators.

4.2.7 Future Plans

Lawrence took some workshops in wikis and started to become familiar with them. As with the other two teachers, he was familiar with wiki technology, but he was not committed to using these in his teaching. He appreciated the technology, but he decided not to use Web 2.0 technologies in his classrooms in the next semester. Also, because it was possible to retire the next year, Lawrence mentioned that he will not teach with TI-Nspire:

And now with the new TI-Nspire calculator which is a new higher end. ... We got the TI-Nspire calculators, but I have not used them yet, and probably I will not use them because I will leave. There is not much software. And Fathom, I taught it in data management. (interview, Dec. 15, 2009)

Because Lawrence was concerned with the high amount of time required for the mathematical assessments, he was interested to start using clickers. In fact, in the next semester he regularly used these devices in his classes to assess student knowledge.

4.2.8 Summary

A long time ago, Lawrence changed his view of the role of the teacher from that of provider of information to that of a guide. Therefore, letting students investigate different problems with technology is an important aspect in this pedagogical strategy for him. Due to his extended teaching experience, his TPACK evolved from the classical PCK, step-by-step, by integrating various technologies.

Lawrence likes to use problems from real life and make different connections between mathematics and different disciplines such as ecology, finance, physics, and mechanics. With respect to the role of technology in teaching mathematics, he believes that the use of computer technology helps his students to explore different ideas, to

model, to try different hypotheses. He relies more than anything else on his pedagogical experience and consequently has high expectations of himself to provide technology as part of his act of teaching, to cover students' instruction from different angles.

Examining Lawrence's TPACK, I noticed that the pedagogy remains the strongest part. Sometimes, Lawrence is a perfectionist as he attempts to know a technology in minute detail before using it in the classroom. Therefore, he is a late adopter of technology, as he delays first to see if a specific technology is confirmed by other teachers and waits a significant amount of time after, in order to master each detail of the software. At the same time, when he uses technology in the classroom, he does this with high accuracy and precision, and he gives very thoughtful lessons.

4.3 Cecilia

4.3.1 Background

Cecilia has an undergraduate degree with a major in mathematics and a minor in geography. She has an East Asian background and completed a Master of Science degree in mathematics. She has been teaching mathematics for over 10 years in secondary schools. For the last 10 years, she has been teaching at George Goodwin School. I asked her to reflect on reasons that made her choose the mathematics teaching profession. She said:

Actually, I enjoyed teaching even when I was student. I was enjoying explaining to my classmates. I found that it helps me to learn. And that it is something that I tried to incorporate in my own teaching to have students explain how they find an answer. I think it helps them to process the mathematics. (interview, Nov. 18, 2009)

She is as a long time computer user since she had access to computer technologies at home while she was student. During her undergraduate program in mathematics, she increased her skills in technology by taking some computer science courses. After graduation, she used computer skills in order to work as a researcher in statistics and Geographic Information Systems (GIS):

When I was doing my undergraduate degree I did some computer science courses so this is part of my background. I also spent about four years working as a researcher for a geography professor who was analyzing data for Statistics Canada. And so I did a lot of programming, a lot of GIS. And spreadsheets were something I grow up with because my father was an accountant. (interview, Nov. 18, 2009)

When I asked if she had professional development experiences that helped her to learn computer technologies, she mentioned that she learned most of the actual technologies by herself, from her previous experiences as a researcher in geography, or by discussing with the mathematics teachers from her school:

In Fathom, occasionally we had workshops for teachers. This would be close to ten years ago, to participate in 10 days of workshops in learning Fathom [software]. We had some formal training for teachers almost 10 years ago. (interview, Nov. 18, 2009)

In the semester of fall 2009, she was teaching two Grade 12 Data Management (MDM4U) classes and another Grade 10 (MPM2D) mathematics academic course. Teaching the Data Management course together with Mark, she was determined to redesign the course so that more real-life problems and simulations would be possible: “This year, I am working with Mark and we decided to push it a little bit farther, to have simulations in the process, so I have to relearn a lot” (interview, Nov. 18, 2009).

I chose to observe her teaching in the two Grade 12 Data Management classrooms, since, in the third course, she was involved in assisting a teacher candidate and it was not possible to observe her for the purposes requested in this research. She acknowledges a collaborative culture in the department of mathematics. Likewise, she appreciates the strong support received from other mathematics teachers in her department:

The Math Department here at this school is really good. We discuss with each other what is happening in courses so that we teach in common. We discuss all the skills the students do or do not learn as they move through high school. I think that we've been good in supporting each other with the technology... Sharing the work that we do... and Mark [the department head] has been really good in making sure that we have all we need in terms of equipment. (interview, Nov. 18, 2009)

When I asked her about support from administration, she mentioned that the administration has helped them to organize their computer lab for mathematics. She said that the numeracy committee was very helpful for mathematics teachers: "I think our numeracy committee has an amount of work to identify what we need in the school in terms of getting support for technology" (interview, Nov. 18, 2009).

I noticed that in the Mathematics Department, the teachers often asked each other for advice on various topics: policies, issues with different students, computer technologies, and mathematics curriculum. Working as a team was very important for the teachers, in particular for Cecilia, as the students registered in this course have a complex project to accomplish at the end of the course.

4.3.2 Overall Strategies of Integrating Technology in Classrooms

Cecilia is integrating technology in multiple ways. For instance, she is using technology in teaching and assessments. She gives opportunities to students to verify

their mathematical ideas by using computer tools. In addition, she is interested in collecting digital resources for her students, in order to provide them with opportunities to consult additional instruction for the course.

When I asked her about the role of technology in teaching mathematics, she believed that technology helps students to explore different ideas and hypotheses. More exactly, she felt that the technology helps students to engage in the classroom:

I think that the technology is important because our students are exposed to it in their everyday life. It is one way of helping to engage students in the classroom and it enables them to explore an idea. We can ask a question ‘what if we did this‘? Because the students ask these; the students have the opportunity to ask different questions. (interview, Dec. 16, 2009)

Students had permanent access to personal computers. In this lab, there were many posters and manipulative materials for statistics and calculus, and around 30 Dell personal computers and connections to the Internet. The teacher had another personal computer connected to the SMART Board. Students used computer technologies such as Fathom, and Microsoft Office tools such as Excel, PowerPoint and Word. Most of the time, the teacher was using the computer lab for mathematics as a regular classroom.

Cecilia is convinced of the utility of computer technology in mathematics education, in particular in her class in statistics. When I asked her to describe the role of technology in improving students’ learning, she mentioned:

I think it [technology] helps. In the curriculum, there are a lot of mathematic processes and I think technology really helps in that area. How do you prove? How do you reason? Technology enables students to do that. Apparently, intuitively. Where it can be misused is when the students use technology as a substitute for understanding mathematics. (interview, Dec. 16, 2009)

Cecilia likes to use digital resources such as video clips, PowerPoint, and small-size documents in PDF or Word format, as resources for students’ when they want to

learn at their own pace. By using these digital resources, Cecilia wants to make available to students different sources of information available for the course. For her, the teacher is not a collector or processor of information. She is not supposed to have and control the information related to a specific curricular area. Rather, she would make the resources available to the students. Students were invited to consider their own perspectives and choose which resources were more comfortable for them. They are encouraged to select any information they need and take the path they feel suitable for their learning.

Asked about the usefulness of computer technology in improving students' performances, from the initial starting point and the trajectory that the mathematical skills have undergone, she gave me a recent example about the use of technology that improved students' skills. By using Excel spreadsheets that contained statistical functions recently taught, students could practice recent notions and types of statistical distribution:

In one of the lessons, early in this week, they were working with a spreadsheet that I developed. It showed the relations between the binomial distribution and the normal continuous function. And, they [tried different approaches] because it was a dynamic spreadsheet. A lot of functions changed some parameters; one parameter constant and changing another; and they can see these connections and what happens if we make these changes. So, I am hoping that these exercises help to reinforce the notions that we have been discussing throughout the course. And I guess we will see whether or not this actually works. (interview, Dec. 16, 2009)

In the normal classes, students could use Fathom or Excel to analyze the data for the current unit.

4.3.3 Using Technology in Teaching, Interactions and Communications

Cecilia mentioned that the focus of her teaching is on practical applications: "My focus, my teaching, tends to be on the applications. I search for applications that would help me explain things clearly for my students or will help them" (interview, Dec. 16,

2009). She relies on her various technical and mathematics experiences and skills. She does not forget any detail or leave any inaccuracy. Observing her classes, I feel that every second and every detail is well planned ahead of time.

In teaching, Cecilia regularly uses the SMART Board as she considers it to be important to help students focus. The teacher also used PowerPoint presentations in order to emphasize the main ideas and concepts from the current lesson. She liked to use the technology to help students focus on good ideas:

I think if we take the SMART Board as a specific example, the SMART Board has been really good in enabling me to focus on what the mathematics is. There are concepts in mathematics that I see in my thinking about mathematics. The SMART Board allows me to show students in a different way. So, I am finding that, when I plan my lessons and I am planning to use SMART Board, I am not just planning what the steps are that the students have to do to answer the questions. I am thinking of what is the mathematics behind the steps, and using the SMART Board to help me highlight those ideas because the SMART Board allows me to keep something hidden until it is ready to be used. Also, it allows me to annotate and emphasize things in a way that is more dynamic than just chalk on the board. (interview, Dec. 16, 2009)

Cecilia uses the SMART Board to reflect on her presentations. The SMART Board allows her to save previous lessons and helps her reflect on and understand what was successful and what was not:

It enables me to have different versions for my classes. For instance, with the data management course where I have two different classes; if I have a longer lesson, I can save it and restore it for the next class, where that particular class left off. And I can also save these lessons. For instance, I took up two files and compared what annotations I made, and sometimes one is better than the other in terms of my explanations and what I have written. Then, I try to amalgamate them and post them on the course website, so that the students can go back on their own. (interview, Nov. 18, 2009)

An important aspect of using a SMART Board was that the writing and drawings are cleaned off when the users select a new slide. According to each individual context,

this might produce benefits in clearing the board, or, on the contrary, might impede the teaching intentions, when something considered essential is deleted. She carefully differentiated the requirement for both SMART Boards and whiteboards, and used both in her teaching in order to present details to her students:

When I plan my lessons, I always use both [SMART Board and whiteboard]. I tend to use SMART Board for presenting the questions, and the solutions usually go on the whiteboard. The reason is that the SMART Board is good, except that it is only 4 by 6 feet in terms of size. Once I turn the electronic page, what was visible before is now gone. So, for questions, we can use the SMART Board to make highlights and questions. For presentations, we can summarise and clarify what remains to be done. But the actual solution I tend to write on the whiteboard because that will still be there when I move to the next slide. So for the students who might need more time in copying things into their own notes, it is still there for them to see. And then later on, a student might come back to me and ask a question and I still might have some slides of what we have done. So, ideally for me, I would have a “smartwall” instead of SMART Board, it would be better. (interview, Nov. 18, 2009)

In summary, Cecilia combines drawing and handwriting when using the SMART Board. Further, the limited space of the electronic board is problematic at times and she usually needs more space. This forces her to find adequate solutions and combine writing on whiteboards and SMART Boards.

I write easily. The problem is when I try to put too much on SMART Board. Then my handwriting deteriorates and it gets too crowded. This is the reason for using a combination between whiteboard and SMART Boards, because I want to keep the SMART Board relatively clear but also there are a lot of solutions when you need more space. (interview, Nov. 18, 2009)

Indeed, I saw her using the SMART Board and whiteboard in the manner she described. The teacher was separating very clear questions and main ideas, which were presented on the SMART Board, and the solutions and demonstrations of the problems, which were presented on the regular whiteboards.

I noticed that Cecilia was interested in collecting digital resources relevant for her coursework such as video clips or documents. The advantage of having video clips related to probabilities was obvious: students could see, at any time, examples of mathematical problems and skills. With digital resources, students could use them to repeat any aspect of the lesson, at any time, without any effort from teachers. Some disadvantages exist too though, because these videos were difficult to locate, and most of them did not have good quality that could recommend them for educational purposes. Some of the videos were based on unreliable web addresses and might disappear without notice. Considering all of the pros and cons, Cecilia still felt that collecting video clips was useful for her and her students.

In addition, she developed a website that contains digital resources for students looking for additional information linked with the Data Management course. This website helps her to maintain and provide digital resources for her and her students. The selection is based on her judgement about significant experiences that these digital resources could provide and on students' work from the course.

What was different from other teachers in the Mathematics Department was that Cecilia designed this website in order to support her teaching activities. This was done as an individual pursuit with help from her family. The website was uploaded on a web server with help from a family member. For several years, she had maintained the content of the course on the website. The teacher uploaded different presentations, handouts, and assignments. In addition, she uploaded work done by some of her students on her website. Her colleagues or the school board were not able to offer any technical

assistance with the website. For designing and maintaining the website, Cecilia used Microsoft Publisher and Microsoft Front Page, two web design tools from Microsoft.

I do it myself; I do this with Microsoft Publisher. The FrontPage is basically reminders for kids. I put the dates for the upcoming tests and the deadlines for any part of the project. And it is updated regularly. There is a copy of the outline there. There is a daily calendar, so a day-by-day record of what we have done in class, which sections were covered in textbooks, what homework was assigned, copies of handouts, presentations... Sometimes, students were asked to teach a concept and to use a PowerPoint presentation. So, I collected the PowerPoint presentations and I also added these to the course web site. There is a page that is dedicated to the casino project that we are working on. So, copies of the rubric, some samples of work. One more page for the course overview where I posted some tests, quizzes, and I posted also some solutions from these tests. (interview, Dec. 16, 2009)

Sometimes, even when technology is not useful for students or not offering them opportunities to experiment with mathematical ideas in the classroom, Cecilia still uses it in teaching. Indeed, observing her classes, she daily relies on using SMART Board, PowerPoint, and regularly uploads new updates and lessons to the course website. When I asked her where the technology is not required, she mentioned:

For example, we had a unit in the middle of the semester in which we used very little technology. It really required the students to analyze the problem and think about what steps are required in order to find the answers to the questions. And technology cannot help to improve learning in this unit. I think that the unit was not requiring students to use technology. But I still use them in my teaching. So, I was using SMART Board and presentations software to help me to get the point across, but the students themselves did not use the technology to help them get the answers. (interview, Dec. 16, 2009)

4.3.4 Technological Knowledge in Curriculum Planning and Assessment

According to the Ontario Ministry of Education, the curriculum in mathematics for the Grade 12 Data Management course should provide opportunities to understand concepts in probabilities and statistics and collect, investigate and interpret data using these concepts. More exactly, this course:

broadens students' understanding of mathematics as it relates to managing data. Students will apply methods for organizing and analysing large amounts of information; solve problems involving probability and statistics; and carry out a culminating investigation that integrates statistical concepts and skills. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. Students planning to enter university programs in business, the social sciences, and the humanities will find this course of particular interest. (Ontario Ministry of Education, 2007, p. 111)

For this course, the teacher relies on a textbook recommended by the school board. This textbook contains many applications that explicitly detail multiple ways to solve a large number of problems by using personal computers or graphic calculators. The textbook presents applications from different areas: finance, natural science, and social sciences.

When asked to comment on the ways the curriculum is designed, she mentioned that some topics were removed from the Ontario curriculum in order to make it more accessible, let students be more exploratory, improve learning skills, and make them more problem-solving oriented. She wanted to help her students to develop their skills in problem solving:

The current Data Management curriculum is less content. I think it is more focused on the development of skills like thinking mathematically, reasoning, proving, problem solving. I think that there is a lot of time and opportunity for teachers to help students to explore so that they get the sense where the mathematics came from. It's not about telling them 'here is the rule'! (interview, Nov. 18, 2009)

Cecilia did not use graphic calculators often. In fact, during my observation in her class, she never brought in TI-84 or TI-85 calculators for her students. In her classroom, students bring their own graphic calculators most of the time. However, Cecilia introduced the TI-Nspire in order to assist students working on their final course project.

In general, Cecilia encouraged students to work with personal computers and use statistical software packages such as Fathom or Excel.

I asked Cecilia why she uses personal computers and avoids using graphic calculators. She mentioned that the old graphic calculators had limited functionality. In addition, there are many free software packages that can be easily installed on personal computers and simulate various graphic calculators. Another reason was that for personal computers, the school board offers technical maintenance. Also, personal computers offer more security than graphic calculators:

We occasionally get questions whether they understood the need to have graphic calculators. We are lucky that if we want to use graphic calculators, we are able to use them in class. But for graphic calculators there is a lot of software [for personal computers] now that can replicate the same thing. We are able to say to the students “you can go to this website and it is good.” Software that you can use in most cases ... we recommend software that is free. So the students don’t have to purchase the graphic calculators for themselves. And the security issues around the computers. Because these computers are stationary, as the school board tries to maintain these computers for us, it makes it easier to take care of the technology. (interview, Dec. 16, 2009)

When I asked her how the school is managing the issue of maintenance, she mentioned that some teachers are qualified to do some basic maintenance, while the most difficult aspects are treated by specialists from the school board. “We do have teachers in this school who have some specialized permissions, but for the most part we have a contract with the board. So that the students do not necessarily have to purchase graphic calculators or computers, they are offered here” (interview, Nov. 18, 2009).

In particular for the Data Management course, she gave numerous examples of ways in which computers tremendously help students in the process of understanding statistical phenomena:

Let us take statistics as an example. It is very easy to ask students to calculate means and data deviations just using a regular calculator. It is good for us to verify if there is understanding or not. How those measures are done. But, in order to get into deeper questions such as what the data tells us and why these pieces of information might make sense, you need to have a much larger data set. Or to find some real data is difficult. We have been talking about hundreds of thousands of data and this is where the technology really plays an important role. I know you know how to calculate means. To go that step further, try to put some meaning to the calculations or to the result of these calculations. This really values the technology. (interview, Dec. 16, 2009)

For the final project, students were requested to write the analysis in a Word document and also make a PowerPoint presentation. The simulation for the game was supposed to be done with either Excel or Fathom and all the theoretical aspects, the analysis, and the explanations were to be given in the word document.

They had to use spreadsheets in order to record the data from playing the game. They could either use Fathom or Excel to analyze that data but also to do some simulations. So they will have to use a word processor, to write the analysis. I asked for a paper because for me it is easy to mark it. But for the simulation, it will be an electronic document that they submit it. They have to present how they designed these simulations and the connections between these simulations and the game. (interview, Nov. 18, 2009)

In testing students' knowledge, Cecilia regularly used specific tools called Smart Devices or clickers. These clickers were designed to assess students' understanding in real time. They help students to give their answers in numerical form directly. At the end of the test, they are asked if they are sure. After they confirm that this is their final choice, students can only see the problem but they are not allowed to edit the answers anymore. They can skip questions.

I saw clickers used by Cecilia for assessments. For instance, when students had a quiz in Data Management at the beginning of December, all students were using Smart Technologies clickers that were connected to a server, allowing them to introduce the

results in numerical form. The teacher was able to connect to the server and see at any time how many students completed a specific problem. The evaluation of the results could be done instantly, without any effort from the teacher. I found this tool very useful to save a lot of time and energy for the teacher and students.

4.3.5 Knowledge of Students

When I asked Cecilia about her students' background, abilities, skills and interests, she answered that the students from her two classrooms have different academic backgrounds. The majority of them took the previous course Functions, Grade 11, University Preparation (MCR3U) for the academic profile while some of her students took the Grade 11 course Functions and Applications (MCF3M):

In Data Management students are coming with various strengths. 80 percent took in the 11th year Advanced Functions course which is for the academic stream, and the rest of our students took Functions and Applications, so for them their experience with math is weaker than the others so when we teach data management, we do not assume too much and we have the possibility to do that because Data Management is not so algebraic as the Advanced Functions course. (interview, Nov. 18, 2009)

The present course called Mathematics of Data Management depends on students' previous academic backgrounds. More exactly, if students had previous credits in the academic curriculum, they manage the course more easily, while students from the applied profile struggle to adapt to this course:

It depends on the students. We have some students who are coming from the three year course, the Grade 11 Functions. For these students, this is a very easy part of the course. Working with formulas, doing calculations, they are comfortable with that. For other students, who are from the weaker three year program, students coming from the Functions and Applications course, they are having more difficulties; they need to follow multiple steps. It requires a little more analytical thinking and so for them this part of the course is a struggle. (interview, Nov. 18, 2009)

When I asked about students' skills, Cecilia acknowledged the increase in the number of students with computer technology skills. She said that, "as time goes on, more students came into my course with some experience with technology" (interview, Dec. 16, 2009). She confirms that now the students have increasingly skills in using computers:

When I started teaching this course five years ago, I would have less than half of my students would know how to use spreadsheets before. Now three quarters of my students come with some experience in using spreadsheets and even the ones who did not, tend to pick it up more quickly. (interview, Dec. 16, 2009)

In particular, she found Fathom easily accepted in secondary classrooms, as this product was not only user-friendly but also "classroom-friendly". Although most of the students had not used it before, they were able to learn it fast. "Most students did not use Fathom before. But this product is designed to be user-friendly. So they do pick it up quickly" (interview, Nov. 18, 2009).

Asked about the difficulties that students encounter, she answered that, especially for this project, it is hard to keep them on-track. For her, maintaining a website improves students' focus on tasks and clarifies many ambiguities:

For these students the most challenging is keeping them on-track. They need to be doing work outside of the classrooms in order to be prepared for the next chapters. And so for the findings from one year to the next it depends on the students I got. But some are much better in time management. I can see for some of the students I have right now their time management is a weakness. So this is part of the reason why I maintain the course's website. Because, as I know they must misplace things or they do not have the information they need at their fingertips or in the notebooks. So having a course website as another place where the student can look for information is important to me. There are slides students can look for the information when they are not in class. The reason is to eliminate some reasons to say they forgot these things. (interview, Nov. 18, 2009)

When she was asked about students' expectations, she indicated that sometimes students have problems focusing and becoming engaged in their academic work.

Therefore, computer technology might perform an important role of making students more interested in studying mathematics:

Sometimes I believe that students just want me to entertain them. They want to be engaged in a lesson and I think this is how technology plays a role in being able to see something move on the screen... Somewhat more interesting than watching me drawing diagrams on the board. I think it also enables me to do more examples informally. And also I can do more examples quickly. (interview, Nov. 18, 2009)

She recognized that the parents are important contributors to student achievement, as they are interested in improving mathematics education and are asking that technology be integrated in the course. "[Parents] are pushing for technology", mentioned the teacher. When I asked her about the way community and parents are involved in this school, she mentioned that they were interested in education, and that mathematics education has an important role:

In my interview with parents, I get the sense that they do value math education. They value education in general; they do believe that math is important. Our students take more courses than they are required to graduate. In this school, the number of credits that students complete in mathematics is more than the students we have in the school. (interview, Nov. 18, 2009)

The two Data Management classes regularly go to the computer lab. I asked if there was enough space and she mentioned that the number of teachers currently interested in using computer technology did not exceed the present school's technological infrastructure. She said that there were other labs available:

We have our cross-curricular labs in our school that might be used and also, for any of those teachers who want to bring the class into the computer lab we trade

classrooms. We try to accommodate them when they do want to use it. (interview, Nov. 18, 2009)

4.3.6 Involving Technology in Math Classroom Activities

In this section, I present some activities of integrating technology that Cecilia did with her students in the Data Management course in a normal class and in the final project, where all students from the three Data Management courses were brought together to interact.

4.3.6.1 Teaching an Unit about Statistical Distributions

Usually, the lessons are structured such that Cecilia first gives the theoretical part and then asks students to solve problems related to the current unit. Cecilia not only likes to give mathematical problems related to probability and statistics, but also gives problems from real life. For instance, once she collected pennies and tried to predict with her students how many coins were from Canada and how many were from the US, or how many have been issued after a specific date and how many before. After students made different predictions, the teacher asked her students to check.

The teacher gave the following problem to the class: what is the probability that, throwing a coin 100 times in the air, one would obtain tails 50 times. Students started thinking about the problem. However, someone speculated that 50 is half of 100 and whispered “one half”. The teacher tried to explain rigorously the entire path of possible cases and showed why the result is not one half.

In another hour, students were given problems with different degrees of difficulty about different types of distribution. The problems were presented on the SMART Board and the students tried to solve them. Some of them went to the table and presented by writing on the whiteboard.

4.3.6.2 The Data Management Project

The students were highly motivated to work on the Casino Project and therefore were willing to dedicate a lot of time and energy. In this project, students were required to design a casino game. Each student was asked to design rules, to make it clear the situations of gain and loss in the game. Overall, the game should allow the students to make profit, but only a small amount. The students had to work several weeks in advance in order to make the game's scenario, calculate the cases and probabilities of gain and loss, and demonstrate the theoretical gain of the game.

I asked Cecilia to describe the final project in the Data Management course, in which all three classrooms participated, and provide more details about the project:

The students were asked to design a Casino game. So they had to incorporate some sort of random event. It could be rolling dice, drawing cards or a spinner and then use probability to analyze the game that they designed and then set an appropriate prize for different outcomes so that the game was almost fair. They were supposed to make a little bit of profit, theoretically. In the long run, they just gain a little profit. What you saw on Tuesday, the students physically built the games. We invited other students in the school to come and play these games. (interview, Dec. 16, 2009)

I witnessed the moment when the Data Management students were introduced to TI-Nspire to work on the Casino project. Before, they were familiar with older graphic calculators such as TI-85. There is a great difference between these two products, as TI-Nspire has many additional facilities. For instance, TI-Nspire is able to save the work made by users into files. Also, TI-Nspire allows the user to incorporate fractions and complex algebraic formulae. For working with statistical data, TI-Nspire allows a greater variety of financial functions and also to work with tables and worksheet files.

Cecilia explained the process of collecting the data obtained from playing Casino games and providing it to her students for their project. In this way, students could collect

the data obtained from playing these games and merge the collected results with the data generated randomly. In the final stage, they had to verify if they were about to obtain a small profit. Based on these estimations, they could modify the project so that the final outcome for the game could provide a small profit.

From these TI-Nspires, we could then transfer that information into a spreadsheet format, very easily. I took the information from all the calculators and put it into an Excel spreadsheet where the students could then retrieve it. I filled up a set of files for the students and then put them into what we called pick-up folders. This is on the TDSB server so the students have access to this, they could retrieve their own file. What the students are to do next is to take the experimental data, analyze it, and compare to their theoretical expectation. In some cases, because they are still working with them, they had a relatively small sample. They are also being asked to simulate their games so that they have a larger number of trials, and hopefully either confirms their theoretical calculation or in some cases find mistakes and correct these mistakes. (interview, Dec. 16, 2009)

For unsupervised users, the adaptation from TI-85 to TI-Nspire might take days if not more. However, the instruction of these students from Cecilia's Data Management classes was done pragmatically in a very short period of time. They were first given a sheet of paper containing new commands and options that were necessary for their project. They learned these new opportunities to use them in a practical way and had the opportunity to integrate these new options in their work on the project.

For the Data Management project, students were requested and instructed how to do a PowerPoint presentation. First the students listened to a video clip on the Internet where a person was giving advice about how to make efficient presentations in PowerPoint. The video was shown on the SMART Board so that students could see the presentation on the large screen. However, the teacher and students found that the video presentation did not offer much advice so overall it was not very effective. Also, the quality of the video was low. After the first class was not satisfied with the video

presentation, the teacher gave up on the video and, instead, presented the guidelines herself. Her presentation was much better than the video.

When students experimented creating a PowerPoint writing their projects, they used a large variety of mathematical formulae, fonts, images from gaming and casinos, effects, and statistical models to adapt to the project. The teachers encouraged them not only to experiment with all these different types of content but also to communicate their mathematical ideas in the classroom. After the presentations, Cecilia encouraged students to constructively criticise their peers' projects and gave points to those who gave good critiques.

After the students designed the game and presented their PowerPoint presentations on SMART Boards, they were required to present their work, and to perform the game with other students and teachers in the school. They were also requested to collect data. All three classes that were currently registered in the Data Management course went to the gym together with their teachers, Cecilia and Mark. The atmosphere was typical for a casino. Many tables with vividly coloured posters were spread out in the room. There were various casino objects such as spinners and cards. There were also dice with different numbers on the surfaces (4, 6, 8 or 12) and colours. I describe the activity in my notes:

At the entrance, some students give us some fake paper money to play with. Students were dressed up like casino game dealers and presented themselves as professional players. They had their own amount of money. If someone took a hard look to observe the casino room, he or she would notice something unusual for a casino game: students registered every result in TI-Nspire. Almost all students were behaving very enthusiastically. Other students were joining them to play at the casino. Other teachers, not only from the mathematics department but also from other departments, joined the students to participate in this event. The "game dealers" tried to persuade 'customers' to learn their games' rules and asked us to play. (Field notes, Dec. 9, 2009).

I started playing at each table to lose and gain, depending on each rule. However, I felt like I won too much. Was it because they let me win? I heard when the game dealers commented loudly on the events as they happened. The teachers playing in the casino gave evaluations of each game they tried. Hence, it was very important to imitate the casino atmosphere and especially to collect the data about wins and losses.

In the process of creating the projects, students used the new TI-Nspire graphic calculators. These were essential because they allowed students to save data. Cecilia gave detailed arguments why this model of graphic calculators was important:

The games being played, the students were required to record the outcomes of their game. So we were making use of the TI-Nspire because they have a spreadsheet component on their calculators. They could save the data and also with one calculator you could save multiple files so students could share among them this piece of equipment and yet be able to keep their information separate. (interview, Dec. 16, 2009)

4.3.7 Further plans

Asked what new technology she intends to bring into the classroom, Cecilia mentioned that tutorials such as Gizmos might play an important role in helping them to understand math concepts.

I would like to incorporate more applets and Gizmos. Those have a place in helping the students to review or reinforce some concepts. We discussed [these] in class. The reason I am not using them is because of time. I have not been able to find which application will be fit for my students. (interview, Dec. 16, 2009)

She acknowledges that there are some shortcomings with Gizmos, as these products might offer some tutorials that are not suitable to current students' experiences.

This is right. If I give to my students a list of ten, some of them might consider I've given them too much, and they might not look at any of them. But if I give them, let's say, a short list of two or three, they might look at them and direct them and then I think they will be more able to make more connections between

what we discussed in class. And also they will be more likely to look at it and explore. (interview, Dec. 16, 2009)

Asked how other teachers would start integrating computer technology in the classroom, Cecilia mentioned that for the provincial curriculum, integrating technology is only a recommendation and not mandatory. Therefore, for secondary school teachers considering integrating technology in mathematics classrooms, the textbooks in use already present ways of solving problems by using computers and graphic calculators. Therefore, one of the more affordable solutions might be to start reading and practicing exercises presented already in the textbook:

For the math department, I think we ... there is not an expectation that says everybody must use it. The curriculum is saying that we should be using technology. So, part of this is finding the resources to support our teaching. In our textbooks, there are exercises or activities that are geared for technology. Therefore, for teachers who might not be comfortable with technology they can take some that are already written, prepared and use that for their students and learn at the same time with their students. (interview, Dec. 16, 2009)

I asked Cecilia if she has any preferences for collaborative software such as Web 2.0 technologies. She mentioned that, in her case, because she meets her students each working day, using Web 2.0 collaborative technologies might not be so important. Also, she mentioned that Web 2.0 technologies do not offer adequate support for a public educational setting.

I like the idea of Web 2.0 but I do not have personal experience with Web 2.0 technology. I have been in a workshop where they showed how to contribute to a wiki. In my personal experience, I am not convinced that it is beneficial to use it in my teaching. Because I see my students from Monday to Friday, so if they have a question they might be more likely to talk to me to add something to the discussion. Maybe in the future I will incorporate but not in this moment. (interview, Dec. 16, 2009)

She believed that the current technology might be enough from many points of view. Laptops were not viewed as important, as in Mark's case. For her, maybe another lab with desktop computers was better than a mobile cart brought into traditional classrooms containing laptops. "For laptops, I am not personally convinced. I think it is better to have a stationary lab it is a little bit easier. Because we know where information is". (interview, Dec. 16, 2009)

She also mentioned that she tries to bring some video clips to show to students when learning new concepts and skills is required:

I have, in the back of my mind, other pieces of technology that I would like to bring to the course. Some video clips that are available online in some cases and also some online applications are available. I am always worried about what the benefit is of using another piece of technology and, also, am I overwhelming the students. I always make a decision between what I would use for my benefit, in getting the point across to the students versus whether there is a piece of technology that is needed for them to learn and be able to use on their own. (interview, Dec. 16, 2009)

4.3.8 Summary

Cecilia relies heavily on computer technology for teaching. She uses the SMART Board with PowerPoint presentations in order to give summaries of the lessons for her teaching almost daily. She has excellent skills in time management and classroom management. When I asked her about the role of technology in teaching mathematics, Cecilia believed that technology helps students to explore different ideas and hypotheses. In teaching her students, she relied mostly on personal computers and less on graphic calculators.

She designed and maintained a website in which she developed resources for the current course in Data Management. The technology changes and so does the way students communicate their ideas and results. Students often consult their computer

technology tools, so that when they interact with others they often do so by referring to these tools, be they computers or SMART Boards. Cecilia looks to the Internet in order to provide supplementary materials for her students with various digital resources for learning and assessing. Without reducing technology to digital resources, she sees one of the roles of technology as being able to deliver digital resources so that she will give to students only customized answers. In addition, having students in Grade 12, they are supposed to ask her for clarifications.

Cecilia has a strong computer technology background linked with mathematics. Therefore, her TPACK was formed by combining the three components (mathematical content, technology, and pedagogy). Prior to teaching in this school, she had a graduate mathematical background, experiences using computer technology since she was a student, and opportunities to link mathematics and technological ideas. Her TPACK was developed by using technology in assessments, presentations, and creating student facilities to experiment with their mathematical ideas.

4.4 Mark

4.4.1 Background

Mark is a mathematics teacher with over 15 years of experience. He was born in an Eastern-European country, where he did his undergraduate in engineering. Before coming to Canada, he was a teacher in his native country in elementary and high school where he taught mathematics and science. He had to work hard to become a qualified teacher:

[In my native country,] I was a math teacher for 4 years. I did not graduate from a teaching program. Here is different for the majority of teachers. You first graduate from your major and eventually you go into the Faculty of Education. My case

was similar because I studied engineering, mathematics. After I graduated, I did not find any job that interested me and I started applying for teaching. Soon, I was teaching full time. I was going [to teaching courses] Saturday and Sunday to obtain additional qualifications and to obtain the teaching degree. (interview, Nov. 5, 2009)

After arriving in Canada, he completed his bachelor program in mathematics, obtained a Master's degree in mathematics, and started his doctoral program in mathematics. At the same time, he applied to obtain the credentials for teaching mathematics. After he received a certificate for teaching in Ontario, he decided to become a teacher of mathematics in secondary schools:

This is my 14th year in Canada as a full time teacher but I worked as a part time teacher before being able to get a full time teaching position. I also worked as a teacher assistant at the University of Toronto for a few years before that, and in my home country, for four years as teacher. In university, I got my master in engineering degree in my home country and, originally I started working outside of teaching but not in my actual professional area. Then I switched to teaching. While teaching, I took the equivalent years in the Faculty of Education, which was only weekends there, and I was a teacher in the elementary school. (interview, Nov. 5, 2009)

Coming to Canada, Mark started to go to school, taking the process of certification in teaching from the beginning. Step by step, he updated his credentials in undergraduate mathematics and pursued graduate studies in mathematics. He obtained teaching qualifications in mathematics education at the intermediate/senior level and looked for a teaching position. After he got his first offer, he gave up on his PhD program in mathematics:

After coming to Canada, I tried to find a job as a teacher but it was really hard times at that time. I was not getting any response from the boards. I am not quite sure who told me to do this, but I decided to start it again [going to school]. So I took a master degree in pure mathematics from the University of Toronto. I did a master degree in mathematics; I guess this was because of my background. Before leaving my home country, I did study engineering which is applied mathematics;

so my program wasn't probably so challenging. I finished very fast the master and I decided to register in the PhD program in mathematics at the University of Toronto and I spent three years trying to get there. Meanwhile I was always keeping in mind you know, going back to teaching, and at some point, I accepted the offer to become a full time teacher. And I took it and I have not completed that PhD program in mathematics. (interview, Nov. 5, 2009)

Mark started teaching in this school in 1997, after he obtained a permanent position as a full time teacher in secondary mathematics. Since this time, he taught all secondary school courses, including academic and applied courses. During the data collection, he was teaching mathematics in George Goodwin School. He is also a curriculum leader and the head of the Department of Mathematics. In the semester of fall 2009, he was teaching three different grade 12 courses: Advanced Functions (MHF4U), Data Management (MDM4U) and Calculus and Vectors (MCV4U). All three courses are required credits for university programs in Ontario.

He is a self learner in the use of technology. He started to learn about computers very early on, in 1990, when he arrived in Canada, at a time when the price of computers was prohibitive for many people. Mark became computer literate and started to learn different mathematical software while he was doing his graduate studies in mathematics. At the beginning of his teaching career, he attended a workshop on the Geometer's Sketchpad. Recently, he attended some seminars on SMART Boards and wikis. He feels that he has become very proficient in using technology in classrooms. Consequently, he is one of the school board math teachers' leaders in instructing the integration of technology in mathematics classrooms and offers workshops for inservice teachers for his board.

Mark remembers his first steps in learning the Geometer's Sketchpad in order to help students learn geometry. He remembers how the technology evolved. However,

changes in curriculum cut most of the geometry from the mathematics curriculum, which decreased the usefulness of the Geometer's Sketchpad (GSP):

My first technology learning in teaching mathematics was in a workshop. I took a week long to learn the Geometer's Sketchpad workshop, during summer between my first and second year in teaching here at this school. I have used it a lot since, and the support of technology was not as good like we have here right now. We have [now] our own math computer lab; we did not have that lab before. So it was harder then. In these days, I do come to classes with students. Now most of the geometry was removed. A lot of the Geometer's Sketchpad was not necessary because the context changed. Since then, most of the other technology is my initiative. I want to learn it. It is my initiative. (interview, Nov. 5, 2009)

Mark is well known for his expertise. He has been regularly offering workshops for teachers and has been participating in new training sessions for some time. His name was mentioned as advisor in an Ontario textbook and his opinion was requested by Ontario Ministry of Education officials when the provincial curriculum in mathematics was recently changed.

4.4.2 Overall Strategies of Integrating Technology in Classrooms

For Mark, technology becomes essential when it effectively challenges students' thinking and gives them opportunities to build new knowledge:

I do not necessarily think it is different than nontechnology. Constructing knowledge... Technology, mostly, how I understand, is usually visualisation. You care to cover the goal simpler with technology than you do otherwise. I think that this is part of the answer too, that you can use the most sophisticated approach to knowledge construction. But is it specific for technology? I do not know. I have the general feeling... You do not tell them what to do all the time. You only let them try to discover so they can build the whole knowledge. And the concept they hopefully got is the concept that they wanted. In this way, they challenged themselves and I do not think for me it is technology specific. I like it more in this way. (Mark, interview, Dec. 18, 2009)

An important step in integrating technology is PD activities for teachers. These are useful in informing and motivating teachers to start inquiring about ways of adapting the new software to their classrooms. However, this is a decision that should be taken individually by each teacher. Teachers should decide if the new software is useful and would find ways of integrating it into the present curriculum:

How else can I enhance the classroom with technology? [First,] I attend a workshop. But a couple of hours of workshop are not going to teach you how to use it. It is going to ignite your interest, maybe, and you have to decide whether to try in classrooms. You have to make an effort to learn it enough to try it in the classroom. Many teachers are afraid of using it because they are saying, 'well, I have to master it first'. And I had a hard time convincing teachers to use it for classrooms. You do not need to know it very well. You need to introduce it in the classroom and the students will take it on their own. (interview, Dec. 18, 2009)

He gave the example of Lawrence who, being supported to integrate technology in his classroom, was able to adjust it for his classes, although Lawrence's desire to master every minute detail might delay his plans of integrating new technologies into classrooms:

He is an example in this sense. He was somebody who said some time ago when I tried to convince him to use computer technology in classroom. He was saying 'No, I have to learn it first. But he did make an effort, a few years ago, when I asked him to use Fathom in data management classes. He made the effort but he did it in his way. He learned it first and then he studied it in class. But he realized that I was right in the first place. That he really did not know it as an expert... He needs to know something, a little bit; how to turn it on and how to run the program. That's about it. Now he is doing it. He does not need to be an expert in that specific technology. Nobody needs to be an expert but everyone has to take the risk. (interview, Dec. 18, 2009)

Although he acknowledged the importance of technology in mathematics, he did not see technology as replacing the abstract part of mathematics. He recognized that, due to limited time, it is difficult to link abstract mathematics with real problems. When I

asked if he sees any adjustment between teaching the abstract part of mathematics and using real-life math, he mentioned that “I think that is a problem. We have a limited time” (interview, Dec. 18, 2009). He offered teaching geometry as an example where he sees it as a way of teaching pure mathematics and deplors the severe reduction of the geometry curriculum:

There is a lot of push to use technology which I agree with. There is a lot of push to teach math only in the context of real life. This is where I have a problem. I am not saying that I disagree with teaching the mathematics of real life. I do agree with that. What I disagree with is taking off the art for art part of mathematics, the abstract part. Because there is nothing wrong with teaching the completely pure abstract part either. Geometry is an example. Students these days who are graduating high school almost have no idea about Euclidian geometry. That is an example of what really pure math and art for art is. There are parts of the courses where we are being told that students are not required to prove things mathematically. I think it is wrong. It is okay that not everybody has to do these. But to remove it completely from expectations, I kind of disagree. So I see that how the real math is delivered. That is the push for using the real math. (interview, Dec. 18, 2009)

While I observed Mark teaching, I noticed that the abstract part of mathematics does benefit from technology. For instance, he used a lot of technology when he taught geometry in the course Calculus and Vectors. He taught using Fathom and the Geometer’s Sketchpad software. He was committed to integrating technology and to keeping the most difficult parts of mathematics in the course: abstract math, math as an art, and living as a mathematician. I asked for details about specific ways of teaching, what his goals are, and how he is trying to get these objectives accepted:

I would like to have the whole picture. I want to know what the school is... Where the main efforts are going. ... What the ministry is trying to do. I am trying to be aware of these changes in the curriculum, and changes to approaches of delivering the curriculum. Sometimes, it is a struggle to discuss with parents because some [parents] have not been to school for years. There are also other challenges. To give you particular examples, about ten years ago, the ministry introduced the new assistant evaluation approaches. It was written into the

curriculum how we are supposed to evaluate the students. Not just what the students are supposed to know at the end of each course, but how it is supposed to be delivered and how it is supposed to be evaluated. As I said, this was ten years ago and the debate is still on. There are schools where still this has not been accepted. (interview, Nov. 5, 2009)

Mark acknowledges that the role of technology is very important in helping him to teach mathematics. However, when he tries to convince other teachers to integrate technology in classrooms he encounters resistance:

I think that the use of technology is really great in the classroom. You know that. I love doing it. You see me using technology and I am using a lot. So, the problem though is we are getting bored. You can encourage other teachers to do it. But they have to decide on their own. ‘Yes, I am going to do it!’ Takes time to do this. (interview, Dec. 18, 2009)

Mark considers that technology might greatly assist students in the process of understanding mathematical concepts. Technology saves a great amount of time when students are required to graph functions:

...any concepts; other than for visualising, what concepts really mean. Other than that, it also takes time. I’ll give you a simple example, say about ten years ago, when I had to teach lessons on parabolas in vertex form, when students were investigating how the shifts in the left-right-up-down or the compressions and stretches how they changed that, they needed to graph by hand and it took a long time for students to graph one set of parabolas and then they needed to do it several times to be able to do a conclusion. And now with the technology we can save a lot of time graphing. Yes they do know how to graph by hand, I am not arguing that; but once they know the actual concepts there is no way of going back. This is so much better. (interview, Dec. 18, 2009)

For Mark, the technology is not helpful when someone is trying to substitute the “beauty of thinking” on mathematics with computer technology. However, the technology might help in understanding even the most abstract aspects of mathematic thinking:

What the technology does not discuss is the art part of the mathematics where there’s beauty of just thinking about the mathematics problem and sometimes we

do not want the technology to spoil the thinking, and it is possible. I remember when I was a PhD student, one of the students was using a software called Mathematica to help himself with answering some problems, and I found it a little bit like cheating. Well, I am not so sure now, but all I am saying is that sometimes, there are areas of math where the technology cannot go, or it is not there yet. But the question is: do we need it there? I personally would not do it. (interview, Dec. 18, 2009)

4.4.3 Using Technology in Teaching, Interactions, and Communications

Teaching mathematics in the classroom, Mark uses a variety of styles of teaching. Sometimes he chooses several software packages in the same lesson to explain different mathematical aspects to students, and other times he does not use technology at all and just shows them on the blackboard how to do different proofs. He often uses the SMART Board for short quizzes, when he shows each question to students and then discusses its theoretical aspects thoroughly with the class. Different from Cecilia, he does not use PowerPoint slides to show theoretical aspects and concepts from the current lesson.

He mentioned that step-by-step instruction in teaching technology does not always work. Therefore, he recommends understanding first the global picture of the lesson and mathematics unit. “The instructions step-by-step are there but sometimes these do not work. But the thing is if you use technology regularly, you are going to face these problems regularly, and you are going to understand how to understand these problems” (interview, Nov. 5, 2009).

Mark knows how to use the SMART Board very well. He has both taken and given workshops on using the SMART Board in the classroom. I often saw him using the SMART Board in teaching new concepts or computer techniques to students. Sometimes, he writes on the SMART Board with special markers designed for this purpose. Although he appreciated the possibilities that the SMART Board might offer in order to visualise

concepts and make them more accessible to students, he believes that the SMART Board is prone to be used in a teacher-centered style and not as a student-centered option:

With SMART Boards, I do have some reservations. I love using it but it is mostly for presentations. I make some effort to use SMART Boards in engaging students but it usually ends up with me being the presenter in using SMART Board in a smart way. (interview, Dec. 18, 2009)

Most of the moments when he uses the SMART Board are typically for showing some problems solved by using specific mathematical software commands, or when quizzes requiring clickers are given to the class. He brings a personal computer, a monitor, and a projector on a cart. This cart can be easily transported and this gives a lot of flexibility. It is easy to bring the cart into a traditional classroom and change the possibilities for learning and teaching. Together with a SMART Board, these radically transform a traditional classroom. In all of his three courses he had several periods when he did not use SMART Boards at all.

4.4.4 Technological Knowledge in Curriculum Planning and Assessment

In line with Lawrence's reflections, Mark acknowledged that cutting Grade 13 from the Ontario curriculum left students unprepared for university, in particular for mathematics. Mark described his concern:

The curriculum changes, so it may not be simple to compare the level of achievement but, when I started working in 1997, we still had that five-year program. The students were more mature. Then we had to change the programs. There are now visible changes in our school. (interview, Nov. 5, 2009)

For this reason, Mark recently tried to offer an enrichment preparation in mathematics for Grade 12 students of his secondary school, in order to give more academic content to students focused on studying mathematics. He mentioned that these

enrichment plans were seriously challenged by other curriculum leaders from his school.

He had to insist and give them reasons to accept this plan:

There are challenges, absolutely. One of the challenges is that I cannot simply do what I want to do. Because the bottom line is when we want to do this initiative, which requires the entire school, the administration has to be aware of it. When the administration has to be aware of it, the entire administration team usually has input. Some committees usually want to have input. For instance, we came up with an idea of offering enrichment classes for Grade 12 a year ago. The way we wanted to do it was to offer an advanced placement, calculus and stats courses. We faced a lot of opposition. I am not going into details. For some reason, the other curriculum leaders were strongly opposing them. Not all of them, but part of them seemed to be opposing this idea... We did put a lot of effort into this. The advancement program that we tried to use before, we are offering right now in one class of calculus. But what was challenging was to convince the rest of the curriculum leaders. That this is going to be good for our students, for the benefit of our school. So, the challenges are whenever you come up with an idea, it goes outside of the department. Even if we agree within the department, things are usually bumpy. Challenges are stuff involved in other departments. But these challenges are not always great. But if it involves the rest of the school, the rest of the school is challenging these ideas. (interview, Dec. 18, 2009)

Mark acknowledged that anxiety towards mathematics is widespread, so one of the main concerns was to make students feel comfortable with learning and practising mathematics. Another important goal was to help students achieve their potential:

I try to make sure that students do not mind coming to my class, so they are not stressed; so I try to make them enjoy the class.... People are scared of math and probably there is a reason somewhere during the past. We have a full spectrum of people with abilities and interests. I have somebody with very strong ability and very strong interest; just getting a credit is not success in my opinion. We have on the other end, one who is struggling so we have to make sure that students are getting enough credits. (interview, Dec. 18, 2009)

Mark mentioned that he has witnessed numerous changes in the mathematics curriculum and in the school structure in general. When a new mathematics curriculum was designed, he was consulted in order to give feedback to the provincial governmental committee that authored the curriculum. He was a consultant for one of the textbooks in

secondary mathematics. Although he loves mathematics and teaching mathematics, he sees his personality more as a mathematician. He loves doing mathematics and the art of thinking in mathematics. Having limited time to do abstract mathematics, it is difficult to connect abstract mathematics with the ‘mathematics of real life’.

Therefore, for him, removing the proofs from the mathematics curriculum was problematic, as it might seriously impede potential candidates from selecting a career in mathematics sciences.

I have seen changes in each year... the last time [the curriculum changed] I was involved in giving feedback to the minister so I was able to know [what changed] even before the curriculum would be released. I am a mathematician and I am a teacher. If you are asking me what I am more, I think in my heart, I am more a mathematician than teacher. This does not mean that I do not like teaching. I love teaching but I love math. What I see happening is that the changes take more and more out of math like techniques and the art of math. Unfortunately, for people like me, mathematics becomes less interesting. Potential mathematicians are not going to do it because they are not going to be exposed to know what mathematics really is. (interview, Dec. 18, 2009)

While talking about success in learning mathematics, Mark mentioned that the success is beyond getting credits. Success should not be a rigid standard but a personal evaluation:

I think I have like a continuous perspective of what success is. Success is not very popular in education, in Ontario, at least. We need to deal with students who are having problems accumulating credits. We need to agree that this is very important part of the program of what we are doing or what we should be doing. But we should not be neglecting everybody else. So for me, success is reaching the potential of the kids, the potential of the study of acquiring mathematics knowledge beyond curriculum expectations of what would be successful for that student. If the student is struggling with getting a credit, the success for that student is getting a credit. (interview, Dec. 18, 2009)

Mark attempts to see the connections between various parts of the curriculum, even though these are units from previous grades. For instance, he described a problem

from Advanced Functions directly related with problems of maximum and minimum from Grade 9 and Grade 10:

In Grade 10, we are interested in maximizing and this is a Grade 10 problem. You have to build something and you have to do it by using some materials. So what I do, I use either Fathom or Geometer's Sketchpad or both. Actually, both are able to do that. What we do, or even in Grade 9, we also maximize functions. We build the bottom line of the graph... to gather data that from experiment. And then, what you do is to separate or to juxtapose the parabola, or whatever it takes. Now, we can use the technology to manipulate, in a continuous way, the shape until it fits in the best way with the central data. This is absolutely one of the best ways of using technology. Even better than using the quadratic functions in a calculator... You see the spot where the parabola is until it fits there. Or in data management when you have a scatter plot and you have to predict or to extrapolate and the students seem to perceive right away: 'This is it'! (Mark, interview, Dec. 18, 2009)

Mark alternates the use of SMART Boards with writing on the blackboard. In each course, he takes most of the exercises from textbooks. He mentioned to me that these courses gave him different ways of interacting with students and managing the classroom. For instance, while the Calculus and Vectors course was, in many regards, more structured, the Advanced Functions course allowed him to develop more interactions among students. He describes the courses:

I don't know if you're aware of this but, in Advanced Functions, it is more experiential. Students are more in control when they learn it. In Calculus and Vectors, I'm a little bit more in control of some of the topics, because I'm not introducing the topic. Most of the time it is just the introduction part. ... Also, it is the calculus part, which is just repackaging what was done in Advanced Functions. There is room for them to make a lot of discoveries in the calculus part. I would expect to see as much as in Advanced Functions but not as much as in the Vector part of the course. (interview, Dec. 18, 2009)

Mark also used clickers technology in his classroom. He gives assessments often to his students and most of these require clickers as a fast way to give feedback to students and teacher about students' strengths and challenges. These devices were used in

all of his courses. He also used them to give students short assignments and see whether students understood the current chapter or not. I speak often with Mark when he brings the set of clickers to the classroom about advantages and disadvantages that they have. I asked Mark about his overall impression of clickers. He said:

I used clickers in tests but this is not the main reason why I like using them. Clickers for tests are fine but this is not how I believe that evaluations should be. I only use it for quick evaluations. They do not mean much but where I see the clickers' power is when I am teaching. Then, I can ask questions, the students can answer anonymously, and then we can look at how many students answered correctly. After, I can then make adjustments in the way I teach these concepts. Students are more open to using these in feedback. (interview, Dec. 18, 2009)

Overall, I saw him using clickers very often. He often uses clickers together with graphic calculators. He uses a software emulator for the graphic calculator model TI-85, the graphic calculators that the school is currently using. Coupled with the SMART Board, this emulator can show how the graphic calculator should be used in order to solve a specific problem required in the classroom.

Mark often uses short quizzes to test students' knowledge and then displays information using the SMART Board. Sometimes, the students answer by writing with water-markers on pads. Mark wants to read their answer, evaluate them immediately, and give them the response right away. These come like small TV show contests, where everybody sees the points obtained, giving students the opportunity to check the results. On the other hand, the water-markers became difficult to write with after 10 questions. In addition, to clean these pads properly and in short time is sometimes hard, as every time, the pads get more and more coloured.

In the following sections, I will present some ideas about planning curriculum in the three courses that Mark taught during the semester in which I interviewed him:

Calculus and Vectors, Advanced Functions, and Data Management. The last two will have reduced space as I already presented them when I described the TPACK of the other two teachers.

4.4.2.1 Calculus and Vectors Course

Mark is the only one at the school who teaches the Calculus and Vectors course. Mark told me that this course has changed over the years. Now this course is considered as the most advanced from the secondary mathematics curricula. Because the amount of time allowed for geometry has been reduced in all mathematics curricula, this course inevitably has some difficulties. This course, especially the Vectors part, might pose some serious challenges to his students:

There are difficulties with the Calculus and Vectors course I find ... and I think over the few years... that the students are finding the vectors part more challenging than the calculus. And, in my opinion, that is the easiest part. But maybe this might come back to your question about geometry. Students struggle dealing with concepts, they find it difficult and they don't know why. I find it difficult to explain to them [the reasons for] the right answers. I guess they don't have the background of the geometry. In Calculus and Vectors, I think the main goals were to understand the concepts better. They changed the curriculum ... these are really hard questions. Many students find it difficult when they see the textbook. (interview, Dec. 18, 2009)

Because there is not one software package able to cover the whole curriculum, for this course, Mark uses a variety of software. For the Calculus part of the course, Mark and his students work with graphic calculators. For the Vectors part, the teacher uses several software packages for different units such as the Geometer's Sketchpad, Fathom, Graphmatica, and 3DG. As the teacher recognized, these software offer some facilities for some problems, but none of these software programs offers the possibility to cover

everything. As Mark noted, “I use many other software like Graphmatica and I am still in search of other software for graphics in 3D” (interview, Dec. 18, 2009).

4.4.2.2 Advanced Functions Course

In the Advanced Functions course, Mark often uses the same textbook and graphic calculators as Lawrence. Unlike Lawrence however, he often uses SMART Boards, clickers, and personal computer software, such as Fathom and the Geometer’s Sketchpad. In addition, he uses a software emulator for graphic calculators.

4.4.2.3 Data Management Course

For the Data Management course, Mark uses the same textbook as Cecilia. Similar to Cecilia, he often brings his students into the lab, where they use software such as Fathom and Excel and he uses clickers. Unlike her, he uses graphic calculators (TI-85) and the software emulator for this model of graphic calculator. At the end of the course, he started using TI-Nspire.

Mark was generally satisfied with the present curricular content in Data Management and with the textbook. He often gave assessments in writing, taken mostly from the textbook and used clickers and the SMART Board. The Data Management course was taught in cooperation with Cecilia. Together, they formed a teaching team that succeeded in getting mathematics teachers involved and also other teachers from other departments and students.

4.4.5 Knowledge of Students

This school has a strong academic culture. Mark is very appreciated by the students for his expertise and extensive academic background. He was teaching Grade 12 academic courses, where it is expected that the students will pursue university studies.

Therefore, Mark treated his students as peers. The teacher often talked about the relations between students and teachers and how students should be proactive and speak properly with teachers. In the classroom, he is usually very close to students and often makes jokes with them. However, he did not allow them to do activities that were not related to the current mathematics curriculum. He encouraged his students to collaborate and experiment with different computer options in his presence and ask him for advice. He was always very busy. He sets time outside the regular course periods for consultations and these periods get booked very quickly, as he is known for his high level of expertise in mathematics and integrating technology.

I asked Mark how he sees the support of the community for technology. He acknowledged the interest of the parents from the Parent Council and administration to support education, and in particular mathematics education, by using technology in school. Also, this attitude is reflected in the increasing number of students who have computers at home and are computer literate:

It seems that most students already have computers home... This is how I know that parents are in support. As for parents in council, I do not know because I am not involved in Parent Council. I think we are supported enough by the budget to cover graphic calculators. We got our Math Lab Computers, but I think we need a little bit more. (interview, Nov. 5, 2009)

Being curriculum leader and department head, Mark was involved in teaching mathematics and in curriculum development, in creating technological infrastructure, and in familiarizing other teachers with computer technologies in order to make sure that the integration of technology in the mathematics curriculum is successful.

Mark praises his students' proclivity to learn and use technology. He mentioned that teaching with technology is challenging but students are part of the solution and not

only part of the problem, as their passion for learning technology helps them to figure out various challenges that might occur:

The students seem to perceive it right away! This is it! We don't even need to explain it to them! They will do it right away. The technology will do the part of the graph. How is technology used to predict the future parts of the graph? What is an independent value program? And if I don't know, we figure it out together. (interview, Dec. 18, 2009)

When I asked him what he expected from the students, Mark mentioned that he expects them to know the basics of using computers but that it is not necessary to know the mathematical software. If they know the basics of using computers, most of the products are easy to use. However, other software such as Fathom and Geometer's Sketchpad require specific training:

I want them to recognise what a computer is, but who doesn't these days? It is usually so simple. You don't need to know much about the software but just start using. For Fathom and Geometer's Sketchpad you need to know specific tasks and activities that you are doing. But in most cases everything is straightforward. (interview, Nov. 5, 2009)

I asked him about recent student projects that required the use of TI-Nspire. In the case of the Casino Game Project, he acknowledged the importance of graphic calculators as being the only portable device available to introduce and store information directly. He enjoyed some of the features of TI-Nspire:

I do not know much about TI-Nspire yet. We just purchased them recently. But the students used them in projects in collecting the data...The TI-Nspire can save data in the form of a spreadsheet. Students adapted very fast to this, because the students are technology savvy. This is what I must say. Do not be afraid of using technology. The students will guide you. Because the students are good at it. They were raised in the technology age. This is the way to go. (interview, Dec. 18, 2009)

The problem of technology cannot be limited only to the Department of Mathematics. The approval of technology requires other groups, such as Parents Council,

school administration, and teachers from other departments, to be involved. While the administration and the parents are sympathetic to the needs of purchasing new technology for mathematics, some teachers from other departments have serious reservations towards these initiatives:

Our admin seems to understand, that but there is less understanding among the rest of the staff. The teachers who teach other subjects... they do not understand why math would need technology in teaching while they still have a board and chalk image of mathematics. (interview, Nov. 5, 2009)

4.4.6 Involving Technology in Math Classroom Activities

In teaching, Mark chooses a variety of technologies available. Any technology that was accessible and useful for him was brought immediately into the classroom. Sometimes there are problems when the versions of technology used in the classroom are different from those in the textbooks. “We are sometimes using different versions” (interview, Dec. 18, 2009) mentioned Mark, as the same technology required different commands when the class uses a version different from that in the textbook.

I noticed that the majority of teachers from this school were using graphic calculators. For many hours, I saw Mark using graphic calculators, although he preferred the use of personal computers. I asked him whether he agrees with other teachers’ ways of teaching, for instance teaching exclusively with graphic calculators. He was of the opinion that, due to the increasing multifunctional opportunities of the mathematical software and to a continuous lowering in price of producing personal computers and mathematical software, graphic calculators are likely to disappear:

I always prefer to use computers over using graphic calculators. I still think it is a better way to go. Computers are more versatile and the prices are going low so quickly. I don’t see a future for these graphic calculators. The use of technology is written into the curriculum, and you know I am not against it. Other than graphic calculators, you really have very few choices. Other schools have a shortage of

computers and computer labs. You have to compete with other teachers for computer labs. We are fortunate here, because we have our computer lab but it is still not an ideal situation. So, as I said, I think I see it since computers became popular. (interview, Dec. 18, 2009)

In day-by-day instruction, Mark maintained a balance by being committed to using both personal computers and graphic calculators. At times, Mark was visibly more attracted to personal computers and the mathematical software. He preferred Fathom as educational software purposely designed for classrooms:

For data management, we use a lot of Fathom. I am a big fan of Fathom over Excel, although they can be used interchangeably. My students and I are using both... Well, Fathom is designed to be used in the classroom and Excel was not necessarily designed for working in classrooms as a purpose. Excel was designed to help accounting. But school is nothing what Excel people had in mind. There is nothing wrong with using it, because I am using it. But I prefer Fathom because it's much faster and has examples, for instance mean, media, standard deviation, or line of best fit curve for some sets of the data, the correlation between some sets of data. (interview, Dec. 18, 2009)

I asked him about ways of using the Geometer's Sketchpad and how he started to learn other software products by himself. I was interested to ask him how he succeeded diversifying learning and teaching for various educational software and settings. He mentioned that Fathom is well-suited to secondary school teachers when they use this in data management class:

I used them in both ways [teaching them and letting them experience it]. Then letting them do it would be better than a demonstration. But I didn't use these for demonstrations. First time, I used Fathom in 2001, when the ministry bought the license for Fathom and I think at that time I was teaching Data Management and it was the ideal software. And instead of graphic calculators, I would use Fathom. (interview, Dec. 18, 2009)

Indeed Mark was not satisfied with graphic calculators. While I was observing him in the classroom, I noticed that, although he provided students with graphic

calculators, he criticized the performance of the calculators sometimes. I asked him for details about why he prefers personal computers to graphic calculators. He talked about some moments when graphic calculators are inefficient for some types of graphs:

I do feel comfortable with graphic calculators and I do use them a lot. The problem is that there are a lot of shortcomings. I don't know if you were present when students were using them for logarithmic functions. The graphic calculators don't show very well if there is a vertical asymptote. There are some issues that graphic calculators have; there are some shortcomings. (interview, Dec. 18, 2009)

He mentioned that, although he often uses graphic calculators, personal computers are usually more effective for teaching and learning mathematics. Therefore, he was not sure that he would change much:

I see advantages when using computers. I think it was pretty successful in both cases, Advanced Functions and Data Management [courses]. I am going to do exactly the same next semester. For Data Management, I didn't teach for a couple of years but the curriculum was almost the same. I don't see how I would justify changing what I am going to use in Calculus and Vectors. We are going to use other textbooks, with some changes. I don't find many opportunities to use technology in acquiring knowledge as I do in Advanced Functions. Because it already builds in what you already know in the Data Management course. They already know the changes, so they should already have these in the new course. (interview, Dec. 18, 2009)

4.4.6.1 Advanced Functions Course

As technologies used for his courses, Mark often used graphic calculators and Fathom in order to help students draw graphs of the functions. He discussed exponential functions. In particular, he discussed the exponential function 2^x . He is drawing the graph of the function by using the emulator for the graphic calculator. He asks some questions to the classroom: "why is this function not linear?" Some students discuss reasons and explain why an exponential function cannot be linear. He asks them to choose some particular values and determine the value of the function. He gives some

examples of practical applications of the exponential function 2^x , especially in the case of using computer memories.

The teacher asks, “With the exponential function, what remains constant and why?” A student explains the answer. Afterward, the teacher draws the inverse function. The method used in this case is by reflection of initial function. After this, the teacher gives the students a short test with clickers. “No checking, please! Now, I’m not collecting marks”. He mentioned that this time the correctness of the answers is not important for him but for themselves. Now he is pretending to argue with a student joking that she did not do (again) her job of installing the PC and the SMART Board. After they install the SMART Board, Mark gives students a test with 15 questions. He is flexible with the time of test completion, waits for them to answer, and discusses the correct choices afterward.

One of the problems was: $\log_2 \log_3 \log_4 x = 0$. The solution was not determined by using a multiple choice approach. Students were asked to solve the equation, write the solution, and present each step. They could see their results right away. For those who missed the right answer, Mark explained at the table why $x = 64$ was the right solution. He showed each step of the problem by writing on the SMART Board. When he clicked on the SMART Board to select the next question, the table erased instantly.

Another problem was about the function $Y = 17 f(-3x - 7) + 17$. The question was about whether this specific function has a vertical or a horizontal reflection. The teacher asked two questions:

- a) Is there a vertical reflection?
- b) Is there a horizontal reflection?

After he gave these questions, the software displayed on the SMART Board how students answered. In addition to presenting them each answer, the teacher asked students to provide detailed answers and reasons for their choices. (Field notes, Nov. 3, 2009)

Occasionally, the logarithmic function did not display accurately on the graphic calculators' screens. Mark noticed this situation once in class and asked the students to check their graphic calculators. They noticed that the calculators do not perform well when the graph has a near to vertical asymptote and acknowledged the limitations of the calculators.

The graphic calculators in use are the model TI-85 devices and are known by every student in his class. The graphic calculator simulator associated with TI-85 is running on the computer and displays the graphics on the SMART Board. A student went to the personal computer and used the graphic calculator emulator. The other students followed the sequence of keyboard strokes displayed on the screen. The student finished the job and visualised the graphic by using the scatter-plot option.

Another problem was to study a family of functions for different values in order to approximate a specific function. He gave the general function $Y = a \sin [b(x-c)]+d$ and required students to calculate the parameters for some particular values. After that, he asked them to find the highest point and eventually try a guess. The students reported the following values for the parameters:

$$a = (23 + 14) / 2 = 18.5$$

$$b = 2 \pi / 13$$

$$c = 4$$

$$d = 23 - 18.5 = 4.5$$

After the students finished calculating the parameters, the teacher asked them to enter these parameters and draw these functions by using graphic calculators. The students noticed that the new function drawn is closer from the initial function and offers a better approximation. Mark used the Geometer's Sketchpad software. He writes a generic function on the SMART Board: $G(x) = a \sin[b(x-c)]+d$. He is interested to explore different shapes that the function might take for different parameters. The students discuss some particular shapes of the function when the parameters a , b , c , and d take different values. For instance, when $a=1$, $b=1$, $c=0$ and $d=0$, the function looks like $\sin x$. Now the teacher calculates some limits of functions by writing on the blackboard. Mark uses a lot of humour: "That's tangent but I do not care". Students laugh.

Sometimes Mark prefers to write on the blackboard and not use any computer-related technology. For instance, Mark wrote the whole demonstration to prove that $\lim_{x \rightarrow 0} \sin x/x = 1$ when $x \rightarrow 0$. After this, the teacher completed another demonstration on the blackboard, to demonstrate that the derivative of $\sin x$ is $\cos x$. Four tables were already filled and the class did not use any technological resource. In these cases, Mark discusses directly with the students and tries to show the proof and theoretical aspects of mathematics without involving any computer technology.

In the Advanced Functions course, Mark used mostly Geometer's Sketchpad and Fathom software. These software packages were introduced in Ontario schools more than ten years ago, although new versions have brought additional facilities and requirements:

I use Geometer's Sketchpad and Fathom for graphing. I use Fathom when we talk about trigonometric functions and transformations. I think that the lesson went really well. You observed that lesson and they showed good results in the following tests. We use Fathom pretty well in this case with several parameters.

Therefore, I'm not limiting myself to a specific software program. (interview, Dec. 18, 2009)

4.4.6.2 Calculus and Vectors Course

The lessons from the Calculus and Vectors course were taught in a variety of ways. Sometimes, he showed graphs by using Fathom or the Geometer's Sketchpad. Other times, there were several lessons in a row where no computer technology was used at all. In this case, the teacher was interested in discussing with students the theoretical parts and prove some parts of it. These activities required mainly writing on the blackboard. For instance, in one hour, Mark asked students to open the textbook to page 400 and solve some problems. The teacher explained the problems to students on the blackboard and did some demonstrations without any graphic calculators or the SMART Board. The computers stayed unused. In the last half hour, the teacher gave students graphic calculators. He asked students to calculate the maximum and the minimum of functions by using derivatives. The students, although from the high academic class, did not seem to enjoy these challenges. The teacher drew some graphs at the table and tried to give them more details.

While I was observing his classroom, I noticed that Mark does not limit himself to predetermined software products and is actively exploring other packages to test for other new facilities. For instance, in the last two units in Calculus and Vectors, I noticed that Mark used a new software program, called 3DG. Although he only started to learn about this software recently, he was able the following week to learn it and to explain in the classroom some of the geometrical spatial facilities. For instance, by using this software,

he showed his students how to use and graph planes. Mark showed this program several times to me and we had some time to discuss it.

Indeed, by using it in the classroom, this program helped students to understand how the planes might be used in space and how, by using computing tools, these plans can be drawn easily and with increased accuracy. Overall, he believes that the 3DG software is valuable.

It was something I just downloaded a week ago. I'm just testing right now. I figured that it might be helpful and it's not expensive; about 50 dollars per user. It's called 3DG and I can graph planes in 3D, I can change the perspective, and I can offer a very good 3D perspective effect. I can graph lines, planes, I can demonstrate where the lines of the planes intersect and also, you haven't seen yet, but I'm going to be using it for intersecting planes. I will use GraphCalc and I have been using it for a few years for that. There's a difference between these two. In the first one, I can enter an equation in the parametric form and, in the second one, it can be only in the explicit form. That gives me some limitations. Sometimes, I wish there was one a software for all, to use explicit and implicit forms and the parametric form in the vector way, exactly in the way we needed to do. But it's something for the ministry to do that I mentioned before. Why don't I have somebody hired to put these all together? But maybe it's not something that I'm aware of. (interview, Nov. 5, 2009)

4.4.6.3 Data Management Course

In the Data Management course, I noticed that Mark gave students opportunities to practice either on graphic calculators or on software programs designed for personal computers, such as Fathom and Excel. Most of the time, however, I saw students using Fathom. I noticed that Fathom offered very intuitive links between graphics and data and excellent tutorials. This might explain the preference for Fathom that all these three teachers, and especially Mark, have. Another important point observed in the Data Management course was that the computers made a great difference in collecting, using and maintaining large amounts of data:

With Fathom, the graphic is better and the correlation with data is more flexible. I'm not quite sure now if with more complicated regressions like exponential regression Fathom remains better. Fathom has some limitations but I think that helps, as I was giving you the example with the parabola. Again, we're not spending time on the mindless part that much. We have to plot for 50 or 60 points. This takes only a minute now. Now we can concentrate on the actual problem on the actual thinking part, to make conclusions. From what we are seeing, we don't have to restrict ourselves to a small set of data. We have data that has thousands and now millions of records. We can get it quickly. Also, we can just go to different websites, take the data from there, put it in Fathom and then analyze it. (interview, Dec. 18, 2009)

Sometimes there are some technical problems. Once, Mark had to reset the computer and to teach a part of the lesson writing on the blackboard. Another time, the current test had some ambiguities, so that the file had to be redesigned. Mark corrected the test and asked students to sign in again. Mark asked them to use graphic calculators. The problem/test was presented on the SMART Board. After the students answered the question, Mark wrote some details on the SMART Board, in order to discuss the answers. When Mark pressed the button called "more details", the students had the option to see in detail how they answered. The teacher discussed the questions, noticing how many answered correctly, and students had the opportunity to receive feedback immediately.

These opportunities to collect data would be impossible without using computers. In addition, the old models of graphic calculators, such as TI-85, were not able to store large amounts of data. Therefore, the new technology proved to make a great difference in collecting large amounts of data. This could be seen especially in the final course project. I asked Mark to describe the performances of his students in doing projects. He expects that his students should be able to design an event (in this case a casino game), evaluate the game with a large amount of data, experiment with statistical variables, and improve the accuracy and evaluating their chance of winning the game.

[Students] were required to have knowledge of counting, probability concepts. I think that my class worked really well. I don't know the complete outcome because at this time they completed only some parts. They did the theoretical model before, they did the practical part, and they collected the data now. They will need to analyze the data, find the outcome, and compare it with what they expected. Despite the conclusion, to realize if the practical and the theoretical is close, it's important whether [students] understand now what a variable means, how an expectation went, how a random value is used in making a value for a test, and how to design a game that will make you profit. Obviously they will achieve this because they will acquire this knowledge at the end of the course. (interview, Dec. 18, 2009)

He described the organization of the project: "In the case of the Data Management course, we did not schedule in our computer lab. So they used projections, they used PowerPoint, Word processor. But they use it. They wouldn't be able to do it without Fathom or Excel because otherwise the projects would be very difficult" (interview, Dec. 18, 2009).

4.4.7 Future Plans

Mark has a very clear opinion and strategy about computer technology. While the other teachers were satisfied with the existent technology, Mark tried to negotiate between what is possible and what might be brought in. He mentioned that laptops are one of his first priorities for the coming years. He was concerned with the technical support, so that all the devices could have several years of regular maintenance. Therefore, in order to ensure the necessary maintenance for the technology, he asked the school to pay a supplementary amount of money.

As for the short term perspective, he considered that many of the present courses taught will be continued in the same manner for the next semester, as the curriculum and the technology available in classrooms will largely remain the same:

I taught those courses before. I kind of expect the outcome, so I cannot find much new. I don't know if I would change much. For instance, next semester I expect new textbooks for Calculus and Vectors and I am going to continue the way I use technology as I have been using technology. (interview, Dec. 18, 2009)

I asked him about different software available. He was studying different types of mathematical software such as Graphmatica. "I have not checked out GeoGebra yet. In our department, we're going to have people who know a specific piece of software, showing the others how to do it. GeoGebra, I don't know yet. But I heard good things about it". (interview, Dec. 18, 2009)

When I asked him what he is considering to change in the future, he mentioned that:

I don't think I would. This [Casino games] project was performed by [Cecilia], and she did a great job. I am going to be teaching this course next semester. The satisfaction among students was good, and almost nothing is going to change. One thing maybe is going to change. More for real life, we are going to give them some chapters' from books that we know. Not necessarily from the mathematical point of view. But this is the only change. (interview, Dec. 18, 2009)

I spoke with Mark several times about the possibility of acquiring laptops. He mentioned to me that acquiring laptops is now much more possible, as laptops and notebooks have become more affordable. In addition, they are easy to bring into traditional classrooms and students could easily accommodate as they use similar products home:

There are some things about Internet-based platform. I would like to let the kids see the evaluations. I would like to hand out the instructions that they know how to do, the visual part, and the activity on our websites. I would like Grade 9 teachers to bring them often to see more lab activities which means that we need more labs. I would like to see another lab. I am actually negotiating with the principal for laptops. Not going in a specific lab but bringing laptops in the classroom, the other way around. (interview, Dec. 18, 2009)

Since the school board was heavily involved in using wikis in the process of education, and Mark himself had attended some workshops about using wikis in education, I asked Mark if he intends to use this Web 2.0 technology in the future. He seemed to acknowledge the theoretical importance of the wiki. Indeed, he considered wikis and Web 2.0 a space in which students receive and create new information but he was not ready to use them, yet:

I don't know much about it yet. I have been using wikis in some workshops. I think it's a good idea. This is the communication with students taking the same course, like a forum. Right? Not just a space where they take the information but also to put some information. Things are happening. Time is needed for that but the technology is coming. (interview, Dec. 18, 2009)

4.4.8 Summary

Mark is using a large variety of software in his classroom to provide students with learning opportunities, assessments, and teaching. Student knowledge has many different sources and technology is one of them. When I asked about his personal perspective on helping students to construct knowledge by using technology, Mark mentioned that for him, technology is not an isolated component but rather is integrated with many other mathematical and pedagogical aspects. Therefore, he shows very well-developed computer skills and adapts various techniques to a large category of goals, from nurturing abstract thinking to real-life problems.

In classrooms, he relies on both software for personal computers and graphic calculators. He has extraordinary mobility, being able to learn fast from different mathematical software. Experiencing various types of mathematics software, he is willing to share and present them to his students and to other mathematics teachers. Surprisingly though, Mark considers himself to be a mathematician more than anything

else. He regularly uses clickers to assess students' skills, as these technologies help him to provide fast and individual feedback to his students. Mark knows various ways of using technology and proved to be an early adopter of innovations in integrating new computer technology in mathematics.

4.5 A Summary of the Three Case Studies

In this chapter, I described the settings of the public secondary school in which the study took place. I presented three separate case studies of mathematics teachers integrating technology in classrooms. The table that summarises the findings for each case study is shown below. The cross case analysis and the final results are presented in the next chapter.

Table 1. Cross Case Analysis

Some comparative characteristics noticed after the cross-case analysis

Aspect	Lawrence	Cecilia	Mark
Courses	Advanced Functions (Grade 12)	Data Management (Grade 12)	Calculus and Vectors Advanced Functions
Academic	Applied Math (Grade 9) B. Sc. in Mathematics	Master of Science in Mathematics	Data Management (all Grade 12) Engineering
Background in Mathematics	and Science	Mathematics	Master of Sc. in Mathematics Started the Doctoral Program in Mathematics
Previous Experience in Computer Technology	Self-learner; Teacher in science and computer science; Took PD courses	Self-learner; Coursework in computer science; Computers in research; Took PD courses.	Self-learner; Use of computers in graduate mathematics; Took PD courses.
Teaching Technologies	Video Projector Graphic Calculator, PowerPoint, Word, Gizmos	SMART Board, PowerPoint, Word	SMART Board Graphic Calculator Mathematical Software, Word

Table 1 (continued). Cross - Case Summary

Aspect	Lawrence	Cecilia	Mark
Assessment Technologies Approach to	Gizmos (only with Grade 9 Applied) Cautious	Smart Technologies, Senteo Manager Early	Smart Technologies, Senteo Manager Immediate
Integrating Technology Learning Technologies	Graphic calculators (TI-84)	Graphic calculators (TI- Nspire)	Graphic calculators (TI-84, TI-Nspire)
Teaching Style	Manipulative technologies Short time periods of teaching;	PowerPoint, Word Excel, Fathom Lecturing	PowerPoint, Word Excel, Fathom Lecturing and discussions with the whole class
Preferences	individual attention		
Context	Applied, Academic	Academic	Advanced Academic
Approach to Explaining Problems Classroom	Main Aspects Student controlled	Main Aspects Teacher controlled	Theoretical aspects, Technological bias Teacher controlled
Control Relation to Students Future Technologies	Middle More Gizmos, Clips, Clickers	Distant Gizmos	Close Laptops, Web 2.0 technologies

CHAPTER FIVE: DISCUSSION AND INTERPRETATION OF FINDINGS

5.1 Introduction

The purpose of this study was to trace the role of the Technological Pedagogical Content Knowledge (TPACK) for these teachers, to highlight areas in need of improvement in terms of teaching practices, and to evaluate the usefulness of the TPACK framework generally. Based on this analysis, I identify challenges with using technology in classrooms and modalities designed to improve mathematical teaching skills.

The structure of this chapter is based on four major sections. First, I will answer the research questions. Next, I will summarize the findings of the present research. Based on these findings, I make some suggestions and recommendations for teachers and schools willing to integrate technology. Finally, I will present possible directions for further research.

5.2 Reviewing the Research Questions

This study was designed in order to better understand the way TPACK is represented in secondary school mathematics teachers. This section focuses on answering the main research questions of this study. These questions were:

1. How do secondary school mathematics teachers describe their ways of integrating technology?
2. What difficulties do teachers have when they try to integrate technology into mathematics classrooms?

In the present section, I will detail the answers to these questions based on the findings reported from the case studies described in the fourth chapter.

5.2.1 Research Question 1: How do secondary school mathematics teachers describe their ways of integrating technology?

In order to describe the process of integrating technology with the TPACK framework, I used the dimensions proposed by Niess (2005) and followed by Suharwoto (2006) in his doctoral research for preservice mathematics teachers. These four dimensions are:

- 1) An overarching conception of what it means to teach mathematics that integrates technology,
- 2) Knowledge of instructional strategies and representation for teaching mathematics with technology,
- 3) Knowledge of students' understanding, thinking, and learning with technology in mathematics, and
- 4) Knowledge of curriculum and curriculum materials that integrate technology in mathematics.

The following sections describe the analysis based on these four dimensions mentioned above.

5.2.1.1 Conception of Teaching Integrating Technology in Mathematics

It was mentioned that TPACK is defined as a place where mathematics content, pedagogy, and technology meet each other in various contexts and settings (Mishra & Koehler, 2006). Each of these teaching cases had intersected these domains in various ways. All main components (mathematical content, pedagogy, and technology skills) were well developed and interacting with each other in personalized ways, based on individual contexts, and teachers' previous experiences with computer technology and interactions (Koehler & Mishra, 2008).

The three teachers displayed high competence in teaching the mathematics curriculum. They knew not only the mathematics content required for the secondary school curriculum but also the students and the context of the school. Since each of them had been teaching for over 10 years in this school, they were familiar with the context of the school and with the community in which the school was situated. They were able to acknowledge the difficulties that might appear when their students started to learn these mathematical curricular units and were willing to use computer technology in order to let students have their own way of exploring mathematics. The teachers were aware of various trends and features that the computer software technology was able to offer. This made them able to look for different ways to adapt dynamically to the technological changes and to treat them as opportunities rather than perils (Koehler & Mishra, 2008).

These three components (mathematics content, pedagogy, and technology) were developed differently by each teacher. Cecilia's TPACK could arguably be described as being oriented more toward technology, Lawrence's as having strong pedagogy, and Mark's showing more of a proclivity toward mathematical content.

The teachers were aware of difficulties in purchasing computer equipment. They knew how to adapt their teaching practice to the limitations that a public school might have in purchasing adequate technological equipment. As the head of the Mathematics Department, Mark was able to predict emerging requirements and challenges, and was able to negotiate with the administration and teachers from other disciplines in order to obtain the computer equipment for the Mathematics Department.

As for competence in general technology (TK), the teachers were able to use different computer technologies in classrooms such as graphic calculators, computers, SMART Board, video projectors, Internet protocols and services, and manipulatives. Although very differently used, these technological resources for teaching were adapted efficiently (NCTM, 2000), according to the needs of the teachers and the students.

The teachers showed strong knowledge using technology for pedagogical purposes (TPK). For instance, the usefulness of SMART Boards and video projectors in classrooms has been mentioned before (Glovera, Millera, Averisa & Doora, 2007; Smith, Higgins, Wall & Miller 2005; Ward, 2003) and are good examples to illustrate the TPK component. These case studies showed the possibility of using these devices in a purposeful way. While these devices were used traditionally to help teachers communicate their ideas to students, they also offer additional ways of using in classrooms in a more equitable way. For instance, in Data Management classes, SMART Boards were used by students to help them present their projects to their colleagues and the teacher.

The usefulness of other devices for classroom assessment was also noticed. For instance, Clickers and Gizmos were used for assessments and significantly reduced the

effort and the amount of time dedicated for assessments, confirming the findings of Caldwell (2007) and Lantz (2010). I noticed that these two assessment tools had different requirements and purposes. For Clickers, the assessments might be designed directly by the classroom teacher, while for Gizmos, programmers design the tests. Therefore, the content of Gizmos is not directly available to the teacher. On the other hand Gizmos had its advantages. For instance, Gizmos were used in this study not only to assess students' knowledge but also to tutor them on specific content. Sometimes, this content could be selected not from the current unit but from the elementary mathematics curriculum, content that the teacher felt needed to be repeated. Therefore, I would classify Clickers as being part of the TPK while Gizmos were part of the TPACK.

An important part was the software designed for the mathematics curriculum. The teachers have experience using software that supported their curriculum (TCK) such as Geometer's Sketchpad, Graphmatica, Fathom, Excel, and graphic calculators. Each teacher used these software packages differently, based on the context and requirements of each classroom.

5.2.1.2 Knowledge of Instructional Strategies for Teaching Mathematics with Technology

The process of integrating different instructional strategies and concepts for teaching was not spontaneous. The teachers started integrating computer technology in their mathematics classrooms several years ago and were committed to continuing the process of integration in the future by showing a clear plan of development. They have a long term perspective in updating their pedagogical skills and technologies. These inferences were confirmed from interviews and observations in classrooms.

As Sfard (2008) and Healy and Hoyles (1999) found in their research, this study showed that using software helped students to connect to different dimensions of mathematical concepts. Also, as Kaput, Noss and Hoyles (2008) recommend in their study, computer technology improved support for thinking, simulation and visualisation for students and teachers.

Computer technology was found to be a helpful tool for teaching in this study. Also, the teachers actively used multiple technologies to teach. For instance, Lawrence used a video projector connected with a graphic calculator. In other cases, he used the video projector connected with a personal computer to give short PowerPoint presentations. Mark used SMART Board with Fathom, Excel, Geometer's Sketchpad, and software for geometry programs to show and explain things to students. Computer technology also offered the possibility to assess student performance. In particular, Mark and Cecilia worked with Smart Technology tools to verify assessments, while Lawrence used Gizmos with his Grade 9 Applied students. Technology can therefore be employed to various degrees in many areas of secondary mathematics curriculum.

Technology was a tool to help students understand and visualize concepts in mathematics. Students understood the mathematics and the use of technology in mathematics differently, accordingly to their previous experience and skills (Kaput, Noss & Hoyles, 2008). For instance, in the Grade 9 Applied course, the graphic calculators were provided to students to investigate the visual side of the problems. In addition, Gizmos were used to practice current units and units from the elementary curriculum. In the Data Management course, different software such as Fathom, Excel, and TI-Nspire were used in order to collect large amounts of data and analyze them. For the Advanced

Functions course, graphic calculators, Fathom, and Geometer's Sketchpad were used to help students draw various types of functions: polynomial, trigonometric, exponential, or logarithmic. As a newly designed course, the Calculus and Vectors course was challenging for Mark, as he was trying to select various software packages such as Geometer's Sketchpad, Fathom, GraphCalc, or Graphmatica.

The teachers often used group work in the classroom to help students better understand mathematics concepts. Each class had at least one project that required teamwork and the use of technology was helpful in this process. However, only in the Data Management course did students have the opportunity to work on complex projects. In these cases, computer technology helped students to work in groups. All three teachers interacted with others in order to offer and provide support and to improve their pedagogical and technological activities.

Teachers used technology for various purposes in their teaching and for assessments. Mark and Cecilia used SMART Boards while Lawrence used a similar projector to show on a big scale what graphic calculators might display. In their presentations, Lawrence and Cecilia used PowerPoint slides. Mark and Cecilia often used clickers, which drastically decreased the time and effort required to check the results.

The majority of computer technology offered visual aids to students and teachers. For instance, Lawrence mentioned that graphic calculators helped students to visualize problems. Mark also recognized the role of technology as an important tool to provide students with visual aids. The technologies used tended to be more portable. For instance, the graphic calculators and the clickers were easy to bring into the normal classroom. In addition, the next personal computers that would be purchased were supposed to be

laptops and could be brought into normal classrooms, instead of acquiring more desktops and allocating specific rooms as computer labs.

The school's administration supported the integration of technology, by investing money in computers and instruction, and by hiring new teachers with technology knowledge. They supported acquiring new computer technology for mathematics teachers for the purpose of improving numeracy and EQAO tests. Therefore, we might conclude that the teachers were not only individuals with a sound technological pedagogical content knowledge, but also the administrators supported them to implement computer technology in mathematics instruction.

5.2.1.3 Knowledge of Students' Understanding, Thinking, and Learning with Technology in Mathematics

Smart and Hoyles (2009) recommend adapting teaching and curriculum to each classroom's characteristics and context rather than teaching using a centralized approach. In our case the teachers were able to use technology to do that. This context-specific approach to education is more valuable because different types of students have different needs and therefore requires a tailored instruction.

Suharwoto (2006) mentioned that "student thinking has rarely been a focus of preservice teachers because of their focus on their own performance and teaching" (p. 133). This study had a different outcome. When the mathematics teachers had over ten years experience teaching in the same school, this made a great difference from the previous research on preservice teachers. Consequently, the use of technology was different. Mathematics teachers were aware of their students' academic background, their

potential, their commitment to learn, and their challenges these students might be exposed.

Teachers understood the socio-cultural context of this school, in particular the context of each classroom they were assigned to teach, and knew how to adapt computer technology to these contexts (Kelly, 2008). In particular, they were aware of their students and how to give them appropriate guidance for integrating technology in the learning of mathematics. Each teacher has knowledge about the students' background, academic performance, and possible difficulties (Kelly, 2008). Based on students' skills, the teacher has to devise his or her methods that were deemed to be successful. This was confirmed in our observations and interviews with teachers.

The teachers were able to recognize students' needs and adapted the technology in order to provide various interpretations and perspectives. For instance, Cecilia and Mark acknowledged that Fathom gave a better perspective than Microsoft Excel for their secondary school students. Cecilia knew that some of her classrooms had previous difficulties in Data Management and provided more opportunities for instruction. The teachers were able to provide customized instruction according to specific student needs. In this line, Lawrence had very different teaching approaches between the Grade 9 Applied and the Grade 12 Academic courses in Advanced Functions. This differs from Wenglinsky (1998), who notes that students from vocational courses were given less attention, fewer human resources, and less computer technology.

In this school, the number of students in an applied class had at least 10 fewer students than an academic class. Lawrence offered students access to computers, a video projector and graphic calculators, while in the Grade 12 academic course, he offered only

graphic calculators. The teacher mentioned that graphic calculators were sufficient for the grade 12 class. For Grade 9, he felt that Gizmos offered a great opportunity to tutor and test his students individually with different chapters from grade 9 or from previous years of study. Therefore, he took his Grade 9 students to the specialized computer lab, while the Grade 12 studied in a regular classroom.

The teachers used developmentally appropriate technologies and digital resources. They were able to communicate appropriately how computer technology might be used efficiently as part of their instructional strategies in the mathematics classroom. They practiced responsible use of technology and expected their students to be responsible while using computer technology. For instance, Mark used computer tools to systematically evaluate students' deficiencies in the current unit. Lawrence used computer tools to remediate the mathematics skills deficit and gave his students opportunities to practice these skills. Cecilia helped students to create developmentally appropriate digital resources and encouraged them to use them according to their needs.

5.2.1.4 Knowledge of Curriculum and Curriculum Materials that Integrate Technology in Mathematics

As researchers such as Pea (1997) and Kaput et al. (2008) have mentioned, technology requires teachers to use new pedagogical strategies in their classrooms. The teachers who participated in this research were advanced users of computers, were confident with teaching technology, and took several courses in using computers in mathematics education. The process of integrating technology started a long time ago and well-evolved through several iterations in which they adapted and personalized the computer technology with their teaching style.

All teachers evaluated the accuracy and appropriateness of their curriculum requirements and made decisions about using specific technology. Cecilia acknowledged that her lessons offered different degrees of opportunity to use computer technology. Mark offered different styles of teaching in the Advanced Functions course and the Calculus and Vectors course. In the Advanced Functions course, students were more in control, while in the Calculus and Vectors course, the teacher was more in charge. Mark described a very different approach to teaching different lessons. In some lessons, he used several technologies, such as SMART Board, Geometer's Sketchpad and Fathom, while in other lessons he was interested in demonstrations and therefore he only wrote on the blackboard.

The teachers had a good knowledge of mathematics curriculum and knowledge of how to adapt their curriculum material for their students. For computer technology, they gave students the opportunity to experiment with their mathematical ideas and test their knowledge through computer technology.

The technology was used for communicating presentations and individual and collaborative writing. The teachers used technology for problem solving, self directed learning, and extended learning activities (projects). They were able to determine and evaluate whether the technology was useful and select specific tools and techniques appropriately. Mark was willing to explicitly evaluate the bias, the accuracy relevance and the comprehensiveness of computer tools. Cecilia was willing to contribute with digital resources (video, texts, and PowerPoint presentations) in order to build a class archive. Lawrence asked students to interpret the results and use their common sense observation or information from other disciplines. Teachers were able to use different

procedures for collecting data and integrate the obtained results with other forms of presentation.

5.2.2 Research Question 2: What difficulties do teachers have when they try to integrate technology into mathematics classrooms?

Although integrating computer technology gave them some major advantages in mathematics instruction, these attempts were not without challenges. Teachers had some problems with technology and with instructing students with computers, and these were easily noticeable. The challenges that appeared are classified in the following main categories: a) technological issues; b) teacher instructional difficulties; and c) student difficulties.

Both levels of difficulty mentioned by Ertmer et al. (1999) were found in this study. First, the teachers mentioned the difficulty in obtaining computer resources such as personal computers, software packages, and SMART Boards. At the second level, the teachers had difficulty in acquiring adequate instruction for technology. This was the case, for instance, of Web 2.0 technologies when teachers had access to them but did not feel prepared to use it in classrooms.

5.2.2.1 Technological Issues

Some limitations were due to challenges posed by the technology, either hardware or software problems. For instance, although the prices have dropped significantly for equipment and software, purchasing technology still remains challenging for public schools. In addition, the technology becomes outdated quickly (McCrary, 2006) so other financial efforts are required to purchase new equipment. Some students dropped the graphic calculators on the floor and this might reduce their functionality over time. Some

graphic calculators ran out of batteries and some had deteriorated. Once, the entire school ran out of electricity. The teachers used different versions of the software from that used in the examples in the textbooks. However, these were reasonable challenges and the teachers were able to fix or work with them.

A considerable problem was the lack of computer technology. If all the teachers were determined to integrate technology in their classrooms, the school could not afford to have all these technologies and use them simultaneously. Therefore, as noted in Maor's study (2003), the IT infrastructure represented a serious problem that teachers had to consider.

Some devices were outdated. For instance, monitors with cathode tubes were purchased approximately four years ago and these are now considered outdated. Also, most of the graphic calculators used in these classrooms were TI-85, models produced in the 1990s, and these devices were considered outdated in many respects. Therefore, the fast pace of advance in technology made many of the previously purchased devices now obsolete, as noticed previously by researchers such as Anderson (1992), Kaput (1992), and McCrory (2006).

An important number of challenges were generated by the software. The present software was not always able to help students. For instance, some mathematics units were not facilitated by the use of technology. Cecilia and Mark stated that some units did not offer many possibilities for using computer technology with the students. Each course had some areas where no actual computer technology could have any impact on student's learning.

5.2.2.2 Teacher Instructional Difficulties

Some of the teachers' challenges were caused by the change of software and hardware. When software or hardware used by teachers change, then teachers need time to update their skills (Galbraith, Goos, Renshaw, & Geiger, 2001). For instance, the present version of Geometer's Sketchpad is very different from the older versions on which they were trained. The new version is more complex and has some different features. Because the geometry curriculum has been reduced drastically, the importance of Geometer's Sketchpad has decreased. Another discrepancy was noticed in the Advanced Functions course, where many problems from the current textbook were solved with Geometer's Sketchpad, but Mark chose to solve these problems using Fathom software, which was not mentioned at all in the textbook. An adequate textbook for this course would make the Geometer's Sketchpad software less necessary and would require the inclusion of the Fathom software.

Another example is the use of graphic calculators. There are different graphic calculators in use. As a result, although some commands are common, new models and new features require different ways to use specific commands. For instance, the introduction of a new model, TI-Nspire, presented some challenges for teachers and students. The TI-Nspire graphic calculators have many more features than older models like the TI-84 and TI-85. This required a lot of time and effort on the part of teachers, especially when they had high expectations of themselves, as is in the case of Lawrence.

Some curriculum areas were not covered with current software products. This made teachers feel uneasy. For instance, in the Calculus and Vectors course, there was no software to cover the second half of the course. Therefore, Mark had to use several

software products for different lessons. This approach could not be followed by many teachers as becoming familiar with the content of this course was not covered by any workshop or seminar and was an individual effort.

5.2.2.3 Challenges in Helping Students

Students had specific challenges in the process of integrating technology. Although they were familiar in general with computer technology, the adaptation to specific mathematic tools should not be taken for granted (Ronau et al., 2008). For instance, in the Data Management Grade 12 course, the use of Excel, PowerPoint, and Word presented some challenges when students started to use them for the project. Sometimes, they had difficulties inserting mathematical formulae; sometimes they had difficulty integrating data in their project with previous texts, presentations, and game scenarios.

At times, I felt that teachers did not sufficiently explain to the students the significance of the mathematics outcomes that the computer devices offered. As Christou, Kyriazis, and Pitta-Pantazi (2006) found in their study, mathematics teachers occasionally skip some important explanations. On the other hand, being in a terminal year in a secondary school, this might be explained by teachers' reliance on their students to explicitly make demands and ask specific questions for clarification.

Students also faced challenges in the Advanced Functions course. Similar to what Calvert, Zack and Mura (2001) reported in their study, some of them had problems grasping the mathematical significance of using these tools and were not able to interpret the graphic of the functions that was displayed by graphic calculators. Other students became bored when, throughout their entire course, they used the old models of graphic

calculators. Even when these old graphic calculators could give them adequate support for learning in this course, they were bored with these tools. It was difficult for the students to discuss the possible bias and inaccuracies that these tools might have. For instance, some of them had difficulty in setting the adequate windows, table of values, or to calculate the limits in critical points.

The students from the Calculus and Vectors course had some specific challenges. Being the most advanced pre-university course, it was implied that this course would gather the most advanced students in mathematics. However, not having enough previous opportunities to be involved in geometry, this part of the course was difficult to grasp. In addition, they did not have software able to cover the whole curriculum. Because the content of the course was recently redesigned, Mark was reflecting on these changes and had little time to learn by himself and explore what software would be adequate for specific units.

The Grade 9 Applied students had numerous challenges. Not having previous positive experiences in learning mathematics, the use of computer technology in mathematics did not radically change their mathematical perspective and skills. On some occasions, the students from the Grade 9 Applied course were not able to use graphic calculators for assignments. Instead, they tried to avoid the use of graphic calculators and solve the problems on the paper only. When they were brought into the computer lab, they attempted to use technology for things unrelated to learning mathematics. For instance, some of them preferred to look for games, videos, or music and the teacher had a hard time trying to convince them to keep their focus on working on the mathematical

software. Some students were using the graphic calculators and computers carelessly. This triggered the teachers to ask them to be more responsible when they use technology.

As Calvert, Zack and Mura (2001) observed in their study, I noticed learners in some classes taking visual snapshots and attempting to reduce the effort required to understand mathematics by considering these artefacts as ready-made solutions that do not need additional effort and critical evaluation. In this way, the students were not able to build adequate pathways for understanding and proofs for their mathematical problems. My findings support Hoyles and Noss (2003), who cautioned in their study that students have a tendency to use technology in order to avoid the cognitive load that mathematics requires. As these researchers mentioned, teachers have to raise an important distinction between mathematics learners and mathematics users.

These challenges in teaching mathematics at the secondary level were different for different courses, students, and teachers. They represent important aspects that have to be considered when teachers attempt to integrate technology in mathematics classrooms.

5.2.2.4 Summary

The process of integrating technology in mathematics classrooms posed various types of challenges. Sometimes there were problems with the technology, sometimes the curriculum did not afford much support for learning by using software, and sometimes students or teachers themselves were challenged. Still, the teachers considered that the technology offers opportunities for supporting students' learning and technology is helpful for teaching and assessments. The three teachers displayed a strong understanding of challenges that might appear in the use of technology in mathematics instruction. In

addition, their experience in using technology, in teaching mathematics, and in integrating technology in mathematics gave them confidence and supported their pedagogical efforts to integrate computer technologies in mathematics classrooms.

This evidence was demonstrated in multiple forms: teaching activities, class assignments, interactions between teachers and students, and interviews and discussions about using technology in mathematics. The teachers mentioned that, despite these challenges, the role of technology was still engaging for their students and that computer technology is requested in their classrooms.

5.3 Major Findings and Implications of the Study

TPACK can be valuable when analyzing effective cases of integration of technology for various teachers, classrooms, curricula, and school contexts. This can be done by tracing the connection between mathematical content, pedagogy and technology for the purpose of exploring teachers' performances in integrating computer technology in mathematics classrooms. TPACK is not a "grand" theory but a framework. That is, this model provides a system with coherent concepts, methodologies, and ideas that gives adequate structure for mathematics teachers and educators to guide their efforts in integrating technology in their classroom.

The major contribution in this study was to explore effective integration of computer technology in mathematics education. This study shows three different ways of integrating technology in classroom. The teacher participants in this research have different personal, pedagogical and technical backgrounds. In order to teach their students mathematics, they used different technologies or use the same computer device

differently. Yet, they were able to show, in different ways, that their pedagogical approaches of integrating technology in mathematics classrooms remain successful.

This study makes three main contributions to the research in mathematics teaching:

1. To help teachers develop pedagogical skills and also a conceptual understanding in integrating technology in mathematics classrooms,
2. To learn more about TPACK and how it is designed as a theoretical and practical approach in assisting secondary school mathematics teachers better understand how to use technology tools in teaching mathematical concepts, and
3. To identify secondary school mathematics teachers' challenges with using educational technologies in classrooms, as they investigate pedagogical and technical issues and modalities designed to improve mathematical teaching and learning skills.

Education is not presently a system in which the integration of technology is placed at the highest level of creativity (Dede, 1999). Yet, the present educational system offers some islands of innovation in integrating technology and the TPACK might be described as one of these islands in the public school system where the creative integration of technology takes place. Cuban (2001) cautioned that even though teachers may introduce new technologies in their classroom, few of them actually change their pedagogical strategies in order to adapt to the technologically enriched environment.

My presence in their classrooms changed somehow their activities by putting them in the situation of inquiring about their methods of integrating technology into mathematics curriculum and probably forced them to use technology more often.

However, these issues are not challenging fundamentally the outcome of this research. To summarise, the major recommendations of this study are summarized as follows:

1. Personal skills and attitudes in technology, achieving computer expertise, and learning mathematical software are essential,
2. Teaching alone is not recommended: working in a team, assuring for technical support, and being supported by administration and other teachers is important,
3. Allow time to disseminate the knowledge, and
4. Choose deadlines to implement technology in classrooms.

In the following subsections, I will detail each finding.

5.3.1 Achieving Computer Expertise to Teach Mathematics Is Essential

Technology literacy starts with teachers. We cannot ask students to be technologically literate if their teachers are not. Consequently, mathematics teachers should be proficient in using technology and know the software that enhances mathematics curricula (NCTM, 2000). Therefore, it is their professional responsibility to learn to use different technologies such as SMART Board or Clickers. Teachers have to reflect on how these might be used in teaching and what curriculum areas they might cover. As Koehler and Mishra (2008) state, teachers should reflect on how these courses might be redesigned, what the benefits and new challenges are in each case, and decide what strategies are useful for their classroom and how they would be implemented.

These teachers had previous opportunities to use computers; therefore they have positive attitudes towards computer technology. Their beliefs about technology helped them to actively consider the process of integrating technology in the mathematics classroom. Being a teacher in science and computer science, Lawrence offered numerous

connections with these topics. He was able to provide numerous pedagogical representations with computers and manipulatives in order to provide different angles and aspects of the problem. He was a perfectionist, insisting on rehearsing a long time ahead and mastering all details before coming into the classroom. Cecilia had a background in mathematics and statistics and strong experience in computer technology. She was able to display high commitment to her students and excellent time management skills.

Mark had advanced knowledge of mathematics. His graduate studies in mathematics and engineering gave him a high level of expertise in mathematics, and also familiarity with mathematics software. He has strong skills in technology. In addition, he was able to connect these with the mathematical curriculum. Therefore, he became well-known for his expertise among his students and other teachers.

The teachers were able to make technology available as a support for mathematical investigation and to establish connections with pedagogy and content. The teachers allowed their students to direct their own learning by using the mathematical software. In addition, collaborating and receiving technical support from each other helped the teachers to cope with difficulties of integrating technology in classrooms. The technology was used for assessing student knowledge, a fact that tremendously reduced the time for feedback and the workload of teachers.

One of the important roles of computer technology noticed in this study was to help educators teach their mathematics curricula. As Ward (2003) mentioned in his study, SMART Boards and video projectors were useful to help teachers visualize different educational software. PowerPoint presentations gave teachers the opportunity to

summarize their pedagogical ideas to the students while SMART Boards could flexibly mix images from the previous presentations with new digital resources and handwriting.

The computer technology was used to reduce the time allocated for assessments. For instance, by using clickers, the teachers designed their own tests and the evaluation was done automatically. By using Gizmos, the students could select tutorial programs to initiate themselves in various topics and consider different assessments that are evaluated automatically. Some of these tutorials also offered advice to detect weaknesses or strengths that students achieved. Connected with video projectors or SMART Boards, these results could be shown easily to the entire classroom, so that all students could see the results.

5.3.2 Technology Requires Working in Team

In this study, each teacher collaborated with other teachers. This aspect is very important when teachers are using computer technology (Evoh, 2007). Teachers have additional responsibilities such as sharing technological resources with other mathematics teachers or with teachers from other disciplines. In order to have technical resources, teachers need to convince administration and other teachers about the necessity of investing funds for technology. After they purchase the equipment, teachers need to be sure that the technical support is provided. They need to discuss with other teachers different methods and strategies that might be useful for their classrooms.

Therefore, the process of integrating technology in curriculum, and in particular in mathematics, requires teachers to deepen their collaboration with students, other teachers, technical staff, and administrators. A fair use of these technological resources, a

share in knowledge about instruction, and a commitment to ensure technological maintenance are all important (Dalgarno & Colgan, 2007).

5.3.3 Time to Disseminate the Knowledge Required for Integrating Technology

Besides the collaborative work, a specific time for reflections and commitment for teachers was needed. The teachers started integrating computer technology in their mathematics classrooms earlier in their career. Gradually, the teachers shifted their status from general computer users to mathematics teachers able to use various computer technologies in their instructional activities.

It required a long period of work for teachers to adapt and periodically redesign their strategies and pedagogical approaches, in order to be proficient in using technology in the mathematics classroom. At least ten years ago, the teachers individually started learning specific mathematics software packages such as Geometer's Sketchpad. They needed time to critically reflect on what was suitable, in order to adopt or to reject a specific combination of technology and pedagogy for their classrooms. For instance, Lawrence tended to avoid SMART Boards, while Cecilia was not interested in graphic calculators. Although these devices were useful for the other (e.g. Lawrence heavily used graphic calculators and Cecilia used SMART Board intensively), not all devices were helpful for them. They taught several times by experiencing and teaching with different computer devices and software. This time was required to reiterate and readjust their pedagogical approaches in order to suit their pedagogical style and preferences with the technology available onsite. In addition, they learned many computer aspects by themselves.

5.3.4 Technology Change Classroom and Classroom Management

Computer technologies change the interactions and classroom management when they are heavily used. For instance, artefacts such as computers and graphic calculators change the instruction in class, and their use drastically changed the management of the classroom (Niess, 2005; Pea 2004). Students regularly consulted these devices to experiment with their ideas in order to acquire mathematical skills. These tools gave students the possibility to acquire new information and also change their interactions with the instructor. Therefore computer tools often became an intermediary between the instructor and students.

The role of teachers in instruction and class management changes when they use technology in classrooms. On the one hand, the amount of information delivered for instruction was reduced since the instructor did not have to provide any single piece of information to their students. This role was undertaken now by computers and graphic calculators. On the other hand, the teacher's role became more complex since the instructor is ultimately the one who takes care of students and all these devices (Artigue, 2000).

5.4 Suggestions for Teachers in Integrating Technology in Secondary Mathematics Classrooms

Why are so few teachers currently integrating technology in secondary school classrooms (Cuban, 2001), particularly in mathematics classrooms? This study did not explore the root causes that make technology difficult to be integrated in mathematics curriculum. Rather, this research was focused on successful practices of integrating

technology in mathematics, revealing both individual and institutional efforts that made these attempts possible.

As Mishra and Koehler (2006) stated, “merely knowing how to use technology is not the same as knowing how to teach with it” (p. 1033). Indeed, as this study found, understanding and implementing computer technology in mathematics curriculum requires commitment, collaboration, rigorous planning, and a long term perspective. Based on the results reported in this study, the following suggestions can be made for teachers:

- Look for opportunities to learn in PD activities such as workshops, meetings, conferences, and discussions with experts.
- Work as a team together with other teacher colleagues, administration and technical support staff.
- Learn about their students’ technological expertise and be aware of paths to achieving success in mathematics instruction.
- Build activities, reflections, and motivations to consider integrating innovative technologies as a continuum modality as requested by the present society to reshape their teaching.
- Consider the students’ level of knowledge, preferences in the role of cooperative and collaborative tasks, shifting the control to students and tolerance for noise and disorder.

In the following sections, I will detail these ideas.

5.4.1 Schedule PD Opportunities to Learn Technology

Preparation for teaching mathematics using computers should start from the preservice stage. By learning to use computer technologies in a classroom from this early stage, teachers will later have key knowledge to start their teaching. However, because technology evolves at a fast pace, teachers should be prepared to update their knowledge. Therefore, they should have opportunities to participate in PD workshops. At the same time, many aspects of learning technology remain unclear.

The focus should not remain on technology itself but on reshaping the way we approach the curriculum. As Earle (2002) writes:

Integrating technology is not about technology—it is primarily about content and effective instructional practices. Technology involves the tools with which we deliver content and implement practices in better ways. Its focus must be on curriculum and learning. Integration is defined not by the amount or type of technology used, but by how and why it is used. (p. 8)

Perhaps it might be argued that some software products such as Excel and PowerPoint are easy to use because teachers have been using them in other contexts before. Other software products such as SMART Boards or TI-Nspire Graphic Calculators might be new and require some training. For some specific software products for mathematics curriculum, such as Fathom or the Geometer's Sketchpad, an extensive period of training is required. These are cases when software requires an extensive period of learning, training, and assistance in teaching them in the classroom and therefore teachers should consider them attentively.

PD programs for mathematics teachers should provide more opportunities to help the teachers integrate technology in the classroom. Mark mentioned that the workshops and seminars only trigger an interest in a specific problem. Indeed this is the case for an

experienced math teacher who has already taught using computer technologies in the classroom. But the problem is different for a teacher who has not yet tried to use computer technology in their teaching.

Technical support is very important. For teachers who have strong expertise in computers, it was fine to have technologists from outside the school to support them. However, for teachers who were new to technology, this could be frightening. Therefore, for new teachers, maybe the technological support should be embedded in the school in order to encourage them to efficiently use technology in classrooms.

5.4.2 Create an Adequate Support Group

It is important to have teachers be skilful in mastering computer techniques and be able to show these to students. In this research, I found that it is important to have in a mathematics department a critical number of teachers who are interested in integrating technology in the classroom. Without fostering an adequate support group, teachers have various individual interests and would not be able to implement technology in classrooms. Teaching with technology requires interactions with colleagues and teamwork.

As Gopalakrishnan (2006) states, “Individuals from both educational and technical orientations can support teachers with technology integration as long as they are able to ‘translate’ between the two domains and work with users of varying technical abilities” (p. 54).

Therefore, it is very important to establish mentorship relationships with teachers who are already comfortable with computers in teaching mathematics. It is essential to establish collaboration between technical support staff and teachers in order to solve

various technological issues. The administration of the school should support efforts to integrate computer technologies in mathematics classrooms. Likewise, time should be allowed for designing and integrating technology in classrooms (McDougall, 1997). In addition, a more specific agenda for sharing ideas, skills, and computational resources should be considered.

5.4.3 Knowing about Their Students' Technological Expertise

Teachers should be aware of individualities that their students have. In particular, they should be aware of specificities that their individualities, classrooms, school and community offer. Planning mathematics lessons based on these particularities is crucial to success. Teachers should be aware of what worked in their teaching and what did not. Similar to Harris's (2008) description of experienced teachers linking the integration of technology with spontaneity, Lawrence mentioned:

I always read my students. When I teach a class, if I know that they do not know anything about what I am talking about, I stop teaching that... Because you got to be flexible, you got to be adaptable. You got to read the situation, you got to read the people. (interview, Nov. 6, 2009)

Therefore, learning about their students' technological experiences and motivations and consciously designing pathways of learning and achieving success is critical to mathematics instruction (Villegas & Lucas, 2007).

5.4.4 Courage and Motivation to Schedule Integrating Innovative Technologies as a Continuum Improvement

As the teachers mentioned, they are left alone most of the time to teach. This might explain why other teachers did not succeed in implementing technology in the

classroom. Therefore, in order to be able to integrate new technologies and use them efficiently in front of the class, teachers need to be helped to develop planning, collaboration, and determination. They should be able to reflect on strategies, representations, and visions that make purposeful use of a specific technology for their students.

A major problem is that, although the integration of technology in education has been recommended for almost three decades, in fact this process of integration is still not implemented on a large scale. The process of integrating technology has not penetrated every school as expected. This is due to different reasons. At the beginning, the main reason was the high costs required. Now, it seems to be that the main reason is that the integration of technology is still an option and therefore teachers can delay whenever they wish.

Unfortunately, as Mark and Cecilia mentioned, this process of integrating technology in curriculum still remains at the stage of recommendation. This lack of specific deadlines might trigger a lack of planning and clarification for the specific use of computer technology. This might be because of lack of clear guidelines for integrating technology in mathematics classroom from teachers, administrators, or the school board.

Teaching is an iterative process (Koehler & Mishra, 2008) and, as the models of Rogers (1996) describe, the technology takes different stages of integration. Therefore, I see the process of integrating technology as an important and sustained leadership effort that needs to be carefully planned. More precisely, teachers and administrators should consider a long term perspective in plans to implement technologies into classroom.

The technology is changing fast and therefore it will always remain a challenge implementing in purposeful way in mathematics education. As Grandgenett (2008) recommends, in order to provide adequate training for inservice and preservice teachers, the goals of the instruction should flexibly target teachers to help foster their skills and attitudes in using technology in thoughtful ways.

5.4.5 Changing Classroom Management and Patterns of Communication

As Mishra and Koehler (2007) note, “teachers construct curricula through an organic process of iterative design and refinement, negotiating among existing constraints, to create contingent conditions for learning” (p. 2222). This was also noticed in this study as using technology produced major changes in teaching. For instance, the mode of teaching and the assignment procedures were totally changed from a traditional classroom where the teacher talks and writes on the blackboard. In addition, the technology changed the way students interacted. As a result, the roles of collaborative and cooperative strategies were redesigned.

The technology changed the control and the management of classrooms. For instance, by making digital resources available to students, the students had access to them to learn without any effort from teachers. In their turn, the teachers interacted with students only at critical points when they needed specific advice or coordination. These strategies are developing over time. As Koehler and Mishra (2008) suggest, the preparation of teachers should be a spiral process, starting first with technologies that are simple and familiar to them. They might extend afterward with products of increasing difficulty. Therefore, aspects of and strategies for time management, interaction, and collaboration should be carefully considered in integrating technology.

5.5 Reflections on TPACK

In this section some theoretical aspects about the TPACK frameworks are discussed. More precisely, I will try to present strengths, challenges and weaknesses of the TPACK framework. By using the TPACK framework, this analysis showed ways in which some technologies might be used with specific pedagogical purposes in mind for mathematic classrooms. As Earle (2002) and Grandgenett (2008) mention, technology in classrooms should be focused on curriculum and not on technology per se. The teachers from this study had this approach. In the research, participant teachers integrating technology focused on the mathematics curriculum. With appropriate use of technology, the teachers were able to customize their instruction.

I used Niess's (2005) model of analysis in TPACK, which is similar to Grossman's approach to the PCK framework. The four dimensions of Niess's model, however, more clearly track the way technology is integrated in specific contexts. More exactly, this analysis analyzes the framework by using four important referential layers of integrating technology: general philosophy of integration, instructional strategies, students' learning, and curricular outcomes. This analysis reveals the many ways different computer tools are used.

Professional development strategies should be used in order to facilitate teachers' requirements to keep up with the latest technological trends. As this study shows, teachers learned new technologies by themselves as they appeared. Instead, PD courses should take into consideration the latest trends in order to help teachers adapt these technologies to their specific curriculum content and mathematics classrooms.

5.5.1 Reflections on the TPACK of the Mathematics Teachers Participants

In the following section, I present graphical representations of the TPACK of each of the three teachers and discuss their specific strengths and challenges. As I mentioned, this qualitative research is subjective and might overlook some aspects of pedagogical practices used by teachers in their efforts to integrate computer technology. With all of these, I found it useful to attempt an approximate representation of the graphical justification of the TPACK for each teacher, instead of avoiding any representation in the name of so-called objective representation. I would like to emphasize, however, that these are based solely on observations of the courses, technologies, and pedagogies used during the time of my research.

5.5.1.1 Lawrence's TPACK

I had difficulty in evaluating the level of integrating technology in mathematics classrooms developed by Lawrence. Although he is not using the latest trends of technology, his manner of integrating technology shows full maturity in designing pedagogical goals. I remember what one of Harris' students (2008) said about the integration of the technology, a quotation that might characterize the way Lawrence integrates technology:

A classroom that has successfully integrated technology into the curriculum would be one where you would not really notice it because it would be so second nature. The teacher would not have to think up ways to use whatever tools were available, but would seamlessly use them to enhance the learning of whatever content was being covered. Technology [would be] used to assist in acquiring content knowledge, and the acquisition of technology skills [would be] secondary. (p. 116)

His TPACK is intuitively approximated in the following figure:

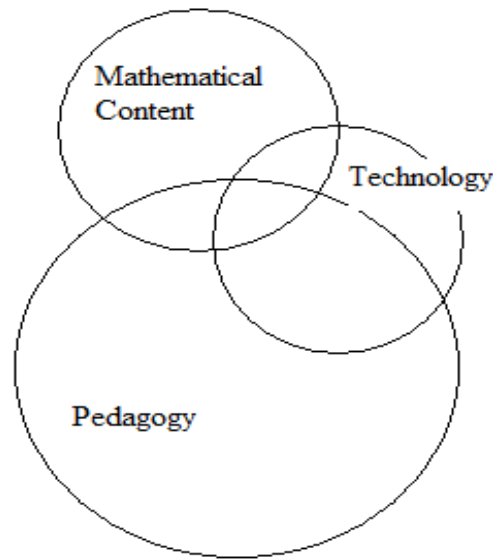


Figure 4: Lawrence's TPACK

Due to his experience in teaching, I felt Lawrence's main strength is pedagogy. While his mathematics studies and his experience in teaching computers gave him enough credit for the other two components of TPACK (Content Knowledge and Technological Knowledge), the Pedagogical Knowledge component remains by far the strongest asset for Lawrence.

5.5.1.2 Cecilia's TPACK

Cecilia showed a lot of initiative in using technology. For instance, she independently designed and maintained a website for the Data Management course, in which she posted resources and details about assignments for the current unit. Also, she coordinated the Casino project, which was enthusiastically received by the students. Given her previous practical and academic experiences in mathematics and computer technology, Cecilia has a strong computer technology background linked with mathematics. Therefore the stronger components of her TPACK were Content Knowledge and especially Technological Knowledge.

Her TPACK is represented intuitively in the following picture:

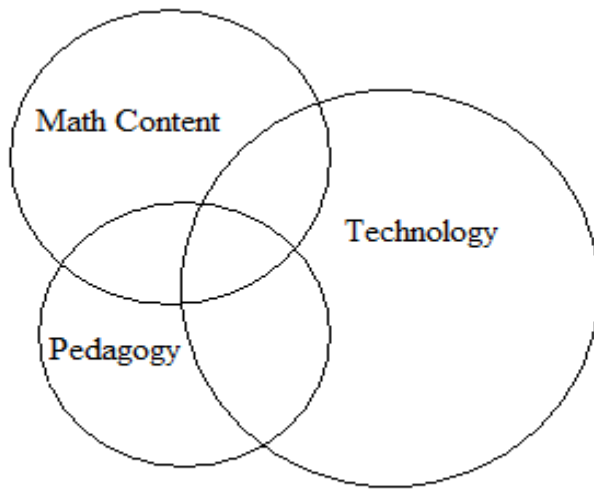


Figure 5: Cecilia's TPACK

As Grandgenett (2008) states:

This talented teacher's ability to imagine potential applications of computer technology and then 'weight' the relative benefits of these various opportunities within the context of a specific mathematical topic might be what TP[A]CK for mathematics teachers is all about in today's classroom. (p. 156)

5.5.1.3 Mark's TPACK

With his openness to look at new technologies in order to find new opportunities offered by technology. With all his extraordinary skills in integrating technology, the discussions with him emphasized more his concerns for the mathematics content.

Therefore, examining Mark's TPACK, I noticed that mathematics (Content Knowledge) is for him his first priority and is the most well developed component.

Mark knows various ways of using technology and proved to be an early adopter of innovations in integrating new computer technology in mathematics. According to Roger's model (1996), his high level of skill in integrating technology in mathematics classrooms puts him at the highest level (advancing stage – confirmation).

His representation in TPACK is represented below

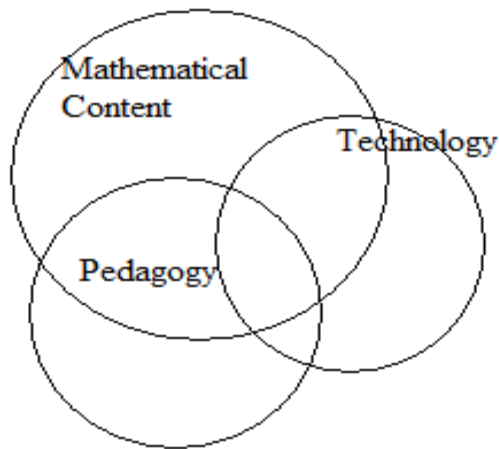


Figure 6: Mark's TPACK

Mark shows a strong TPACK background, or as Grandgenett (2008) would argue:

Mathematics teachers with strong background in TP[A]CK would probably have a relative openness to experimentation with the ever-evolving technological tools available to them in the mathematics classroom. In other words, they will 'try' new technologies-based lessons with their students on a regular and sometimes spontaneous basis, confident that if done thoughtfully and interactively, their students can learn something of value each time they attend something new. (p. 157)

5.5.2 Strengths of the TPACK Framework

Examining the TPACK of each of the three teachers, I noticed that they all had a critical approach to adapting different technologies for their mathematics classrooms.

While Kaput et al. (2007) caution about the passive attitude of many teachers who start integrating technology by uncritically accepting software that is unable to offer adequate educational outcomes, these teachers demonstrated that the contrary can also be true.

To me, the use of Technological Pedagogical Content Knowledge in teaching and research has some important strengths. First, this framework shows consistency in tracing the use of integrating technology in different contexts. By systematically being aware of the three main aspects (technology environment, mathematical content and pedagogy), any activity in the classroom can be analyzed and improved.

The context in which TPACK is studied is very important and, fortunately, I was able to use it for different classrooms and technological settings. In this study, the three mathematics teachers were in the same school but they had different classroom environments. For instance, in the Calculus and Vectors course, Mark had advanced students, while in the Data Management course the students were at the middle academic level and had previously taken mathematics courses in both academic and applied levels. Different from these classrooms, the student from the Grade 9 Applied class had serious difficulties in understanding mathematics. In addition, they missed important units from the mathematics elementary curriculum. Therefore, it is no surprise that the pedagogical approaches in these classrooms were different.

Even for the same teacher, the approaches for different classrooms were different. For instance, Lawrence's approach to teaching Grade 12 Advanced Functions was different from his approach to the Grade 9 Applied courses. When students from the applied classroom offered a rich socio-cultural context that produced challenging outcomes to the curriculum in place, this particular analysis of the classroom made me

aware of the importance of taking socio-cultural contexts more systematically into consideration. Therefore, different cultural and social issues should be considered within TPACK, as Kelly (2008) recommends. Overall, by following the advice from Kelly (2008) and Mishra and Koehler (2006), the use of the TPACK offers awareness in exploring various classrooms' contexts.

5.5.3 Challenges and Weaknesses of the TPACK Framework

While the literature on educational technology acknowledges numerous attempts to create a consistent framework for integrating technology in classrooms, at the time when this study was completed, this framework was not tested enough. Reviewing the literature, I noticed that the TPACK framework is still a relatively new area of research. This framework was proposed for the first time in 2005 by Mishra and Koehler, and few experimental studies specifically in mathematics education have been completed so far. Therefore, more studies are required in order to study the TPACK of mathematics teachers in different academic and sociocultural settings.

Another challenge coming from the design of the TPACK framework is that it did not offer the possibility to systematically take into account teachers' attitudes, opinions, philosophy, and paradigms of teaching. For instance, how does the TPACK show when a teacher is not using constructivist approaches? What is happening when teachers do not consider the social or cultural backgrounds of their students? Another problem that might occur is the lack of clarity in exploring the TPACK contexts. For instance, to me, it was challenging to find consistent ways of using the framework across different contexts.

The greatest unsolved problem in this study, however, is the lack of clarity in determining with accuracy the TPACK components of the teacher. Until this moment,

there is no procedure available to determine, with precision, the level of integrating technology that a teacher displays in his or her classroom. Therefore, using the four dimension framework might facilitate the finer analysis of integrating technology.

There is a lot of discussion about the opportunities that the TPACK framework offers. Is the TPACK framework worth putting all the research in integrating technology in this framework? Should we decide to design and use alternative frameworks? What could these be? Researchers already have different approaches to these questions. While more studies about the TPACK framework are in progress, some researchers have already decided to develop other frameworks (Angeli & Valanides, 2009).

5.6 Implications for Further Research

The present study suggests some consistent advantages in exploring the potential of the TPACK framework. For instance, additional questions are posed: Is this model of integrating computer technology in mathematics curriculum useful? How can the TPACK model help the integration of technology in mathematics classrooms, in particular for inservice secondary teachers? Were these cases of integrating technology in mathematics pathways that might be adapted and implemented by other teachers?

Few studies have been designed to trace the TPACK for inservice secondary mathematics teachers. Therefore exploring the TPACK model for secondary school mathematics teachers might provide some consistent paths of analysis and reveal useful practices and advantages that integrating technology gives to the secondary mathematics curriculum. This study shows cases of an effective use of technology in teaching, assessments, and giving students opportunities to explore mathematical ideas.

Other similar studies could be designed to explore the TPACK of expert elementary mathematics teachers. Changing the context for expert teachers for different places or academic profiles might be useful in exploring different ways that the expertise in integrating technology is shaped by mathematics teachers. Another possibility is to study expert mathematics teachers in integrating technology from both the elementary and secondary levels and eventually study the differences between these two types of expertise.

Another research opportunity is to focus on general characteristics and pedagogical resources of teaching with technologies from various disciplines. More specifically, this type of study would focus on the Technological Pedagogical Content (TPC) component, where there are already some new promising studies focused on exploring the way technology is integrated into common pedagogical practices and teaching skills. The research methodology might be changed to study more subjects and other methods might be justified, such as quantitative methods (Schmidt et al., 2009; Landry, 2010) or mixed methods.

Another possible direction to continue this research is to develop longitudinal research, studying metamorphosis of expertise in integrating technology from novice teachers to intermediate or expert teachers. Here, different methodologies might be possible, such as grounded theory (Glasser & Strauss, 1967), microgenesis theory (Grannot & Parziale, 2002) or skills theory (Fisher, 2000). These might be used in order to track teachers' transformations. This can be done either under a series of workshops, microteaching lessons, or under a multi-year longitudinal study. These studies would be

focused on revealing and documenting major teaching changes at specific stages, and their causes.

Specific perspectives and educational paradigms could be included in the research when teachers attempt to integrate computer technology in classrooms. Such examples might be constructivism, knowledge building, critical theory, activity theory or actor network theory. Also, different methodologies might be used in this study such as co-design research or action research.

It is useful to reflect that the TPACK framework was designed based on the older PCK framework (Shulman, 1986). Therefore, we might want to adapt previous pedagogical models and see how the structure of this model should be adapted in order to explicitly include the integration of technology. Therefore, I believe that comparing PCK with TPACK might inspire us to redesign some pre-existing pedagogical models. For instance, new frameworks could be redesigned such as Technological Pedagogical Practical Knowledge (TPPK), Technological Pedagogical Context Knowledge (TPCxK), or Technological Pedagogical Context Knowing (TPCKing) based on Pedagogical Practical Knowledge (PPK) (Clandinin, 1985), Pedagogical Context Knowledge (Barnett & Hodson, 2001), or Pedagogical Context Knowing (Cochran, DeRuiter & King, 1993). Of course, the creation of any new framework should be very carefully designed and empirically verified in various settings as a new framework always needs a consistent amount of theoretical and empirical study.

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Appendix A: Information and Consent Form to Participate in Research



DEPARTMENT OF CURRICULUM, TEACHING AND LEARNING

E-mail: ctl@oise.utoronto.ca • Telephone: (416) 923-6641 ext. 2601 • Fax: (416) 926-4744

Dear Sir/Madam,

I am a PhD student in the Curriculum Teaching and Learning department at the Ontario Institute of Studies in Education, University of Toronto, working under the supervision of Professor Douglas McDougall. I am conducting a doctoral dissertation study called “Technological Pedagogical Content Knowledge: Secondary School Mathematics Teachers’ Use of Technology” as part of my doctoral program requirements. I am requesting your cooperation as a voluntary participant in this study, which I hope is going to help generate a more in-depth understanding of the growing and changing computer integration in mathematics classrooms in Ontario secondary schools.

Reading the Ontario Secondary school curriculum and the professional development program in mathematics for secondary school teachers, I have noticed how the curricula encourage and ask for integrating computer technology in mathematics education. I am interested to explore modalities to foster the integration of technology in secondary math classrooms. It is my belief that the best way to gain insights into these areas of inquiry is to gather data by interviewing teachers’ who have completed the program of studies.

For this reason, I am inviting you to assist me by agreeing to participate in the study, which will be two interviews of approximately 50 minutes each. At the beginning point, I will be requesting, at a time convenient for you, to conduct a one-on-one interview that will be audio-recorded. The interview will focus on teaching experiences in integrating technology in classrooms. During two units, I will assist you in classrooms for 10 hours, and take notes. After the units will be taught, you will participate at the final individual interview to debrief the challenges in teaching and learning of these units, and your feedback to understanding integrating technology in teaching mathematics.

All data generated during this study will remain confidential. Neither your name, nor the name of your school will be used in the published doctoral thesis and the subsequent articles, and only Prof. Douglas McDougall and I will have access to the primary data. All data, including field notes and audio records, will be digital encrypted, and will be destroyed after five years when the study is concluded. You are free to refuse to answer or skip at any question, if this seems uncomfortable for you. Also, you are free to raise questions or concerns throughout the study, and may withdraw at any time if you choose. Please be assured that you are under no obligation to agree to be observed or to participate in an interview. There are no known risks to you for assisting in the project.

The findings of this study will not only benefit you directly, but by participating in this study, you will be contributing to the production of new knowledge about the effectiveness of the technology in mathematics education, which is becoming increasingly important to teaching and learning mathematics in the Ontario secondary schools. If you request, you will receive a copy of the summary of findings from the study and will have the opportunity to have the digital version of the entire thesis once it is finished.

For questions about your rights as a research participant contact the Office of the Research Ethics from the University of Toronto, at 12 Queen's Park Cres. West, McMurrich Building, 3rd Floor, Toronto, ON M5S 1S8, phone 416-946-3273, fax 416-946-5763 or email: ethicsreview@utoronto.ca

Thank you in advance for assistance and cooperation.

Sincerely,

Dorian Stoilescu

Researcher Signature

Date

Participant Signature

Date

CONTACT INFORMATION:	
<i>Principal Investigator:</i> Dorian Stoilescu Professor Dept. of Curriculum, Teaching & Learning Ontario Institute for Studies in Education 252 Bloor Str. W., Toronto, ON, M5S1V6 Room 11-239A Tel: E-mail: dstoilescu@yahoo.ca	<i>Supervisor:</i> Dr. Douglas McDougall, Assoc. Dept. of Curriculum, Teaching & Learning Ontario Institute for Studies in Education Room 11-250 Tel: E-mail: doug.mcdougall@utoronto.ca

Appendix B: Information and Invitation Letter to School Principals



DEPARTMENT OF CURRICULUM, TEACHING AND LEARNING

E-mail: ctl@oise.utoronto.ca • Telephone: (416) 923-6641 ext. 2601 • Fax: (416) 926-4744

Dear _____,

I am a PhD student in the Curriculum Teaching and Learning department at the Ontario Institute of Studies in Education, University of Toronto, working under the supervision of Professor Douglas McDougall. In my doctoral dissertation called “Technological Pedagogical Content Knowledge: Secondary School Mathematics Teachers’ Use of Technology”, I am investigating techniques, practices and activities that contribute to successful integration of computer technology in mathematics education. The purpose of this research is to contribute to the knowledge base regarding integrating technology in mathematics and the use of the Technological Pedagogical Theoretical Framework (TPACK) framework to guide the improvement in mathematics instruction in Grade 9 Applied mathematics course.

The External Research Review Committee of the school board and the University of Toronto Ethics Office has approved this study. I would like to ask your permission to let three of your mathematics teachers to participate in this project by allowing me to conduct an interview first with her/him. The interview will take about 50 minutes and it will be audio-recorded. I will conduct the interview during the school day and in your

school. Also, the teacher will be observed for approximative 10 hours, during two curricular units. At the end, the teacher will have another interview for around 50 minutes. You and your teacher will also be given an opportunity to receive a summary of the report. I will also give to your teacher, at the end of research, feedback about the improvement of integrating computer technology in mathematics.

I will not use any name or anything else that might identify the school, the students or the teachers in the written work, oral presentations, or publications. The information remains confidential. Your teacher is free to withdraw at any time, even after she/he has consented to participate. She/he may decline to answer at any specific questions. I will destroy the digital recording after the research has been presented and/or published which may take up to five years after the data has been collected. There are no known risks to you for assisting in the project.

Please sign the attached form, if you agree with this research. The second copy is for your records. Thank you very much for your help.

Yours sincerely,

Dorian Stoilescu

PhD Candidate

OISE/University of Toronto

Email: dstoilescu@yahoo.ca

Appendix C: Parent Letter for Information



DEPARTMENT OF CURRICULUM, TEACHING AND LEARNING

E-mail: ctl@oise.utoronto.ca • Telephone: (416) 923-6641 ext. 2601 • Fax: (416) 926-4744

Dear Parent/Guardian,

I am a PhD student in the Curriculum Teaching and Learning department at the Ontario Institute of Studies in Education, University of Toronto, working under the supervision of Professor Douglas McDougall. I am conducting a doctoral dissertation study called “Technological Pedagogical Content Knowledge: Secondary School Mathematics Teachers’ Use of Technology” as part of my doctoral program requirements. Reading the Ontario Secondary school curriculum and the professional development program in mathematics for secondary school teachers, I have noticed how the curricula encourage and ask for integrating computer technology in mathematics education. I am interested to explore modalities to foster the integration of technology in secondary mathematics classrooms.

Three mathematics teachers have been identified. The University of Toronto (UoT) and the school board approved my research proposal to invite the three mathematics teachers to participate in my doctoral dissertation using the Technological Pedagogical Content Knowledge (TPACK) model to integrate technology into their classes. My research is focused on fostering the use of technology in teaching in mathematics. I will interview the teachers only and will observe them in classrooms. I will not interview any student and I will not interfere with classrooms activities.

I received also the criminal verification clearance and have a valid Police Background Check documentation. Although written parental consent is not required for general whole group classroom observations, the TDSB requested this Parent Letter for information purposes only. This letter gives you an overview of my study, explains the purpose of my visits and observations in your child’s classroom, and provides contact information if there are any questions.

Sincerely,

Dorian Stoilescu
PhD Candidate
Department of Curriculum, Teaching and Learning
Ontario Institute for Studies in Education
University of Toronto
Tel:
Email: dstoilescu@yahoo.ca

Appendix D: Initial Interview about Technology and Implications for Mathematics

Teachers

Instructions

Please answer to each of the following questions. Include relevant examples whenever possible. There are no “right” or “wrong” answers to the following questions. I am only interested in your opinion on a number of issues about mathematics and technology.

These questions aim to elicit your views concerning mathematics education as it is practiced within your own teaching classroom. Please consider this authentic context in your responses.

Questionnaire

Initial/Professional information

1. Tell me about your background:
 - a. What is your major in teaching?
 - b. How many years have you been teaching?
 - c. What is your experience in teaching mathematics?
 - d. Why did you decided to teach mathematics?
 - e. What are your skills in computer technology?
 - f. How do you see the use of computers in mathematics education?

2. **Mathematics Education**

About students:

 - a. What are the skills your students need in mathematics in your classrooms?
 - b. What are students' mathematical expectations from you?
 - c. About the curriculum:
 - d. How do you feel about the Ontario curriculum for Grade 9 applied math?
 - e. What does the curriculum emphasize?

3. About school/community support:
 - a. How do you feel the collegial support in mathematics education?
 - b. How do you feel the administration support in mathematics education?
 - c. How do you feel the parental/community support in mathematics education?

4. **Integrating Technology in Mathematics Education**
5. About integrating technology in mathematics education:
 - a. Do you think that technology is important to improve math education?
 - b. What does the technology emphasize in mathematics education? What things does not discuss?
 - c. When you teach mathematics do you feel technology separated from practical and social considerations? How do you think students would feel this?

6. About students:
 - a. What students' skills do your students have in computer technologies?

- b. What are students' technological expectations from you? What are students' technological expectations about using technology in mathematics education?
 - c. What do you expect from students as skills/knowledge when using technology?
 - d. How do you help students to construct their own knowledge about mathematics by using technology?
7. About school/community support:
- a. How do you feel the collegial support in implementing technology in mathematics classrooms?
 - b. How do you feel the administration support in implementing technology in mathematics classrooms?
 - c. How do you feel the parental/community support in implementing technology in mathematics classrooms?

Appendix E: Final Interview about Technology and Implications for Mathematics Teachers

Name: _____

Date: _____

Instructions

Please answer each of the following questions. Include relevant examples whenever possible. There are no “right” or “wrong” answers to the following questions. I am only interested in your opinion on a number of issues about mathematics and technology. These questions aim to elicit your views concerning mathematics education as it is practiced within your own teaching classroom. Please consider this authentic context in your responses.

Questionnaire

Discussing whether initial expectations in math education were accomplished

We will discuss about your initial expectations in teaching these curricular units in mathematics, what went different, and what would you change for each unit:

1. What did you experience when you taught these units in classroom?
2. How were the students’ skills in mathematics initially? How did they change during teaching the unit?
3. What mathematical goals were achieved? What mathematical goals still need more work in order to achieve them?

Integrating Technology in Mathematics Education

For each unit we will discuss what did you expect that technology in place would help, what helped (or not) teaching mathematics and what would you change for each unit:

4. About students:
 - What computer techniques did students practice in this unit?
 - What was different in students’ technological expectations from you?
 - What did you expect from students as skills/knowledge when using technology? How was in the final?
 - How did you feel students should be helped to construct their own knowledge about mathematics?
 - What it would be different in your teaching if you would teach this unit again?
5. About teaching:
 - How did the technology in place helped in teaching mathematics education?
 - What did not technology cover?
 - When teaching did you feel technology linked with practical and social considerations? How did students feel this?
 - How technology could be used differently to improve teaching and learning place? What technological changes would you suggest?